Output Feedback Adaptive Decentralized Control of Cooperative Robots

Nasser Sadati and Ehsan Elhamifar

Electrical Engineering Department, Sharif University of Technology, Tehran, Iran
sadati@sina.sharif.edu , elhamifar@ee.sharif.edu

Abstract—In this paper a decentralized control scheme for multiple cooperative manipulators is developed to achieve the desired performance in motion and force tracking, in the presence of uncertainties in dynamic equations of the robots. To eliminate the effects of uncertainties in the closed-loop performance, a new adaptive control algorithm is proposed. Based on the Lyapunov stability method, it is proved that all the error signals in the closed-loop system which is composed of a robot, an observer and a controller asymptotically converge to zero. Also to avoid the difficulties of using velocity sensors within the cooperative system, an output feedback control scheme with a linear observer is used.

I. INTRODUCTION

The problem of controlling cooperative robots has attracted the attention of many researchers during the last decade [1]-[7]. This is mostly because of the development of industries that utilize robots for complicated tasks such as space and underwater robotic applications, handling and transforming objects and assembly applications in electronic industries.

There are some differences between the control of a single robot and the cooperative ones that cause conventional centralized algorithms can not be efficient from practical point of view because of the system complexity and time consuming calculations. So we seek some efficient methods such as decentralized control algorithms [2]-[5], [8] that can reduce these problems.

For control of robot manipulators it is necessary to use the information of sensors mounted on the robots. For position control, the encoders which have an acceptable range of accuracy are used. But for the velocity measurement, tachometers are used that are not very accurate because of the presence of noise in the measured signal which is amplified by increasing the control feedback gains. Also using velocity sensors increases the weight of each link that is undesirable from practical points of view. So we seek efficient methods like utilizing velocity observers that guarantee the velocity estimation without error by using only the position information [10], [11].

Most of the works in the control of cooperative robots that have used the output feedback method are based on the exact knowledge of the dynamic equations of the robots [3], [5]. On the other hand, decentralized control algorithms that have considered uncertainties in the dynamic equations of robots have proposed control inputs that are based on the

knowledge of the velocity and sometimes the acceleration terms of the robots [1], [2].

In this paper an adaptive decentralized control algorithm for cooperative manipulators without velocity measurement and by the assumption of uncertainties in the dynamic equations of the robots is proposed. For this purpose, a control input with an adaptive term in conjunction with a linear observer in the controller structure is used to estimate the values of velocities. Finally, based on the Lyapunov stability method, the stability of the closed-loop system with the proposed control input is investigated.

The structure of this paper is as follows. In section II, the dynamic equations and some properties of the robot structures are introduced. In section III by introducing observers to estimate the velocities of robots, a control input for each robot using its local information is proposed. In section IV the Lyapunov based stability theorem is investigated. In section V the performance of the proposed algorithm by simulation of a cooperative robotic system is shown. Finally, the paper concludes in section VI.

II. PROBLEM STATEMENT

Consider a system of m cooperative robots holding a common object. The goal of the system is to 1) move the object center of mass along the desired trajectory and 2) exert the predefined force on the object at the contact points, [9].

The dynamics of robot i can be expressed as:

$$M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + D_i \dot{q}_i + G_i(q_i) - J_i^T \lambda_i = \tau_i \quad (1)$$

where q_i is the vector of generalized coordinates of robot i with dimension n_i , $M_i(\cdot)$ is the symmetric positive definite inertia matrix, $C_i(\cdot)$ is the Coriolis/centrifugal matrix and D_i is the positive semi-definite matrix relating to the joint viscous friction coefficients of robot i . Also $G_i(\cdot)$ represents the gravity vector, J_i is the Jacobian matrix, λ_i is the force exerted on the end-effector of robot i by the object and τ_i is the vector of torque exerted on the joints of robot i .

Assumption 1: The Jacobian matrix of robot i , J_i , is full rank; $\text{rank}(J_i) = O_i$, where O_i is the number of constraints available at the contact point of robot i with the object. Also we assume that $n_i > O_i$.

Assumption 2: The cooperation model is holonomic. In other words, the constraints at the contact between the object and each robot are described by O_i dimensional equations

$$\begin{aligned}\varphi_i(q_i) &= 0 \\ J_i &= \nabla \varphi_i(q_i) \quad ; i=1,2,\dots,m\end{aligned}\quad (2)$$

Also the constraint velocity is defined as $\dot{p}_i = J_i(q_i)\dot{q}_i$. So according to (2), we have

$$\dot{p}_i = 0 \quad , \quad p_i = \int_0^t J_i(q_i)\dot{q}_i dv = 0 \quad (3)$$

where p_i is the constraint position for the i th robot.

For decoupling the position from force control, the joint space orthogonalization is used [3]. By defining

$$\begin{aligned}J_i^+ &= J_i^T (J_i J_i^T)^{-1} \\ Q_i(q_i) &= (I - J_i^+ J_i)\end{aligned}\quad (4)$$

we can write $\dot{q}_i$ as

$$\dot{q}_i = Q_i(q_i)\dot{q}_i + J_i^+(q_i)\dot{p}_i \quad (5)$$

Now, before going to the next section, some properties of the robot structures are described.

Property1: $M_i(q_i)$ has the bounded minimum and maximum eigenvalues $0 < \underline{\lambda}_i \leq \bar{\lambda}_i$.

Throughout this note we use $\underline{\lambda}(\cdot)$ for minimum eigenvalue and $\bar{\lambda}(\cdot)$ for maximum eigenvalue of $(\cdot)$.

Property2: Using the Christoffel symbols, the matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew-symmetric.

Property3: $C_i(q_i, a)b = C_i(q_i, b)a$, where a and b are n_i dimensional arbitrary vectors.

Property4: $\|C_i(q_i, a)\| \leq \bar{k}_i \|a\|$ where $0 < \bar{k}_i < \infty$ and a is the arbitrary vector of dimension n_i .

Property5: According to the property of linearity in the parameters we can write

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + D_i\dot{q}_i + G_i(q_i) = Y_i(q_i, \dot{q}_i, \ddot{q}_i)\theta_i \quad (6)$$

where Y_i is the regressor matrix and θ_i is the p_i dimensional vector of parameters for robot i .

III. DECENTRALIZED CONTROL ALGORITHM

First, it is necessary to define the following variables:

$$\Delta p_i = p_i - p_{di} \quad (7)$$

$$\Delta \lambda_i = \lambda_i - \lambda_{di} \quad (8)$$

$$\tilde{q}_i = q_i - q_{di} \quad (9)$$

$$z_i = q_i - \hat{q}_i \quad (10)$$

where p_{di} is the desired constraint position and must satisfy (3), λ_{di} is the desired force applied to the end-effector of robot i , q_{di} is the desired bounded trajectory of the joints

of robot i and must satisfy (2), $\tilde{q}_i$ is the tracking error and finally z_i is the observation error. Throughout this paper,

$\hat{(\cdot)}$ is used as the estimate of $(\cdot)$. Also it should be noted that $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)$.

The auxiliary variables can also be defined as:

$$\begin{aligned}\dot{q}_{ri} &= Q_i(q_i)(\dot{q}_{di} - \Lambda_i(\hat{q}_i - q_{di})) \\ &\quad + J_i^+(q_i)(\dot{p}_{di} - \beta_i \Delta p_i + \xi_i \Delta F_i)\end{aligned}\quad (11)$$

$$\begin{aligned}s_i &= \dot{q}_i - \dot{q}_{ri} = Q_i(q_i)(\tilde{\dot{q}}_i + \Lambda_i \tilde{q}_i - \Lambda_i z_i) \\ &\quad + J_i^+(q_i)(\Delta \dot{p}_i + \beta_i \Delta p_i - \xi_i \Delta F_i)\end{aligned}\quad (12)$$

$$\dot{q}_{oi} = \dot{\hat{q}}_i - \Lambda_i z_i \quad (13)$$

$$r_i = \dot{q}_i - \dot{q}_{oi} = \dot{z}_i + \Lambda_i z_i \quad (14)$$

where ξ_i and Λ_i are diagonal positive definite matrices with appropriate dimensions, and β_i is a positive constant. Also

$$\Delta F_i = \int_0^t \Delta \lambda_i(v) dv \quad (15)$$

Now, to continue the mathematical calculations, the derivative of (11) must be obtained

$$\begin{aligned}\ddot{q}_{ri} &= \dot{Q}_i(q_i, \dot{q}_i)(\dot{q}_{di} - \Lambda_i(\hat{q}_i - q_{di})) \\ &\quad + \dot{J}_i^+(q_i, \dot{q}_i)(\dot{p}_{di} - \beta_i \Delta p_i + \xi_i \Delta F_i) \\ &\quad + Q_i(q_i)(\ddot{q}_{di} - \Lambda_i(\dot{\hat{q}}_i - \dot{q}_{di})) \\ &\quad + J_i^+(q_i)(\ddot{p}_{di} - \beta_i(\dot{p}_{di} - \dot{p}_{di}) + \xi_i \Delta \lambda_i)\end{aligned}\quad (16)$$

According to the fact that $\dot{q}_i$ is not measurable, we can not obtain $\ddot{q}_{ri}$ from (16), so we have to use the approximation of it which is given by

$$\begin{aligned}\ddot{\hat{q}}_{ri} &= \dot{\hat{Q}}_i(q_i, \dot{\hat{q}}_i)(\dot{q}_{di} - \Lambda_i(\hat{q}_i - q_{di})) \\ &\quad + \dot{\hat{J}}_i^+(q_i, \dot{\hat{q}}_i)(\dot{p}_{di} - \beta_i \Delta p_i + \xi_i \Delta F_i) \\ &\quad + Q_i(q_i)(\ddot{q}_{di} - \Lambda_i(\dot{\hat{q}}_i - \dot{q}_{di})) \\ &\quad + J_i^+(q_i)(\ddot{p}_{di} - \beta_i(J_i(q_i)\dot{q}_{oi} - \dot{p}_{di}) + \xi_i \Delta \lambda_i)\end{aligned}\quad (17)$$

where $\hat{\hat{A}}(q_i, \dot{\hat{q}}_i)$ is defined as

$$\hat{\hat{A}}(q_i, \dot{\hat{q}}_i) = \begin{bmatrix} \frac{\partial a_{kj}}{\partial q_i} & \dot{q}_{oi} \end{bmatrix}$$

So the dynamics equation of robot i can be written as

$$\begin{aligned}M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i &= -M_i(q_i)\ddot{\hat{q}}_{ri} - C_i(q_i, \dot{q}_i)\dot{\hat{q}}_{ri} \\ &\quad - D_i(s_i + \dot{q}_{ri}) - G_i(q_i) + J_i^T \lambda_i + \tau_i\end{aligned}\quad (18)$$

Also equations (16) and (17) can be related by

$$\ddot{\hat{q}}_{ri} = \ddot{q}_{ri} + e_i(r_i) \quad (19)$$

$$\begin{aligned}e_i(r_i) &= -\ddot{\hat{Q}}_i(q_i, r_i)(\dot{q}_{di} - \Lambda_i(\hat{q}_i - q_{di})) \\ &\quad - \dot{\hat{J}}_i^+(q_i, r_i)(\dot{p}_{di} - \beta_i \Delta p_i + \xi_i \Delta F_i) \\ &\quad + \beta_i J_i^+(q_i)J_i(q_i)r_i\end{aligned}\quad (20)$$

where $\ddot{A}(q_i, r_i) = \dot{A}(q_i, \dot{q}_i) - \dot{A}(q_i, \dot{\hat{q}}_i)$.

Assumption 3: The inertia matrix of each robot is perfectly known, but the matrices $C_i(\cdot)$, D_i and $G_i(\cdot)$ are not exactly available.

Now, using (19) and the *Property 5*, we can write

$$M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i = -M_i(q_i)\ddot{\hat{q}}_{ri} - Y_i(v_i)\theta_i - C_i(q_i, \dot{q}_{ri})s_i + M_i(q_i)e_i(r_i) - D_i s_i + J_i^T \lambda_i + \tau_i \quad (21)$$

where $v_i = [\dot{q}_i^T, \dot{\hat{q}}_{ri}^T]^T$.

The input vector τ_i can also be chosen as

$$\tau_i = M_i(q_i)\ddot{\hat{q}}_{ri} + Y_i(\hat{v}_i)\hat{\theta}_i - K_i(\dot{q}_{oi} - \dot{q}_{ri}) - J_i^T(q_i)(\lambda_{di} - k_{fi} \Delta F_i) \quad (22)$$

where k_{fi} is a positive constant and K_i is a positive definite matrix. More over $\hat{v}_i = [\hat{q}_i^T, \hat{\dot{q}}_{ri}^T]^T$.

By using $\dot{q}_{oi} - \dot{q}_{ri} = s_i - r_i$ and (21), we have

$$M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i = M_i(q_i)e_i(r_i) - D_i s_i - C_i(q_i, \dot{q}_{ri})s_i + Y_i(\hat{v}_i)\hat{\theta}_i - Y_i(v_i)\theta_i - K_i(s_i - r_i) + J_i^T(q_i)(\Delta\lambda_i + k_{fi} \Delta F_i) \quad (23)$$

Now according to the *assumption 3*, we are ready to define the observer with the dynamic equations as follows

$$\dot{\hat{q}}_i = \hat{q}_{oi} + \Lambda_i z_i + \delta_i z_i \quad (24)$$

$$\ddot{\hat{q}}_{oi} = \ddot{\hat{q}}_{ri} + \delta_i \Lambda_i z_i + M_i^{-1}(q_i)J_i^T(q_i)(\Delta\lambda_i + k_{fi} \Delta F_i) \quad (25)$$

where δ_i is a positive constant. By substituting (19) in (25) and taking the first derivative of (24) we obtain

$$\dot{s}_i = \dot{r}_i + \delta_i r_i + e_i(r_i) + M_i^{-1}(q_i)J_i^T(q_i)(\Delta\lambda_i + k_{fi} \Delta F_i) \quad (26)$$

Now by multiplying $M_i(q_i)$ by (26) and using (23) and the *property 3*, after some manipulations we obtain

$$M_i(q_i)\dot{r}_i = -\delta_i M_i(q_i)r_i + K_i r_i - C_i(q_i, \dot{q}_i)r_i - C_i(q_i, s_i + 2\dot{q}_{ri})s_i + C_i(q_i, s_i + \dot{q}_{ri})r_i - D_i s_i - K_i s_i + Y_i(\hat{v}_i)\hat{\theta}_i - Y_i(v_i)\theta_i \quad (27)$$

Also by adding and subtracting the term $Y_i(\hat{v}_i)\theta_i$ to the right hand side of (23) and (27), we can get

$$M_i(q_i)\dot{s}_i = -C_i(q_i, q_i)s_i - C_i(q_i, q_{ri}) - D_i s_i + J_i^T(q_i)(\Delta\lambda_i + k_{fi} \Delta F_i) - K_i(s_i - r_i) + M_i(q_i)e_i(r_i) + Y_i(\hat{v}_i)\tilde{\theta}_i + Y_i(\tilde{v}_i)\theta_i \quad (28)$$

and

$$M_i(q_i)\dot{r}_i = -\delta_i M_i(q_i)r_i + K_i r_i - C_i(q_i, \dot{q}_i)r_i - C_i(q_i, s_i + 2\dot{q}_{ri})s_i + C_i(q_i, s_i + \dot{q}_{ri})r_i - D_i s_i - K_i s_i + Y_i(\hat{v}_i)\tilde{\theta}_i - Y_i(\tilde{v}_i)\theta_i \quad (29)$$

where $Y_i(\tilde{v}_i) = Y_i(\hat{v}_i) - Y_i(v_i)$.

Assumption 4: It is assumed that $\|Y_i(\tilde{v}_i)\theta_i\| \leq \varepsilon_i \|r_i\|$, which is a reasonable assumption according to the definition of $\tilde{v}_i$.

IV. STABILITY ANALYSIS

Define the states of the closed-loop system as follows

$$x_i = [s_i^T \quad r_i^T \quad \Delta F_i^T]^T \quad (30)$$

by the assumption that $\|x_i\|$ and q_{di} are bounded, we can conclude from (20) that the bounded constant $\bar{h}_i$ is available, that is

$$\begin{aligned} \|e_i(r_i)\| &\leq \bar{h}_i \|r_i\| < \infty \\ \|M_i(q_i)e_i(r_i)\| &\leq M_{ei} \|r_i\| < \infty \end{aligned} \quad (31)$$

where $M_{ei} = \bar{\lambda}(M_i)\bar{h}_i$.

Considering the candidate Lyapunov function of the system to be

$$\begin{aligned} V_i(x_i) &= V_{i1}(x_i) + V_{i2}(x_i) \\ V_{i1}(x_i) &= \frac{1}{2} x_i^T T_i(q_i) x_i \\ V_{i2}(x_i) &= \frac{1}{2} \text{tr}(\tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i) \end{aligned} \quad (32)$$

where $T_i(q_i) = \text{block-diag}\{M_i(q_i), M_i(q_i), \xi_i\}$ and Γ_i is a positive definite matrix, $V_{i1}(x_i)$ must satisfy the following inequality

$$\lambda_{i1} \|x_i\|^2 \leq V_{i1}(t) \leq \lambda_{i2} \|x_i\|^2 \quad (33)$$

$$\lambda_{i1} = \frac{1}{2} \min_{\forall q_i} \underline{\lambda}(T_i) \quad (34-a)$$

$$\lambda_{i2} = \frac{1}{2} \max_{\forall q_i} \bar{\lambda}(T_i) \quad (34-b)$$

Now we are ready to define the main theorem of this paper.

Theorem: Consider the cooperative robot system (1) with constraint (2) and the control input (22), with the observer (24)-(25). If we assume that q_{di} , p_{di} and their first and second derivatives are bounded, then there exist suitable controller gains that under the parameters updating law in (47), the states, $x_i(t)$ and the force tracking error, $\Delta\lambda_i(t)$ asymptotically converge to zero, as time increases.

In this way it is necessary that

$$\|C_i(q_i, \dot{q}_{ri})\|_{x_i \leq x_{i\max}} \leq \mu_{i1} \quad (35-a)$$

$$\|C_i(q_i, s_i + \dot{q}_{ri})\|_{x_i \leq x_{i\max}} \leq \mu_{i2} \quad (35-b)$$

$$\|C_i(q_i, s_i + 2\dot{q}_{ri})\|_{x_i \leq x_{i\max}} \leq \mu_{i3} \quad (35-c)$$

Also the controller gains must satisfy the following inequalities

$$\underline{\lambda}(K_i) \geq 1 + \mu_{1i} \quad (36)$$

$$\delta_i \geq \frac{\psi_{i1}^2 + 4\psi_{i2}}{4\lambda_{i1}} \quad (37)$$

where ψ_{i1} and ψ_{i2} are defined as

$$\psi_{i1} = M_{ei} + \mu_{i3} + \varepsilon_i + \bar{\lambda}(D_i)$$

$$\psi_{i2} = \mu_{i2} + \varepsilon_i + \bar{\lambda}(K_i)$$

Proof: Rewrite the Lyapunov function defined in (32) as

$$\begin{aligned} V_{i1}(t) = & \frac{1}{2} s_i^T M_i(q_i) s_i \\ & + \frac{1}{2} r_i^T M_i(q_i) r_i + \frac{1}{2} \Delta F_i^T \xi_i \Delta F_i \end{aligned} \quad (38)$$

By taking the derivative of the above equation, using (28)-(29) we get

$$\begin{aligned} \dot{V}_{i1} = & -k_{fi} \Delta F_i^T \xi_i \Delta F_i + s_i^T M_i(q_i) e_i(r_i) \\ & - s_i^T [C_i(q_i, \dot{q}_i) + D_i + K_i] s_i \\ & - r_i^T [\delta_i M_i(q_i) - C_i(q_i, s_i + q_{ri}) - K_i] r_i \\ & - s_i^T [C_i(q_i, s_i + 2q_{ri}) + D_i] r_i \\ & + (s_i + r_i)^T [Y_i(\hat{v}_i) \tilde{\theta}_i + Y_i(\tilde{v}_i) \theta_i] \end{aligned}$$

using the norm properties and *assumption 4* we obtain

$$\begin{aligned} \dot{V}_{i1} \leq & (\mu_{i1} - \underline{\lambda}(K_i)) \|s_i\|^2 \\ & + (-\delta_i \lambda_{i1} + \varepsilon_i + \mu_{i2} + \bar{\lambda}(K_i)) \|r_i\|^2 \\ & + (M_{ei} + \varepsilon_i + \mu_{i3} + \bar{\lambda}(D_i)) \|s_i\| \|r_i\| \\ & - k_{fi} \underline{\lambda}(\xi_i) \|\Delta F_i\|^2 + (s_i + r_i)^T Y_i(\hat{v}_i) \tilde{\theta}_i \end{aligned} \quad (39)$$

Also by taking the first derivative of V_{i2} , defined in (32) we have

$$\dot{V}_{i2} = \text{tr}(\hat{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i) \quad (40)$$

by consideration of the last term in the right hand side of (39) and according to (40) we can obtain the parameter update law as

$$\dot{\hat{\theta}}_i = \Gamma_i Y_i^T(\hat{v}_i) (s_i + r_i) \quad (41)$$

Note that in (41), the values of s_i and r_i , according to (12) and (14), are not measurable. We show later in the proof that we can modify (41) to be able to calculate $\hat{\theta}_i(t)$ values.

Now using (39)-(41) we can write for $\dot{V}_i = \dot{V}_{i1} + \dot{V}_{i2}$,

$$\begin{aligned} \dot{V}_i \leq & -k_{fi} \underline{\lambda}(\xi_i) \|\Delta F_i\|^2 \\ & - (\underline{\lambda}(K_i) - \mu_{i1} - 1) \|s_i\| \\ & - (\|s_i\|^2 - \psi_{i1} \|s_i\| \|r_i\| + (\delta_i \lambda_{i1} - \psi_{i2}) \|r_i\|^2) \end{aligned} \quad (42)$$

As it can be seen, by using (36) and (37) we have a negative semi-definite $\dot{V}_i$ in (42). So from the Lyapunov stability method we can conclude that

$$\begin{aligned} \lim_{t \rightarrow \infty} s_i(t) = 0, \quad \lim_{t \rightarrow \infty} r_i(t) = 0 \\ \lim_{t \rightarrow \infty} \Delta F_i(t) = 0, \quad \lim_{t \rightarrow \infty} \tilde{\theta}_i = 0 \end{aligned}$$

When s_i and r_i are small enough, from (28) we have

$$M_i(q_i) \dot{s}_i = Y_i(\tilde{v}_i) \theta_i + J_i^T \Delta \lambda_i$$

by multiplying both sides of the above equation by $J_i M_i(q_i)^{-1}$, we get

$$J_i \dot{s}_i = J_i M_i(q_i)^{-1} (Y_i \tilde{\theta}_i + J_i \Delta \lambda_i) \quad (43)$$

Using (12) and the property that $J_i Q_i = 0$, we also obtain

$$J_i \dot{s}_i + \dot{J}_i s_i = -\xi_i \Delta \lambda_i$$

By neglecting the second term of the left hand side of the above equation and using (43) we have

$$\Delta \lambda_i = \Delta_i Y_i(\tilde{v}_i) \tilde{\theta}_i \quad (44)$$

$$\Delta_i = -(\xi_i + J_i M_i(q_i)^{-1} J_i^T)^{-1} J_i M_i^{-1}(q_i)$$

Finally using *assumption 4* and (51), we can get

$$\|\Delta \lambda_i\| \leq \|\Delta_i\| \|Y_i(\tilde{v}_i) \theta_i\| \leq \varepsilon_i \|\Delta_i\| \|r_i\| \quad (45)$$

since $\|r_i\|$ asymptotically converges to zero, it is concluded from (45) that

$$\lim_{t \rightarrow \infty} \Delta \lambda_i = 0$$

Now, we obtain an update law that is applicable in real time.

Preposition: for the function $u(t)$ with the dynamic equation $\dot{u}(t) = x(t) y(t)$ we can obtain $u(t)$ from the following nested integrals

$$\begin{aligned} u(t) = & \int_0^t x(v) y(v) dv \\ = & x(t) \int_0^t y(v) dv - \int_0^t \left(\int_0^v y(\eta) d\eta \right) \dot{x}(v) dv \end{aligned} \quad (46)$$

According to (12) and (14), $\int_0^t (s_i(v) + r_i(v)) dv$ is measurable. So by using the above *preposition* we can change the parameters update law obtained in (41), to the following updating rule

$$\begin{aligned} \theta_i(t) = & \Gamma_i Y_i^T(\hat{v}_i) \int_0^t (s_i(v) + r_i(v)) dv \\ & - \int_0^t \left(\Gamma_i \dot{Y}_i(\hat{v}_i) \int_0^v (s_i(\eta) + r_i(\eta)) d\eta \right) dv \end{aligned} \quad (47)$$

Q.E.D.

V. SIMULATION RESULTS

To verify the validity of the proposed control algorithm we consider for numerical simulation, two three links planar robots cooperatively manipulating a common object. Here, the center of mass for each link is assumed to be located at the end of each link. The length of the first and second links; l_1 and l_2 , and the held object are equal to 1m. The length of the third link is sufficiently short and contained into the object.

Let q_{1j} and q_{2j} ; $j=1,2,3$, denote the joint angles of the two robots. The configuration of the held object is

denoted by the position coordinates x and y , and the orientation ϕ . Since all of the end-effectors are rigidly attached to the common object, we have $\alpha_1 = \beta_1$ and $\alpha_2 = \beta_2$, where the forward kinematics α_i and coordinate transformation β_i , from the object center of mass to the end-effectors of the i th robot are given by

$$\begin{aligned}\beta_1 &= \begin{bmatrix} x - 0.5 \cos(\phi) \\ y - 0.5 \sin(\phi) \end{bmatrix} \\ \alpha_1 &= \begin{bmatrix} \cos(q_{11}) + \cos(q_{11} + q_{12}) \\ \sin(q_{11}) + \sin(q_{11} + q_{12}) \end{bmatrix} \\ \beta_2 &= \begin{bmatrix} x + 0.5 \cos(\phi) \\ y + 0.5 \sin(\phi) \end{bmatrix} \\ \alpha_2 &= \begin{bmatrix} 2 + \cos(q_{21}) + \cos(q_{21} + q_{22}) \\ \sin(q_{21}) + \sin(q_{21} + q_{22}) \end{bmatrix}\end{aligned}$$

Thus the holonomic constraints ($O_i = 2$) are formed by

$$\varphi_i(q_i) = \beta_i - \alpha_i = 0, \quad i = 1, 2$$

The dynamic matrices for the object are also given by

$$\begin{aligned}M_o &= 0.7 \text{ diag}\{m_o, m_o, I_o\} \\ C_o &= 0 \\ g_o &= 0.7 \begin{bmatrix} 0 & m_o g & 0 \end{bmatrix}^T \\ m_o &= 1, \quad I_o = .25\end{aligned}$$

Also the dynamic matrices for each robot are as follows

$$M_i = 0.7 \begin{bmatrix} a_{i1} + 2a_{i2} \cos(q_{i2}) + a_{i3} & a_{i2} \cos(q_{i2}) + a_{i3} & a_{i4} \\ a_{i2} \cos(q_{i2}) + a_{i3} & a_{i3} & a_{i4} \\ a_{i4} & a_{i4} & a_{i4} \end{bmatrix}$$

Fig. 1. Object center of mass tracking .

$$\begin{aligned}C_i &= 0.7 \begin{bmatrix} -a_{i2} \sin(q_{i2}) \dot{q}_{i2} & -a_{i2} \sin(q_{i2})(\dot{q}_{i1} + \dot{q}_{i2}) & 0 \\ a_{i2} \sin(q_{i2}) \dot{q}_{i1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ g_i &= 0.7 \frac{g}{l} \begin{bmatrix} a_{i1} \cos(q_{i1}) + a_{i2} \cos(q_{i1} + q_{i2}) \\ a_{i2} \cos(q_{i1} + q_{i2}) \\ 0 \end{bmatrix}\end{aligned}$$

with

$$\begin{aligned}a_{i1} &= (m_{i1} + m_{i2} + m_{i3}) l_1^2 = 5 \\ a_{i2} &= (m_{i2} + m_{i3}) l_1 l_2 = 3 \\ a_{i3} &= (m_{i2} + m_{i3}) l_2^2 + I_{i3} = 3.05 \\ a_{i4} &= I_{i3} = 0.05\end{aligned}$$

where $m_{i1}, m_{i2}, m_{i3}, I_{i3}, m_o$ and I_o are system parameters.

The desired trajectories for the object center of mass and exerted forces are also given by

$$\begin{aligned}\begin{bmatrix} x_d \\ y_d \\ \phi_d \end{bmatrix} &= \begin{bmatrix} 1 + 0.25 \cos(\pi t / 2) \\ 1 + 0.5 \sin(\pi t / 2) \\ \pi / 6 \sin(\pi t / 2) \end{bmatrix} \\ \lambda_{d1} &= -\lambda_{d2} = 40 \begin{bmatrix} \cos(\phi) \\ \sin(\phi) \end{bmatrix}\end{aligned}$$

The simulation results illustrate the effectiveness of the proposed algorithm to obtain the desired objectives. As depicted in Fig. 1, the tracking error of the object center of mass tends to zero as the time increases. Also the result of the force tracking in Fig. 2 shows that the proposed method causes exerted forces tend to their desired values as time elapses. In Fig. 3, residual errors (s_i) of the robot 1 has been shown to tend to zero. Also it is obvious from Fig. 4 that the control inputs for robot 1 have a reasonable range of variations.

Fig. 2. Force tracking for robot 1.

Fig. 3. Residual errors for robot 1.

Fig. 4. Control inputs for robot 1.

VI. CONCLUSION

In this paper a decentralized position/force control scheme for cooperative robot manipulators is presented which can be effectively applied to every redundant cooperative system. It is assumed that the dynamic equations of each robot are not exactly known. It is shown how by using an adaptive control input, the effects of these uncertainties on the performance of each mechanical system have been eliminated. The control input for each robot uses only local information available through the local sensors, mounted on the robot. In addition, using a linear observer, velocity measurement of the joints is avoided. The proposed control input causes the overall closed-loop system to be stabilized which is proved by the Lyapunov stability method.

The simulation results show that the proposed approach works well in the presence of uncertainties in the dynamic equations of the robots.

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