RELATIVE MOTION IN EUCLIDEAN SPACE AND TIME

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ABSTRACT

This is an expository paper of a new approach to the Special Theory of Relativity. Rewriting Lorentz transformation equations in differential form, the basic equations of unrestricted motion are formulated in the euclidean 4-space-time, showing that such a formulation is possible, with the advantage of cancellation of all paradoxes and giving a clear physical picture of the space and time, as well as the meaning of the relativistic transformation in the free space.

Starting with the principle of momentum conservation, the dynamics of the free particle is also considered and, by following the proposed new line of attack, quite unusual results are derived, such as the volume of the Universe or the mass (and energy) invariance principle.

I. INTRODUCTION: EINSTEIN'S APPROACH TO THE FREE PARTICLE RELATIVISTIC DYNAMICS.

We begin with briefly reviewing the relativistic dynamics of a particle in free motion, as it comes from Einstein's theorie and to point out some of its drawbacks.

It has been experimentally verified and universally accepted that the performance of matter (in fact, any amount of matter) is governed by the *Principle of Least Action* (Maupertius, 1740), according to which, the action integral

$$S = \int_{t_1}^{t_2} \mathcal{E}(x,y,z;v_x,v_y,v_z,t)\, dt \qquad (1.1)$$

has a stationary value. In (1.1), t_1 and t_2 are two fixed instants of time and $\mathcal{E}$ is the total energy of the system under consideration. The most obvious consequence of the Least Action Principle is the straight-line path of the particle.

In the relativistic (Einstein's) space-time "continuum", the concept of "distance" gives way to that of "interval" and, consistently with Minkowski geometry, the "straight line" is replaced by the "longest interval", which amounts to compute the extremum of the integral

$$S_{rel} = - K \int_a^b ds \qquad 1.2)$$

where K is a constant and s the interval between two fixed points, a and b.

An alternative approach in the classical Mechanics is to apply the well known *Hamilton Princple*:

$$\delta S_1 = - \delta\int_{t_1}^{t_2} ds = 0 \qquad (1.3)$$

formulated in terms of the *Lagrange function L*. In the Special Relativity (see [7]), p. 41 and [8], p. 297), the action S_{rel}, as defined in (1.2), is replaced by S_1 from (1.3). It leads to the equation

$$- K\int_a^b ds = \int_{t_1}^{t_2} L\, dt \qquad (1.4)$$

From *Lorentz transformations* [cf. (2.1)] we have:

$$ds = c(1 - v^2/c^2)^{1/2}dt$$

Then, from (1.4):

$$L = - Kc(1 - v^2/c^2)^{1/2} \qquad (1.5)$$

For a free particle and at low velocity, L in (1.5) represents the kinetic energy of the particle: $\mathcal{E}_k = \frac{1}{2} m_o v^2$, m_o being the rest mass. Also from (1.5),

$$L \cong - Kc + \frac{Kv^2}{2c} + \ldots \qquad (1.6)$$

To identify L in (1.6) with $\mathcal{E}_k$, the constant term $- Kc$ must be disregarded (in spite of the fact that the classical Mechanics limit is achieved letting either $v \to 0$ or $c \to \infty$). Then, the substitution of (1.6) into (1.4) gives $K = m_o c$ and, from (1.2) and (1.5), we get:

$$S_{rel} = - m_o c \int_a^b ds \qquad (1.7a)$$

$$L = m_o c^2(1 - v^2/c^2)^{1/2} \qquad (1.7b)$$

Continuing with the classical approach, let p_i, q_i be the generalized Lagrange coordinates of the system. A well known from Analytical Mechanics relationship states:

$$p_i = \partial L/\partial \dot{q}_i$$

For a single particle, we take p as the particle momentum and $\dot{q}$ as the velocity. Then,

$$p = \frac{\partial L}{\partial v} = \frac{mv_o}{(1 - v^2/c^2)^{1/2}} = mv$$

with

$$m = \frac{m_o}{(1 - v^2/c^2)^{1/2}} \tag{1.8}$$

which is the celebrated Einstein's formula for the mass of a particle in motion.

Furthermore, let $H(p,\dot{q},t) = \sum p \dot{q} - L$ be the Hamilton function, which can be interpreted as the total energy of the system. For the single particle at hand we find:

$$H = mv^2 - L = mc^2 \tag{1.9}$$

with m given in (1.8).

The last equation is another, probably the most famous relationship proposed by Einstein. The main purpose of this article is to question its validity. In fact, several objections can be formulated with respect to the statement (1.9), when combined with (1.8). The first one is obvious: if both relationships were true, the energy of an isolated system can become unbounded and the very concept of energy, let alone the conservation law, would be bereft of their physical meaning [6]. Besides, there are other considerations that show otherwise. This is the case, for example, of eq.'s (1.7) for three reasons:

a) There is no objective justification in switching, in the relativistic case, from the meaningful **Least Action Principle** to that of *Hamilton*, only because it proved to be useful in Classical Mechanics.

b) As it has been mentioned earlier, it becomes difficult to justify the dropping of the constant (possibly of infinite value in the classical approximation).

c) As it will be proved later on, the main drawback of Einstein's theory that eventually leads to a number of inconsistencies and contradictions in both Special and General Relativities is to interpret the interval *s* as a sort of "distance" in a static space-and-time "continuum". Such a space can admittedly exist as a mental exercise, but it can never be our Universe, complying with the laws of physics and subject to a constant expansion, as it is supposed to have been proved beyond any reasonable doubt [3], [4].

II. EUCLIDEAN VERSUS MINKOWSKI SPACES

An universally accepted assumption is that *the only vector of intrinsic significance is the velocity vector*. Quite naturally, this assertion can be used as the basis for the relativistic transforms. To this end, rewrite the Lorentz transformations in the differential form:

$$\begin{aligned} dt &= R\,(d\hat{t} - Vc^{-2}d\hat{x}) \\ dx &= R\,(-V\,d\hat{t} + d\hat{x}) \\ dy &= d\hat{y} \\ dz &= d\hat{z} \end{aligned} \tag{2.1}$$

where $d\hat{t}$, $d\hat{x}$, $d\hat{y}$, $d\hat{z}$ are refered to the rest frame, $R = (1 - V^2c^{-2})^{-1/2}$ and V is the (constant) moving frame velocity in the (common) $\hat{x}$ and x direction.

The first consequence of writing the Lorentz transformations in the differential form (2.1) or, which is the same, to interpret them as equations of motion, is the surmounting of a conceptual difficulty that lies at the very root of Einstein's Special Relativity, namely, the one related to the notion of simultaneity. In effect, after integrating the first equation (2.1) over a finite interval (t_1,t_2) of time t associated with the moving system, we get:

$$t_2 - t_1 = R\left[(\hat{t}_2 - \hat{t}_1) - Vc^{-1}(\hat{x}_2 - \hat{x}_1)\right]$$

where $(\hat{t}_2,\hat{t}_1)$ and $(\hat{x}_2, \hat{x}_1)$ are some finite intervals along the axes $\hat{t}$ and $\hat{x}$, respectively in the fixed reference system. Incidentally, the last equation is the classical result normally quoted when discussing the Lorentz (non differential) transformations and interpreted in the sense that, for $\hat{t}_1 = \hat{t}_2$ and $\hat{x}_1 \neq \hat{x}_2$ (two events occuring simultaneously at two different points in the fixed system) the interval $t_2 - t_1$ does not vanish, which means that the simultaneity has been lost. It happens, however, that, according to the present discussion, this conclusion is false, because the condition $\hat{t}_1 = \hat{t}_2$ ($d\hat{t} = 0$) implies $d\hat{x} = 0$ as well, owing to the fact that every motion ($d\hat{x} \neq 0$) is necessarily time consuming ($d\hat{t} \neq 0$).

It is also interesting to observe that, unlike the foregoing result, the converse is not true, that is, the condition $d\hat{x} = 0$ does not imply $d\hat{t} = 0$, which means that dx does not (in general) vanish. This lack of symmetry between the (normal) space and the time in the relativistic space-time is quite meaningful and suggests that the time should not be conveniently considered as an independent component (4th direction) in the 4-dimensional space but some sort of norm associated with a linear (vector) space [5]. This suggestion is further supported by the non negativeness of $d\hat{t}$ (complying with the *principle of causality* which, to our best knowledge, holds in the observed Universe) and shall be ultimately established in what follows.

As the next step, it will be helpful to define the new functions, $\psi(v)$, $\hat{\psi}(\hat{v})$ and $\theta(V)$ by the relationships: $\tanh \hat{\psi} = \hat{v}/c$, $\tanh \psi = v/c$, $\tanh \theta = V/c$, with

$$v = \left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right]^{1/2}$$

etc. Using these definitions it is not difficult to prove the following relationships:

$$R = \cosh^{-1}\theta \tag{2.2a}$$

$$R = \hat{\psi} + \theta \qquad \text{(scalar velocity composition)} \tag{2.2b}$$

$$\frac{d\hat{x}}{\sinh \hat{\psi}} = \frac{dx}{\sinh \psi} = d\chi \quad \text{(Lorentz contraction, modified)} \tag{2.2c}$$

$$\frac{d\hat{t}}{\cosh\hat{\psi}} = \frac{dt}{\cosh\psi} = \frac{d\chi}{c} \quad \text{(time transform)} \tag{2.2d}$$

with $d\chi$ invariant under Lorentz transformations (2.1). Observe that *sinh* ψ is an odd function and *cosh* ψ an even function of ψ (time-like transforms).

A very illuminating geometric interpretation can be given to (2.2). To this end define

$$dw = c\,dt$$

$$\alpha = \arccos(1/\cosh\psi) = \arctan(\sinh\psi) = \arcsin(\tanh\psi)$$

Suppose also $dy = dz = 0$ (the x-directed rectilinear motion). It gives $d\chi^2 = dw^2 - dx^2$, which is the definition of the *relativistic interval*. Rewrite it (returning to the euclidean space [5]) as

$$dw^2 = d\chi^2 + dx^2 \tag{2.3}$$

Equation (2.3) simply states the pythagorean theorem, as depicted in Fig. 1 and interprets the Lorentz transforms (2.1) as a rotation in the (time space χ) x (pure space x) plane (or hyperplane if a more than one-dimensional motion is involved). In fact, a generalization for any motion in an arbitrary r direction is straigtforward. Observe also that in this model the trajectory of the light is *normal* to the world-line of the emitting object (not tilted 45°).

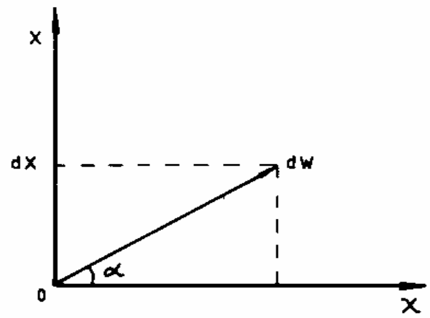

Fig. 1. Rectilinear motion in the relativistic 2-space plane interpreted as rotation.

III. SOLUTION OF THE TWIN-CLOCK PARADOX

An important consequence of the return to the euclidean space is the disappearance of the famous Einstein's twin-clock paradox. To prove that there is no such a paradox, consider the object *B*, in motion relative to the object *A*, as depicted in Fig. 2. For simplicity it will be supposed that *B* follows the solid (broken-line) path, although a more realistic situation will be one shown with the dotted line.

Before starting with the relative speed $c\tanh\theta_1 = c\sin\alpha_1$, the object *B* synchronizes the twin-clock with A. At B_2, B's clock shows $t_{b1} = d_1/(c\cos\alpha_1)$. *B* changes there its course and starts the return path toward A. At A_2, the system *B* merges with A agin. After returning home, whereas B's clock says that the travelling time lasted

$$t_b = \frac{d_1\cos\alpha_1 + d_2\cos\alpha_2}{c}$$

for A, object *B* has been absent

$$t_a = \frac{d_1 + d_2}{c}$$

seconds. The situation would be reversed if, for example at A_2, system A had decided to change its quiescent course alongt χ and join B, reaching it at B_3'. In this case, A's clock would lag

$$\Delta t = \frac{B_3'' - B_3'}{c}; \quad (A_1B_3'' = A_1A_2 + A_2B_3')$$

Thus, there is no paradox at all. Moreover, in the light of the previous discussion, the bewildering (but currently accepted as true) statement that in the relativistic space-time "the straight line is the longest path between two points" [7] (in opposition to the euclidean space where it is the most obvious shortest path) can be simply reformulated in equally obvious terms by saying that **moving along a straight line one travels the longest way.** The essence of this restatement lies in the fact that Lorentz equations (2.1) refer to the motion, not to a static universe. Incidentally, the same is true for Maxwell's equations, which - as it will be proved elswhere - fulfil eq. (2.1); and conversely: it can be shown that it is possible to derive from (2.1) alternative (not Einstein's) general relativistic equations, compatible with the maxwellian electrodynamic theory.

IV. VELOCITY VECTOR IN EUCLIDEAN 4-SPACE.

As it has been already mentioned, eq. (2.3) is readily generalized (by simply rotating the x,y,z system) to an arbitrarily oriented 4-radius vector in euclidean 4-space (which from now on will be abreviated to E-4D). Consequently, if $dl = (dx^2 + dy^2 + dz^2)^{1/2}$ is an element of arc in the ordinary space refered to a set of orthonormal coordinates, any 4-dimensional arc (4-arc) squared can be written as

$$dw^2 = d\chi^2 + dl^2 = d\chi^2 + dx^2 + dy^2 + dz^2 \tag{4.1}$$

with

$$dw = c\,dt = \cosh\psi\,d\chi \tag{4.2}$$

As it has been proved in Sec. II, $d\chi$ is constant, which allows to interpret χ as a parameter and, as such, it will be relabeled ξ. Then, the quotients

$$r_x(\xi) \equiv \frac{dx}{d\xi}\;;\; r_y(\xi) \equiv \frac{dy}{d\xi}\;;\; r_z(\xi) \equiv \frac{dz}{d\xi} \tag{4.3}$$

($\equiv$ stands for "equal by definition") can be interpreted as derivatives of the space coordinates with respect to the parameter, and the integral

$$\Delta s = \int_a^b (r_x^2 + r_y^2 + r_z^2 + 1)^{1/2}\, d\xi \qquad (4.4)$$

as the distance (or interval) between two fixed points (events) in E-4D.

According to (2.2c), for a rectilinear, parallel to x-axis motion, that is to say, for $r_y = r_z = 0$, we have $r_x = \sinh\psi$. In a general case, (2.2c) must be replaced by

$$r_x^2 + r_y^2 + r_z^2 = \sinh^2\psi$$

and from (4.1),

$$\left(\frac{d\psi}{d\xi}\right)^2 = r_x^2 + r_y^2 + r_z^2 + 1 = \cosh^2\psi \qquad (4.5)$$

After multiplying r_x, r_y, r_z by

$$v_\xi \equiv \frac{c}{\cosh\psi} = \frac{d\chi}{dt} \qquad (4.6a)$$

we get

$$u_x \equiv \frac{c\, r_x(\xi)}{\cosh\psi} = \frac{dx}{dt}$$

$$u_y \equiv \frac{c\, r_y(\xi)}{\cosh\psi} = \frac{dy}{dt} \qquad (4.6b)$$

$$u_z \equiv \frac{c\, r_z(\xi)}{\cosh\psi} = \frac{dz}{dt}$$

Eq.'s (4.6) can be interpreted as the components of the 4-velocity vector in E-4D. They also fulfil an important relationship

$$u_\chi^2 + u_x^2 + u_y^2 + u_z^2 = c^2 \qquad (4.7)$$

in accordance with (4.1) (to be compared with a somewhat similar equation quoted in the current literature for Minkowski space [7]: $u^i u_i = 1$, where Einstein's summation convention has been applied; notice, however, that neither u^i nor u_i are the normalized values of u_x, etc. In fact, they have no definite physical meaning).

Strictly speaking, the 4-velocity components (4.6) have been derived for a rectilinear, arbitrarily oriented motion.

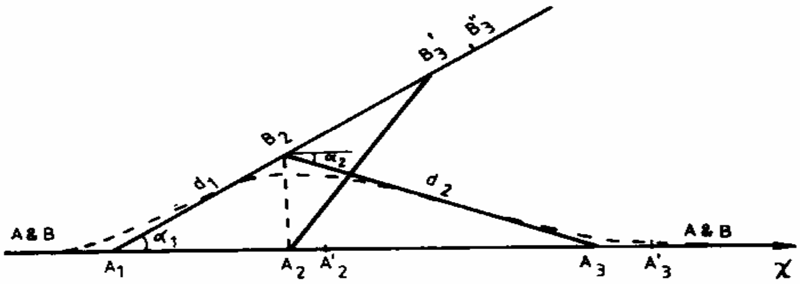

Fig. 2. An object, B, running away from - and returning to - A. No clock paradox is registered.

However, because of the fact that a vanishingly small element of a (continuous and differentiable) arc can be approximated by a straight-line segment, equation (4.7) becomes indpendent of the form of the trajectory. On the other hand, it will be proved elswhere that, in the general case (not restricted to a straight-line motion), the coefficients r_x, r_y, r_z are functions of both ψ and ξ.

Equation (4.7) reveals the fundamental fact that the speed of light in vacuum is at the same time the absolute velocity of every object or system in E-4D, regardless of its behaviour in the ordinary space (recall the Michelson-Morley experiment). In particualar, with $\psi = 0$ (the system at rest), u_x, u_y, $u_z = 0$ and $u_\xi = c$ (the speed of light in time!). On the other hand, when $\psi \to \infty$, we have:

$$u_\xi = 0,$$

$$(u_x^2 + u_y^2 + u_z^2)^{1/2} = c \tanh\psi|_{\psi=\infty} \to c$$

and the particle velocity approaches the speed of light in the normal space (orthogonal to χ axis, in agreement with the discussion of Sec. II).

V. HYPERSPHERICAL COORDINATES; VOLUME OF THE UNIVERSE.

Equation (4.7) also has an important cosmological implication, namely it allows to evaluate the volume of the present-day Universe, provided its age is known, which is seemingly the case.

Indeed, according to (4.7), the very space, independently of the behaviour and amount of the matter scattered across it, may be conceived as a 3-dimensional spherical hypersurface in the 4-space defined by the set of orthonormal coordinates u_ξ, u_x, u_y, u_z. The radius of the sphere is c, as depicted symbolically in Fig. 3, where v represents the normal space velocity axis in the direction (expressed in the usual notation):

$$v = u_x i + u_y j + u_z k.$$

However, we shall resort for convenience to the *hyper - spherical* coordinates, defined (with reference to Fig. 3, and c replaced for generality by ρ) as follows:

$$x_o = \rho \cos \alpha$$
$$x_1 = \rho \sin \alpha \cos \theta \qquad (51)$$
$$x_2 = \rho \sin \alpha \sin \theta \cos \phi$$
$$x_3 = \rho \sin \alpha \sin \theta \sin \phi$$

where θ, ϕ are the usual latitude and longitude in spherical coordinates [observe that definition (5.1) can be naturally extended to a more-dimensional space].

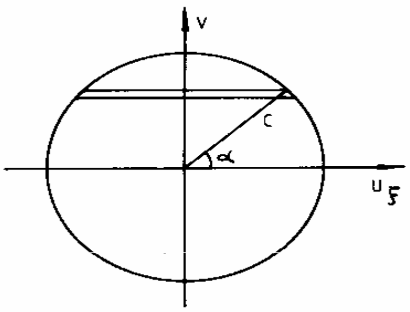

Fig. 3. A symbolic picture of a 4-sphere in the 4-velocity space.

The coordinate system $(\rho,\alpha,\theta,\phi)$ defined in (5.1) is orthogonal; thus, the element of arc is given by

$$ds^2 = g_{oo}d\rho^2 + g_{11}d\alpha^2 + g_{22}d\theta^2 + g_{33}d\phi^2$$

Where g_{ij} is the fundamental metric tensor with the only non zero components:

$$g_{oo} = 1;\ g_{11} = \rho^2;\ g_{22} = \rho^2\sin^2\alpha;$$
$$g_{33} = \rho^2\sin^2\alpha\,\sin^2\theta$$

as it can be easily verified by differentiating (5.1). The volume of the constant-ρ hypersurface is computed by using the well known formula from vector analysis (see for example [1], [2]):

$$d\Sigma_\rho = g_\rho^{1/2}d\alpha\, d\theta\, d\phi \qquad (5.2)$$

where g_ρ is the cofactor of g_{oo} in $\det(g_{ik})$:

$$g_\rho = \begin{vmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{vmatrix} = (\rho^3\sin^2\alpha\,\sin\theta)^2 \qquad (5.3)$$

By substituting (5.3) into (5.2) and integrating over $\alpha \in \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$, $\theta \in [0,\pi]$, $\phi \in [0, 2\pi]$, we get:

$$\Sigma_\rho = 2\pi^2\rho^3$$

In our case, $\rho = c \cong 3 \times 10^8$ m/s. To compute the desired space volume it is sufficient to replace ρ by ct, t being the age of the hypersfere (the Universe). Assuming $t \cong 1.5 \times 10^{10}$ years, $\cong 4.8 \times 10^{17}$s, we finally get:

$$\Sigma_{ct} = 2\pi^2(3 \times 4.8 \times 10^{25})^3 \cong 6 \times 10^{79}\ m^3 \qquad (5.4)$$

Equation (5.4) can be considered as a remarkable result in view of the fact that presently it is still considered as an unsolved question whether the Universe is finite or infinite. Related to it there is also the problem of the unaccountable extreme flatness of the Universe. The restatement of the Special Theory of Relativity under discussion gives the answer to both questions, although we shall not dwell on this point any longer here. It may also be pointed out that the relativistic Doppler effect and the computation of Hubble constant problems are easily solved in the same way.

VI. RELATIVISTIC MASS AND FORCE TRANSFORMS

We are ready now to return to the question of the relativistic mass transform taken on in Section I and, related to it, the momentum and energy conservation laws. To this end let $P_1(m_1,v_1)$ and $P_2(m_2,v_2)$ be two particles with the rest masses m_{o1} and m_{o2}, respectively. According to eq. (1.8) and the definition of ψ we have:

$$m_1 = m_{o1}\cosh \psi_1 \qquad m_2 = m_{o2}\cosh \psi$$
$$v_1 = c \tanh \psi_1 \qquad v_2 = c \tanh \psi_2$$

and the momentum conservation law (which we shall consider provisionally valid even in the relativistic world) requires the fulfilment of the equation

$$cm_{o1}\sinh \psi_1 + cm_{o2}\sinh \psi_2 = \text{const.}$$

Differentiating:

$$cm_{o1}\cosh \psi_1\, d\psi_1 + cm_{o2}\cosh \psi_2\, d\psi_2 = 0 \qquad (6.1)$$

After multiplying both terms in (6.1) by c and dividing by $d\xi$ (both constant), we arrive at:

$$c^2m_{o1}\cosh \psi_1 \frac{d\psi_1}{d\xi} + c^2m_{o2}\cosh \psi_2 \frac{d\psi_2}{d\xi} = 0 \qquad (6.2)$$

The reson for multiplying (6.2) by c is to dimensionally equate both terms to a force. It allows to interpret (6.2) as the relativistic form of the 3rd Newton law: $F_1 = - F_2$, with

$$F_i = c^2m_{oi}\cosh \psi_i \frac{d\psi_i}{d\xi}\ ;\ i = 1,2$$

or, generically,

$$d\psi = \frac{F(\psi,\xi)\ d\xi}{c^2m_o\cosh \psi} \qquad (6.3)$$

In (6.3), $F(\psi,\xi)$ has been supposed (a quite logical supposition) both ψ - and ξ-dependent. To be specific, we shall provisionally assume

$$F(\psi,\xi) = F_1(\xi)\ \cosh^3\psi \qquad (6.4)$$

where $F_1(\xi)$ (nothing to do with F_i for i =

= 1) can be regarded as a force, function of ξ only; that is to say, invariant in the relativistic transformation (rotation in E-4D). The definition of $F_1(\xi)$ given in (6.4) can be considered at this point as axiomatic. Its ultimate justification will be infered from the consequences of paramount importance in the Relativistic Dynamics, relayed to a further report.

Using (6.4) we can eliminate $F(\psi,\xi)$ in (6.3), with the result:

$$F_1(\xi) = \frac{c^2 m_o}{\cosh^2\psi}\frac{d\psi}{d\xi} \qquad (6.5)$$

To properly interpret (6.5) we shall use the classical definition of force as the time derivative of the momentum. For example, in the case of the x-axis directed motion we have:

$$F_{1x} = \frac{d(m_o v)_x}{dt} = \frac{c^2 m_o}{\cosh^3\psi}\frac{d\psi}{d\xi} \qquad (6.6a)$$

where the results of Sec. II have been used. Combining (6.6a) with (6.5) we can also write:

$$F_{1x} = \frac{F_1(\xi)}{\cosh\psi} \qquad (6.6b)$$

The last equation shows that F_{1x} is equal to $F_1(\xi)$ when $\psi = 0$, that is to say, represents the force applied to the particle at rest, perpendicularly to the trajectory (χ-axis). $F_1(\xi)$ does not depend on ψ, which also means that the perpendicularity between the trajectory and $F_1(\xi)$ holds for any speed (or ψ) and, thus, can be considered as a fundamental fact in both Special and General Relativity. This subject will be treated in more detail later on.

Now, consider the energy function. To this end we combine both equations (6.6) and get:

$$F_1(\xi)\, d\xi = \frac{F_{1x}}{\tanh\psi}\, dx = c^2 m_o\, d(\tanh\psi)$$

and, after rearranging terms,

$$F_{1x}\, dx = c^2 m_o \tanh\psi\, d(\tanh\psi) = m_o v_x\, dv_x \qquad (6.7)$$

Suppose now that the particle has been emitted from the origin in χ - x plane with the initial velocity v_o in the x-direction. Integrating (6.7) over x we get:

$$\int_o^x F_{1x}\, dx = \frac{1}{2} m_o (v_x^2 - v_o^2) \qquad (6.8)$$

equation that represents the live-force law. For $x \to \infty$, v_x vanishes, and eq.(6.8) gives:

$$\int_o^\infty F_{1x} dx + \frac{1}{2} m_o v_o^2 = 0 \qquad (6.9)$$

which simply states that the kinetic energy of the particle, abandoned to itself, is entirely spent in the work done by the same particle. Incidentally, after averaging over a system of particles, it agrees with the well known virial theorem.

The term $m_o c^2$ naturally appearing in the previous equations gives, as it is expected to, the rest energy of the particle, postulated by Einstein. There is a drastic difference though between Einstein's statement and the present point of view, namely that equations (6.8), (6.9) that express the energy conservation law are formulated in terms of the rest mass and, thus, remain invariant in relativistic transformations. Yet the fact that the mass of the particle increases with its speed according to Einstein's law (1.8) has been experimentally verified beyond any reasonable doubt. Or does it? It happens that the only fact we know for certain is that the inertia of a mechanical system increases with its velocity which, by itself, allows different interpretations.

In effect, according to Newton's 2nd law, which we have no reason to suppose not to hold in the relativistic mechanics, the acceleration in the rectilinear motion is determined by the ratio between the force and the mass, providing that the former is co-directional with the acceleration. Let the motion be oriented along x-axis. Then the equation

$$a_x = \frac{F_1(\xi)}{m} = \frac{F_{1x}}{m_o} \qquad (6.10)$$

satisfies the relativistic requirement of the increasing inertia and, at the same time, keeps the system mass constant. In the discussion of the Relativistic Dynamics, equation (6.10) will be formally derived by other means.

To better underestand the meaning given here to the relationship between the force and the trajectory, a segment of the latter for a particle (assumed pointlike) in the ξ-x plane is represented in Fig.4. The force, by assumption, is perpendicular to the trajectory which forms angle α with the x-axis.

From the vector diagram in Fig. 4, the following relationships are immediately derived:

$$F_{1x} = F_1(\xi)\cos\alpha = \frac{F_1(\xi)}{\cosh\psi} \qquad (6.11a)$$

$$F_{1\chi} = -F_1(\xi)\sin\alpha = -F_1(\xi)\tanh\psi \qquad (6.11b)$$

where α vs. ψ transformations, defined in Sec. II, have been used.

Equation (6.11a) has already been derived in (6.6), whereas $F_{1\chi}$, defined in (6.11b), can be interpreted as the *"brake force"* responsible for the relativistic dilation of time, associated with the motion (cf. Sec. III). In the limit, when the particle (hypothetically) reaches the speed of light *c*, the whole force is "spent" in keeping the trajectory perpendicular to the χ-axis (and, by the same, confined to the "normal" space, exclusively). In this situation, the entire energy of the particle – which is invariant and amounts to m_oc^2 – is bound to be transformed into radiation, according to Max Planck law:

$\mathcal{E} = h\nu$.

All that leads to the important conclusion

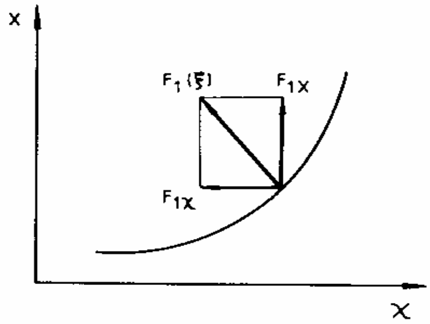

Fig. 4. Trajectory of a particle accelerated in χ–x plane and vector diagram of the accelerating force.

that *only the light* (actually electromagnetic radiation in general) *propagates with the speed of light.*

Equations (6.11) will be used in further reports devoted to the Particle Dynamics (General Relativity). As for now, we shall briefly return to the question of the momentum and the Least Action Principle, considered in Sec. I.

VII. ENERGY, LAGRANGE FUNCTION AND MOMENTUM IN E-4D.

In Sec. I, a number of inconsistencies have been found when applying the Hamilton Principle to the relativistic space-time continuum. According to the Theory of Relativity reformulated in the subsequent sections, such difficulties are mainly due to the rôle of "distance" confered by Einstein to the interval *s*. Thus, by replacing the Least Action Principle by the Hamilton Principle and the fixed-time integral limits in the former by the extremums of the interval, any track of physical meaning of the transformed integral has been lost.

The alternative approach which is followed here is the return to the *Least Action Principle*: $\delta S = 0$, *S* being given by the integral defined in (1.1), and computed along the trajectory in E-4D, with the time as the integration variable. Consequently, according to the discussion carried out in Sec. II, equation (1.2) must be replaced by

$$S_{rel} = K \int_{t_1}^{t_2} \frac{ds}{(1 - v^2/c^2)^{1/2}} \qquad (7.1)$$

Then, with the mass of the particle constant, everything trivially falls into place. In fact, with repect to the meaning of the Lagrange and Hamilton functions, our reasoning is as follows.

In Classical Mechanics, the potential energy $\mathcal{U}$ contains an indetermined constant whose absolute value is allowed to become as large as we are pleased. The same is true for the Hamilton function $H = \mathcal{E}_k + \mathcal{U}$ ($\mathcal{E}_k$ = kinetic energy) interpreted as the total energy of the particle, where both $\mathcal{E}_k$ and $\mathcal{U}$ may be unbounded. In contrast, from the relativistic point of view, the total energy of the particle is m_oc^2, a scalar, and, as such, unaffected by the rotation of the reference frame; that is to say, by the relativistic transformation, that is, by the speed. We shall still call the particle energy *H* and, as usual, *L* the Lagrange function and use the well known from Classical Mechanics equation $L = 2\mathcal{E}_k - H$. In our case,

$$L = m_o(v^2 - c^2) = m_oc^2(\tanh^2\psi - 1) = - m_ou_\zeta^2 \qquad (7.2)$$

where the last equation is drawn from the definition of u_ζ (4.6a). On the other hand, the particle velocity is tangent to the trajectory at each point. Then, by (7.2), the Hamilton Principle can be interpreted in the sense that, in any motion, under the action of any force – conservative or not – the χ-projection of the trajectory is kept minimum under specific constraints. In other words, avery trajectory is the geodesics on a given surface (or hypersurface). For instance, the foregoing discussion fully justify why the light propagates orthogonally to the χ-axis in the free space (and, thus, the absolute flatness of the Universe) and explain the fact that the Hamilton action – such as it has been defined here – is independent of the speed.

The vector momentum is in our case

$$p = m_ov \qquad (7.3)$$

and also remains bounded. It is interesting to observe that, by this assumption, it can be proved [6] that the gravity centre of a mechanical system is conserved in the relativistic transformations, which fails to be true if we replace in (7.3) m by the supposedly "relativistic" mass (1.8).

VIII. CONCLUSION

The new approach to the Special Theory of Relativity summarily described in this paper offers a more consistent picture of the relativistic space-time than the one given by Einstein. In particular, there are no paradoxes left and the energy conservation law holds in all circumstances. When applied to the cosmological problems the theory at hand gives a plausible explanation of many – till now considered as mysterious – performances of the Universe.

It will be domonstrated elswhere that the extension of this method onto the General Relativity realm is straightforward and leads to other interesting consequences.

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