A New Model For Induction Motor With Induced Saliencies

Edinson Franco IEEE Member, Martha Amaya and José Ramírez, IEEE Member.

Abstract—This paper develops a formal and general model for an induction motor with periodic variations of the rotor slot; this induction motor with induced saliencies is commonly used in sensorless control applications. In order to obtain the model, first the analysis of the stator and rotor inductances modified by induced saliencies is carried out; second, the electromagnetic and electromechanical equations in ABC frame are developed and third, the model is represented in the natural orthogonal a-b frames. The obtained model is suitable for control design and compatible with the classical ab model for induction motors without saliencies.

Index Terms - Induction motor, induced saliencies, sensorless control.

I. INTRODUCTION

In the past and present decade, the control for Induction Motors (IM) without mechanical shaft sensors has led to researchers and manufacturers to include sensorless vector drives.

The main problem in the sensorless control of induction motors is around zero speed or zero excitation frequency. In order to solve the problem, several works have been developed for modelling the phenomena in the stator and rotor slots; other ones more complex, include a detailed tensor model of the IM; however, these approaches are something complex for control.

The design of high performance sensorless drives of induction motors include an observer for the rotor/stator flux and one estimation function for the mechanical speed; some problems have been reported and can be summarized as: loss of observability around zero excitation frequency, incorrect flux/torque estimation induced by errors in the stator and rotor resistances, steady-state instability at low speed, particularly under regeneration. About solving the loss of observability at zero excitation frequency, one possible alternative solution is the Induction Motor with Induced Saliencies (IMIS).

The saliencies of the IM used for sensorless control have been classified as: slot harmonics, designed asymmetry (induced saliencies), saturation and dynamic eccentricity. The induced saliencies can be described as changes in the rotor slots geometry.

The sensorless control of the IMIS, for position tracking and speed control including both saliencies and input high frequency signals, has been reported by Holtz [1], [2], Degner[3] and Jansen [4] with promising results; Quan[5] has reported the high performance near to zero speed.

In this paper only changes in the rotor slots width are considered, then for obtain the model, the flux variations induced by the saliencies must be took into account; other important assumptions are: a) the variation of the rotor slots width can be represented for a periodical (with a pole pitch) and sinusoidal permeance; b) the squirrel-cage motor is approximated by an equivalent polyphase wound rotor with the same pole number and equivalents turns Nr and resistance Rr; c) the stator and rotor windings may be approximated as sinusoidal distributed windings; d) the stator and rotor steel have a high permeability; and e) the air gap is assumed uniform and the Carter factor is modified to approach the saliencies phenomena.
II. AIR GAP CORRECTION

In Fig. 1, the slot rotor pitch (td) and the air-gap (δ) are constants; in a) the rotor slot width b is constant and in b) the rotor slot width is position dependent b(x). Although the permeability of the gap region is constant, it is bounded on either side by iron surface which far from being smooth, is indented with slots in the circumferential direction, introducing variations in the air gap permeance[6]. It is possible to suppose that the actual slotted surface can be replaced by an equivalent unslotted surface having the same cross-section but with modified “equivalent” air gap δKc, where Kc the so called Carter factor, is the relation between the equivalent air gap permeance and the actual air gap permeance.

Fig. 2. IMIS stator and rotor scheme.

For the IMIS, the periodical variation of the rotor slot width, superpose a new space variation on the air gap flux density (Fig. 2). It is possible to consider a new correction factor for the sinusoidal modulation of the slot wide rotor, called “Corrected Carter Factor” and it is denoted here as Ks.

III. WINDING INDUCTANCES

A. Stator Inductances

The currents in balanced and steady state conditions are

\[
\begin{align*}
\mathcal{I}_A &= \sqrt{2} I_s \cos(w_s t + \phi_s(0)) \\
\mathcal{I}_B &= \sqrt{2} I_s \cos(w_s t - 2\pi/3 + \phi_s(0)) \\
\mathcal{I}_C &= \sqrt{2} I_s \cos(w_s t + 2\pi/3 + \phi_s(0))
\end{align*}
\]

Then the magnetomotive forces by the stator windings are respectively:

\[
\begin{align*}
\mathcal{F}_A &= i_A N_s \cos(\nu \phi_s) \\
\mathcal{F}_B &= i_B N_s \cos(\nu \phi_s - 2\pi/3) \\
\mathcal{F}_C &= i_C N_s \cos(\nu \phi_s + 2\pi/3)
\end{align*}
\]

Using the Amper’s Law and the new air gap factor, the phase air gap flux densities are:

\[
\begin{align*}
B_A(\nu \phi_s, \theta_r) &= \mu_0 \frac{\mathcal{F}_A}{\sigma_s(\nu \phi_s, \theta_r)} = \frac{\mu_0 N_s}{\sigma_s(\nu \phi_s, \theta_r)} i_A \cos(\nu \phi_s) \\
B_B(\nu \phi_s, \theta_r) &= \frac{\mu_0 N_s}{\sigma_s(\nu \phi_s, \theta_r)} i_B \cos(\nu \phi_s - 2\pi/3) \\
B_C(\nu \phi_s, \theta_r) &= \frac{\mu_0 N_s}{\sigma_s(\nu \phi_s, \theta_r)} i_C \cos(\nu \phi_s + 2\pi/3)
\end{align*}
\]

Remark: In absence of the rotor slot modulation, δ2 = α2/δ = 0, then δs = δKs, so, Ks = Kc and δs = δc.
The phase flux and flux linkages are:

\[ \psi_{sx}(\nu, \theta_r) = \int_{\phi_s}^{\phi_s+2\pi/2\nu} B_x(\nu, \theta_r) r d\xi \]

(6)

\[ \Psi_{sx}(\nu, \theta_r) = \nu \int N_x(\phi_s) \psi_{sx}(\nu, \theta_r) d\phi_s \]

(7)

where the sub index “x” is for the A, B or C windings; “i” and “r” are respectively, the axial length and the mean radius at the air gap of the machine, “ξ” is the integration variable and \( N_x(\phi_s) \) represents the sinusoidal distributed winding.

So, to the phase A,

\[ \Psi_{SA} = \nu \int N_A(\phi_s) \int_{\phi_s}^{\phi_s+\pi/\nu} B_A(\nu, \theta_r) r d\xi d\phi_s \]

(8)

The mutual inductances are calculated following the same procedure as the mutual inductance \( L_{AB}, L_{AC}, L_{BC} \) and \( L_{BA} = L_{AB}, L_{CA} = L_{AC}, L_{CB} = L_{BC} \).

Neglecting the phase leakage inductance \( L_{sx} \), the stator inductances are:

\[ L_{AA} = \frac{N_s^2}{2\pi} \pi \mu_0 r l \{ \delta_1 - \nu \cos(2\nu \theta_r) \} \]

\[ L_{BB} = \frac{N_s^2}{2\pi} \pi \mu_0 r l \{ \delta_1 - \nu \cos(2\nu \theta_r - \frac{2\pi}{3}) \} \]

\[ L_{CC} = \frac{N_s^2}{2\pi} \pi \mu_0 r l \{ \delta_1 - \nu \cos(2\nu \theta_r + \frac{2\pi}{3}) \} \]

(10)

The direct and quadrature axis components of the “a” phase rotoric current are:

\[ i_{ad} = i_a \cos(\nu \theta_r) \; ; \; i_{aq} = -i_a \sin(\nu \theta_r) \]

\( \theta_r \) is the angle between the “A” stator phase axis and “a” rotor phase axis.

The flux linkage, created by a direct \( i_{ad} \) and quadrature \( i_{aq} \) components are respectively:

\[ \psi_{ad} = i_{ad} L_{ad} \; ; \; \psi_{aq} = i_{aq} L_{aq} \]

Assuming non saturation, the total flux linkage for a rotor phase “a” is the addition of the flux linkages \( \psi_{ad} \) and \( \psi_{aq} \),

\[ \psi_{aa} = \psi_{ad} \cos(\nu \theta_r) - \psi_{aq} \sin(\nu \theta_r) \]

and,

\[ L_{aa} = \frac{\psi_{ad}(\nu \theta_r) - \psi_{aq}(\nu \theta_r)}{i_{ad} L_{ad} \cos(\nu \theta_r) - i_{aq} L_{aq} \sin(\nu \theta_r)} \]

\[ = \frac{i_a \cos(\nu \theta_r) + i_a \sin(\nu \theta_r) L_{aq} \sin(\nu \theta_r)}{L_{ad} \cos(\nu \theta_r) + L_{aq} \sin(\nu \theta_r)} \]

\[ = L_{ad} \cos^2(\nu \theta_r) + L_{aq} \sin^2(\nu \theta_r) \]

\[ = L_{ad} \cos^2(\nu \theta_r) + L_{aq} \sin^2(\nu \theta_r) \]

\[ = L_{rad} \cos^2(\nu \theta_r) \]

with

\[ L_{rad} = \frac{1}{2}(L_{ad} + L_{aq}) \; ; \; \Delta L_{rad} = \frac{1}{2}(L_{aq} - L_{ad}) \]

Similarly, to the other phases,

\[ L_{bb} = L_{rb} - \Delta L_{rb} \cos(2\nu \theta_r + 2\pi/3) \]

\[ L_{cc} = L_{rc} - \Delta L_{rc} \cos(2\nu \theta_r + 4\pi/3) \]
The mutual inductances \( L_{ab}, L_{ac}, \) and \( L_{bc} \) are calculated in the same way,

\[
L_{ba} = \frac{\psi_{ba}}{i_a} = -\frac{L_r}{2} - \Delta L_r \cos(2\nu \theta_r + 4\pi/3) \\
L_{ca} = \frac{\psi_{ca}}{i_a} = -\frac{L_r}{2} - \Delta L_r \cos(2\nu \theta_r + 2\pi/3) \\
L_{bc} = \frac{\psi_{bc}}{i_a} = -\frac{L_r}{2} - \Delta L_r \cos(2\nu \theta_r)
\]

and

\[
\mathbf{L}_{rr} = \begin{bmatrix}
L_{aa} & L_{ab} & L_{ac} \\
L_{ba} & L_{bb} & L_{bc} \\
L_{ca} & L_{cb} & L_{cc}
\end{bmatrix}
\]

C. Stator-Rotor Mutual Inductance

In phase “a” the magnetomotive force in the equivalent winding is:

\[
\mathcal{F}_a = N_s i_a \cos(\nu \phi_r)
\]

Considering the new factor \( K_s \) and the initial condition \( \phi_s(0) = 0 \), the phase “a” flux linkage is:

\[
\psi_a(\nu \phi_r, \theta_r) = \mu_0 \sum_{\delta, \xi, \eta} N_s i_a \cos(\nu \phi_r)
\]

The mutual flux between stator phase “A” and rotor phase “a” is then:

\[
\psi_{Aa}(\nu \phi_r, \theta_r) = \int_{\phi_s}^{\phi_s+2\pi/2\nu} \psi_a(\nu \phi_r, \theta_r) r dl \xi
\]

Assuming that the windings have a sinusoidal distribution, the total mutual flux linkage is,

\[
\psi_{Aa}(\nu \phi_r, \theta_r) = -\nu \int_{\phi_s}^{\phi_s+2\pi/\nu} \sum_{\delta, \xi, \eta} N_s i_a \cos(\nu \phi_r) r dl \xi d\phi_s
\]

\[
= \frac{N_s N_r}{\nu \mu_0} \mu_0 \pi a r l \left( \delta_1 + \frac{\delta_2}{2} \right) \cos(\nu \theta_r)
\]

and,

\[
L_{Aa} = \frac{\psi_{Aa}}{i_a} = L_{sr} \cos(\nu \theta_r)
\]

where,

\[
L_{sr} = \frac{N_s N_r}{\nu \mu_0 \pi a r l} \left( \delta_1 + \frac{\delta_2}{2} \right)
\]

The other mutual inductances are obtained following the same procedure for calculating \( L_{Aa} \).

\[
\mathbf{L}_{sr} = \begin{bmatrix}
\cos(\alpha) & \cos(\alpha + 2\pi/3) & \cos(\alpha - 2\pi/3) \\
\cos(\alpha - 2\pi/3) & \cos(\alpha) & \cos(\alpha + 2\pi/3) \\
\cos(\alpha + 2\pi/3) & \cos(\alpha - 2\pi/3) & \cos(\alpha)
\end{bmatrix}
\]

\[
\alpha = \nu \theta_r
\]

IV. ELECTROMAGNETIC CIRCUITS EQUATIONS

The three-phase induction motor model with distributed windings, in the stator frame, is

\[
V_s^{ABC} = \mathbf{R}_s i_s^{ABC} + \mathbf{L}_s^{ABC} \psi_s^{ABC}
\]

\[
V_r^{ABC} = \mathbf{R}_r i_r^{ABC} + \mathbf{L}_r^{ABC} \psi_r^{ABC}
\]

\( \mathbf{R}_s \) and \( \mathbf{R}_r \) are diagonal matrices in \( \mathbb{R}^3 \), \( V_s^{ABC} \) and \( V_r^{ABC} \) are column vectors describing the stator and rotor voltages respectively.

The flux linkage equations are:

\[
\psi_s^{ABC} = \mathbf{L}_{ss}^{ABC} i_s^{ABC} \quad (17)
\]

\[
\psi_r^{ABC} = (\mathbf{L}_{sr})^T i_s^{ABC} + \mathbf{L}_{rr} i_r^{ABC} \quad (18)
\]

The rotor expressions are referred to the stator frame, using the appropriate transformation.

V. ELECTROMECHANICAL EQUATIONS

The electromechanical equations are:

\[
M \dot{\nu}_r = -f w + T_e - T_i
\]

The electrical torque \( T_e \) is:

\[
T_e = \frac{1}{2} (i_s^{ABC})^T \mathbf{R}_s (i_s^{ABC}) + \frac{1}{2} (i_r^{ABC})^T \mathbf{R}_r (i_r^{ABC}) + \frac{1}{2} (\mathbf{L}_{ss}^{ABC})^T i_s^{ABC} \mathbf{L}_s^{ABC} i_s^{ABC} + \frac{1}{2} (\mathbf{L}_{sr})^T i_s^{ABC} \mathbf{L}_{rr} i_r^{ABC}
\]

VI. ORTHOGONAL FRAME EQUATIONS

Due to the polyphase model complexity, is normal to make simplifications using methods of transformation \([8]\): the basic Concordia transformation replaces the symmetrical three-phase variables\(^1\) into symmetrical bi-phase equivalent variables. It is denominated the ortoghonal or “ab” model\(^2\).

\[
x^{ab} = \sqrt{2/3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} x^{ABC}
\]

where:

\( x^{ABC} \rightarrow \) three-phase variables; and

\( x^{ab} \rightarrow \) orthogonal bi-phase variables.

So, the variables transformations can be expressed according to the relationships in Table I. Consequently, the mathematical expressions of stator and rotor voltages and flux linkages are:

\[
V_s^{ab} = \mathbf{R}_s i_s^{ab} + \psi_s^{ab}
\]

\[
0 = \mathbf{R}_r i_r^{ab} + \psi_r^{ab}
\]

\[
\psi_s^{ab} = \mathbf{L}_{ss}^{ab} i_s^{ab} + \mathbf{L}_{sr} (\nu \theta_r) i_r^{ab}
\]

\[
\psi_r^{ab} = \mathbf{L}_{sr} (\nu \theta_r) i_s^{ab} + \mathbf{L}_{rr} (2\nu \theta_r) i_r^{ab}
\]

\(^1\)Defined as a set of equal amplitud sinusoidal which are displaced by 120 degrees.

\(^2\)Also referenced as “dq0”, we use the “ab” to distinguish of others references frames.
with:

\[
\frac{\partial}{\partial \theta_r} (T_c L_{ss} T_c^{-1}) = 2 v c \Delta L_s U^T (2 \nu \theta_r) I - J \]

\[
\frac{\partial}{\partial \theta_r} (T_c L_{sr} T_c^{-1}) = v L_{sr} U^T (\nu \theta_r) J
\]

\[
T_c = \nu c \Delta L_s (i_s^{ab})^T U^T (2 \nu \theta_r) I - J i_s^{ab} + v c \nu L_{sr} (i_s^{ab})^T U^T (\nu \theta_r) J i_s^{ab}
\]

A. State Space Model

Considering the stator current \(i_s^{ab}\) and the rotor flux \(\Psi_r^{ab}\) as state variables, the model in the natural reference frame is calculated. From (24),

\[
i_s^{ab} = L_{rr}^{-1} (2 \nu \theta_r) [\Psi_r^{ab} - L_{sr} U (\nu \theta_r) i_s^{ab}]\]

Replacing (26) in (22)

\[
\dot{i}_s^{ab} = R_s L_{rr}^{-1} (2 \nu \theta_r) [L_{sr} U (\nu \theta_r) i_s^{ab} - \Psi_s^{ab}]
\]

To obtain \(i_s^{ab}\), replacing (26) in (24) and calculating

\[
\dot{i}_s^{ab} = -P_s (\theta_r, w) i_s^{ab} - Q_s (\theta_r, w) \Psi_s^{ab} + U_s^{ab}
\]

where:

\[
P_s (\theta_r, w) = P (\theta_r, w) / Det_s
\]

\[
Q_s (\theta_r, w) = Q (\theta_r, w) / Det_s
\]

\[
U_s^{ab} = \sigma^{-1} (\nu \theta_r) V_s^{ab}
\]

Again, the electromechanical equation is given by (19), and the transformed torque equation is:

\[
T_c = -\nu c_1 L_{sr} (i_s^{ab})^T J I - U (2 \nu \theta_r)
\]

\[
[2 \Delta L_s U (2 \nu \theta_r) + \Delta L_s] i_s^{ab}
\]

\[
- \nu c_1 L_{sr} (i_s^{ab})^T J I - U (2 \nu \theta_r) \Psi_s^{ab}
\]

\[
+ \nu c_1 (i_s^{ab})^T.
\]

\[
\{ L_r U^T (\nu \theta_r) J + 2 \Delta L_r J I - U (3 \nu \theta_r) \} \Psi_s^{ab}
\]

\[
+ \nu c_1 \Delta L_r (\Psi_s^{ab})^T J I - U (3 \nu \theta_r) i_s^{ab}
\]

B. Equivalence among models

If it is considered the case of the “classical” IM, i.e., without “induced saliencies”, the parameters \(\Delta L_s\) and \(\Delta L_r\) are adjusted to zero; in this particular situation it is possible to prove that the IMIS converge to the IM model doing the factors \(\Delta_i\) in the IMIS equal to zero.

\[
\dot{i}_s^{ab} |_{\Delta L_s=\Delta L_r=0} = -\gamma_i^{ab} + a \eta (a I - \nu w J) U^T (\nu \theta_r) \Psi_s^{ab}
\]

\[
V_s^{ab}
\]

\[
\dot{\Psi}_r^{ab} |_{\Delta L_s=\Delta L_r=0} = -a \Psi_r^{ab} + b U (\nu \theta_r) i_s^{ab}
\]

\[
T_c |_{\Delta L_s=\Delta L_r=0} = \nu c_1 L_{sr} (i_s^{ab})^T U^T (\nu \theta_r) J \Psi_r^{ab}
\]

\[
\dot{\theta}_r = w
\]

Which is the classical \(ab\) model found in the induction motors literature[8].

VII. CONCLUSIONS

A new general model to the IMIS was developed which is computed in natural orthogonal components. The model is suitable for control design and compatible with the classical \(ab\) model for induction motors without saliencies. A similar model for control by signals injection at high frequencies was reported by Jansen et al.[4], but the model for the IMIS presented here is more general.

It is observed a complexity in the components of the currents by the presence of the \(\Delta_i\), variations and their implicit dependence with the rotor position \(\theta_r\). Note that the rotor position dependence in the model can not be avoided by rotor variables transformations (\(\alpha - \beta\) model).

Current work is to research the stability and observability properties of this model, analyze the induced harmonics in the torque and currents, among others.

REFERENCES

\[ L_L = L_n - c_1 L_{sr} L_r = L_n - \frac{L_{sr}}{L_{rd} L_{rq}} L_{sr} L_r \]

\[ P(\theta_r, w) = \begin{bmatrix} R_s + c_1^2 (L_r^2 + 2\Delta L_r^2) \frac{2L_r}{4\nu \theta_r} I \n + 2L_r \Delta L_r U^T (4\nu \theta_r) I^T - 2\nu w (\Delta L_s I + 2c_1 L_{sr} \Delta L_r U^T (2\nu \theta_r)) 
\cdot U^T (2\nu \theta_r) I \end{bmatrix} \]

\[ Q(\theta_r, w) = -R_c c_1^2 (L_{sr}^{-1}) (L_s^2 + \Delta L_r^2) U^T (\nu \theta_r) - 2R_c c_1^2 (L_{sr}^{-1}) L_r \Delta L_r U^T (3\nu \theta_r) I^T + \nu w [c_1 L_r U^T (\nu \theta_r) J + 3c_1 \Delta L_r U^T (3\nu \theta_r) J I^T] \]

\[ \sigma^{-1}(\nu \theta_r) = \frac{1}{\text{det}_{\sigma}} \begin{bmatrix} L_L I + \Delta L_s U^T (2\nu \theta_r) I \n + c_1 L_{sr} \Delta L_r U^T (4\nu \theta_r) I \end{bmatrix} \]

\[ \text{det}_{\sigma} = (L_L^2 - \Delta L_s^2 - c_1^2 L_{sr}^2 \Delta L_r^2 - 2c_1 L_{sr} \Delta L_s \Delta L_r \cos (2\nu \theta_r)) \]

\[ \text{Lrr}^{-1}(2\nu \theta_r) = \frac{1}{L_{rd} L_{rq}} (L_r I + \Delta L_r U^T (2\nu \theta_r) I) \]

**APPENDIX**

**Definitions and properties**

\[ U(\cdot) = \begin{bmatrix} \cos(\cdot) & \sin(\cdot) \\
-sin(\cdot) & \cos(\cdot) \end{bmatrix} \]

\[ I = \begin{bmatrix} 1 & 0 \\
0 & 1 \end{bmatrix} ; \quad I^T = \begin{bmatrix} 1 & 0 \\
0 & -1 \end{bmatrix} ; \quad J = \begin{bmatrix} 0 & -1 \\
1 & 0 \end{bmatrix} \]

\[ U^T(\cdot) = U(-\cdot) \]

\[ I^T I^- = I ; \quad JJ = I^- ; \quad I^- = I^{-1} \]

\[ JJ^- = -I^- J = I^- J^T = \begin{bmatrix} 0 & 1 \\
1 & 0 \end{bmatrix} \]

To the IM,

\[ \sigma_j = 1 - \frac{L_{sr}^2}{L_s L_r} \]

\[ a = \frac{R_s}{L_r} \]

\[ \gamma = \frac{R_s L_{sr} \sigma_j}{L_r^2 L_{sr} \sigma_j} \]

\[ \eta = \frac{L_s L_{sr} \sigma_j}{a L_{sr}} \]

\[ b = \frac{L_s}{a L_{sr}} \]

To the IMIS,

\[ L_r = \frac{L_{sr} + L_{sq}}{2} \]

\[ \Delta L_r = \frac{L_{sr} - L_{sq}}{2} \]

\[ L_{rd} = L_r - \Delta L_r \]

\[ L_{rq} = L_r + \Delta L_r \]

\[ L_s = \frac{L_{sd} + L_{sq}}{2} \]

\[ \Delta L_s = \frac{L_{sd} - L_{sq}}{2} \]

\[ L_{sd} = L_s - \Delta L_s \]

\[ L_{sq} = L_s + \Delta L_s \]

\[ L_{rd} L_{rq} = \frac{L_s^2 - L_{sq}^2}{2} \]

\[ L_{sd} L_{sq} = \frac{L_s^2 - L_{sq}^2}{2} \]

\[ c_1 = \frac{L_{sr}}{(L_{rd} L_{rq})} \]

\[ L_L = L_n - c_1 L_{sr} L_r \]