

# RELATIVISTIC DYNAMICS AND ELECTROMAGNETIC FIELD

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## Abstract

This is the concluding paper, out of a series of three, compiled under a general title *A New Approach to the Theory of Relativity*, two of them already published in the previous issue of these Proceedings. In this contribution, the equations of motion in the relativistic space are further analyzed, proving that they can readily be interpreted as the general field equations. In particular, the compatibility between the General Relativity and Maxwell equations has been established, showing that both acquire an identical form in the limiting case of propagation of the system under consideration (a field of force in our case) with the speed of light. Besides, when using the formalism of the Maxwell equations, the effect of strong acceleration comes to light leading to a new interpretation of the quantum transitions of small particles and, by the same, to reconciliation of the Quantum and Relativistic Mechanics.

Another consequence of the analysis reported here is the geometric interpretation of the mass and gravitational field, allowing an easy understanding of the seemingly "strange" behaviour of some celestial bodies, such as the quasars and black holes.

## I. INTRODUCTION

In two previous papers [1], [2], a relativistic 4-space (E-4D) has been defined by the coefficients  $r^k(\psi, \xi)$ , related to the 4-velocity vector  $(u^0, u)$  by the equation

$$u^k = c \frac{r^k(\psi, \xi)}{\cosh \psi} \quad (1.1)$$

In (1.1),  $r^0 = 1$ , and  $u^0 = c/\cosh \psi$  is the component of the velocity along the relativistic time-axis  $\chi$ .  $\xi$  is the "arc length" along  $\chi$ , related to the usual (non relativistic) time  $t$ , through the differential equation:

$$d\xi \equiv d\chi = \frac{c dt}{\cosh \psi} \quad (1.2)^*$$

and  $\psi$  is found from the relationship:  $\tanh \psi = \|\vec{v}\|/c$ , where the second term represents the modulus (norm) of the velocity vector  $\vec{v}$  in the ordinary (3-dimensional) space, normalized with respect to the speed of light. It has been also found that the following relationship holds in E-4D {cf. [1], (4.7) and [2], (2.3b)}:

$$u^i u_i = c^2 \quad (1.3)$$

where Einstein's rule of summation has been used and {cf. [2], (2.3a)}:

$$\sum_{\nu=1}^3 (r^\nu)^2 = \sinh^2 \psi \quad (1.4)$$

Let  $\Gamma_{jk}^i$  be the Christoffel symbols (cf. [2], APPENDIX A). Then, the following functions, closely related to the definition of the geodesics {cf. [2], (1.6) and (2.6)} have been established:

$$R_{\xi}^k = \frac{\partial^2 x^k}{\partial \xi^2} + \Gamma_{\alpha\beta}^k r^\alpha r^\beta \quad (1.5)$$

Furthermore, whenever  $r^k$  vary with  $\psi$ , there also exist non-zero valued coefficients:

$$R_{\psi}^k = \frac{\partial r^k}{\partial \psi} \quad (1.6)$$

An important result brought to light in [1] and drastically contrasting with a well known Einstein's proposition is that the mass of the particle or, in general, of an object, is invariant in the relativistic transformations (does not vary with its speed). To reconcile this supposition with the velocity dependent inertia of the material objects it has been found {cf. [1], (6.1a)} that the space component of a force is given by the equation

$$F_{1x} = \frac{F_1(\xi)}{\cosh \psi} \quad (1.7a)$$

i.e., decreases, in the 3-dimensional space, with the speed  $v = \|\vec{v}\|$  of the object to which the force is applied as  $1/\cosh \psi = (1 - v^2/c^2)^{1/2}$ . In (1.7a),  $F_1(\xi)$  is the relativistic transformation-invariant force.

Likewise, the time-component of the force has been defined as {cf. [1], (6.11b)}:

$$F_{1\chi} = -F_1(\xi) \tanh \psi \quad (1.7b)$$

giving the obvious relationship:

$$F_{1x}^2 + F_{1\chi}^2 = F_1^2(\xi) \quad (1.8)$$

An important consequence of the foregoing statement is that the overall energy (the hamiltonian) of a particle with the mass  $m$  is invariant and equal to  $mc^2$ . When the particle is in motion, a part of its energy is transformed into kinetic and electromagnetic energies. In the limiting case of a particle acquiring (hypothetically) the speed of light, its energy becomes the radiation energy  $h\nu$

(according to the famous Planck - Einstein equation). In the intermediate state ( $0 < v < c$ ), the particle becomes partially matter and partially wave, complying with the DeBroglie Wave Mechanics.

Using all the previous statements and definitions, the general equation of motion has been written in the form [cf. (2). (2.11)]:

$$\frac{dp^k}{dt} = m \left[ \frac{c^2}{\cosh^2 \psi} R_{\xi}^k + \frac{F_1(\xi)}{m} (R_{\psi}^k - r^k \tanh \psi) \right] \quad (1.9)$$

where  $p^k = mu^k$  is the momentum 4-vector in E-4D.

The main purpose of what follows is to relate equation (1.9) to the Maxwell equations. It will be shown that both are compatible, which is to say, can be derived one from another [the maxwellian equations from (1.9) and vice - versa].

On the other hand, it has been pointed out in the concluding section of [2] that a sufficiently strong acceleration can "neutralize" the "normal course" of the

$$\frac{dp^k}{dt} = \frac{c^2 m}{\cosh^2 \psi} \left[ \frac{\partial r^k}{\partial \xi} + \Gamma_{\alpha\beta}^k r^{\alpha} r^{\beta} + \left( \frac{\partial r^k}{\partial \psi} - r^k \tanh \psi \right) \frac{d\psi}{d\xi} \right] \quad (2.1a)$$

natural phenomena and the Quantum Mechanics laws begin to prevail. The acceleration is thus another quantity conditioning the behaviour of the matter and has to be taken into account using a rather different treatment. An important consequence of the present approach to the Relativistic Dynamics is that the Equivalence Principle [3] - the corner-stone of Einstein's Theory of Relativity - does not generally hold, which is to say that the relativity is no longer true for the accelerated bodies. This statement will be considered in more detail in the next sections.

Now, a few words about the coefficients  $R_{\xi}^k$  appearing in (1.5) and their relationship with the equation of the geodesics:

$$\frac{d^2 x^i}{ds^2} + \Gamma_{jk}^i \frac{dx^j}{ds} \frac{dx^k}{ds} \quad (1.10)$$

referred to in [1], eq.(1.6), and rewritten here for commodity. First of all, it has to be noted that  $\xi$  in (1.5) and  $s$  in (1.10) are the same variables, although related in a different way to the notion of "distance" in E-4D ( $\xi$ ) and in the Minkowski space ( $s$ ). In addition, both the second member in (1.5) and the first one in (1.10) represent the trajectory in a "disturbed" space. The main difference between them consists in the fact that  $R_{\xi}^k$  enter into the equation of motion multiplied by a convenient convergence factor, such as  $c^2/\cosh^2 \psi$ , which becomes vanishing small as  $R_{\xi}^k$  run to infinity ( $v \rightarrow c$ ), whereas in (1.10) there is no such a fortunate circumstance and, thus, the equation of the geodesics does not apply to the objects propagating with the speed approaching  $c$  (in fact, it does not

strictly apply anywhere). Furthermore, eq.(1.10) is homogeneous. Thus, when Einstein tried to identify the components  $g_{ij}$  of the fundamental metric tensor entering the Christoffel symbols with the driving force (gravitational field) potentials, he interpreted some terms as both the cause and the effect [cf. [3] and [2], Sec. I]. In contrast, in (1.9), the driving forces,  $dp^k/dt$ , appear in the first members of the equations, the second members being the components of the "opposing force", in agreement with the general principle of action and reaction presiding all the "well behaved" physical laws.

## II. ELECTRIC AND MAGNETIC FIELDS IN THE RELATIVISTIC SPACE.

To apply the general equations of motion (1.9) to the electric charges driven by an electromagnetic field, the term  $F_1/m$  has to be replaced by  $(c^2 m / \cosh^2 \psi) \frac{d\psi}{d\xi}$  [cf. [1], eq. (6.5)]. In addition,  $R_{\xi}^k$  can be eliminated using (1.5) and (1.6). After all these substitutions we get:

where  $r^k = dx^k/d\xi$  [cf. (1.1), with  $u^k = dx^k/dt$ ].

Frequently, when deriving (2.1a), the normalized velocity components  $r^k$  can be rendered explicitly independent of  $\psi$  by using constraints (1.4). It allows to drop the factor  $R_{\psi}^k$ , replace the partial derivatives  $\frac{\partial r^k}{\partial \xi}$  by the total ones,  $\frac{dr^k}{d\xi}$ , and write (2.1a) in a slightly simpler form:

$$\frac{dp^k}{dt} = \frac{c^2 m}{\cosh^2 \psi} \left( \frac{dr^k}{d\xi} + \Gamma_{\alpha\beta}^k r^{\alpha} r^{\beta} - r^k \tanh \psi \frac{d\psi}{d\xi} \right) \quad (2.1b)$$

On the other hand, it is possible to sometimes express  $r^k$  independently of  $\xi$ , in which case  $R_{\xi}^k$  vanish. Such is, for example, always the case in the rectilinear motion, as it has been shown in [2], Sec IV.

In eq.'s (2.1), some terms depend explicitly on  $r^k$  or their derivatives, whereas some other appear in the sum  $\Gamma_{\alpha\beta}^k r^{\alpha} r^{\beta}$ , with at least one of the indices  $\alpha$  and  $\beta$  not equal to  $k$ . After separating both contributions, eq. (2.1a) can be rewritten as

$$\frac{dp^k}{dt} = \frac{c^2 m}{\cosh^2 \psi} \left[ \frac{dr^k}{d\xi} + (T^{\alpha} - \tanh \psi \frac{d\psi}{d\xi}) r^{\alpha} \right] + \frac{c^2 m}{\cosh^2 \psi} K_i^k r^i \Big|_{i \neq k} \quad (2.2)$$

where the index  $x$  has the same meaning as  $k$ , except not being subject to the summation rule.

The ensemble of equations (2.2) with the index  $k$  running through the (ordinary space) values 1, 2, 3, can be written more concisely in the form:

$$\frac{d\vec{p}}{dt} = \frac{c^2 m}{\cosh^2 \psi} (\vec{M} + \vec{r}_x \vec{K}) \quad (2.3)$$

$$g_{kk} K_j^k = \frac{1}{2} \left[ \frac{\partial g_{kk}}{\partial x^j} \frac{dx^k}{d\xi} - \frac{\partial g_{jj}}{\partial x^k} \frac{dx^j}{d\xi} \right] = (\text{curl } \vec{A})_i \quad (2.9a)$$

$$g_{jj} K_k^j = \frac{1}{2} \left[ \frac{\partial g_{jj}}{\partial x^k} \frac{dx^j}{d\xi} - \frac{\partial g_{kk}}{\partial x^j} \frac{dx^k}{d\xi} \right] = -(\text{curl } \vec{A})_i \quad (2.9b)$$

which represents the general motion equation for a particle (charged or otherwise) driven by either electromagnetic or mechanical force in the relativistic space.

To prove this important statement, the motion will be considered, for simplicity, to be confined to a plane defined by the orthogonal coordinates  $x^k, x^j$ . Using the well known relationship between the Christoffel symbols and the metric tensor  $g_{ik}$  (cf. [3], [4]), the term  $\Gamma_{\alpha\beta}^k r^\alpha r^\beta$  can be expanded as follows (the repetition of the indices  $j, k$  - which stand for particular values of  $\alpha, \beta$  - does not imply summation):

$$\Gamma_{\alpha\beta}^k r^\alpha r^\beta = \frac{1}{2} g^{kk} \left[ \frac{\partial g_{kk}}{\partial x^k} r^k + \frac{\partial g_{kk}}{\partial x^j} r^j \right] r^k + \frac{1}{2} g^{kk} \left[ \frac{\partial g_{kk}}{\partial x^j} r^k - \frac{\partial g_{jj}}{\partial x^k} r^j \right] r^j \quad (2.4)$$

The first term between brackets in (2.4) can be interpreted as a contravariant vector component:

$$T^k = \frac{dg_{kk}}{d\xi} r^k \quad (2.5)$$

Furthermore, let us define for convenience a mixed 2<sup>nd</sup> order tensor  $K_\alpha^\mu$ :

$$K_\alpha^\mu = \frac{1}{2} g^{\mu\mu} \left[ \frac{\partial g_{\mu\mu}}{\partial x^\alpha} r^\mu - \frac{\partial g_{\alpha\alpha}}{\partial x^\mu} r^\alpha \right] \quad \mu, \alpha = j, k \quad (2.6)$$

Then, (2.3) follows with

$$M^k = \frac{dr^k}{d\xi} + \left[ T^k - \tanh \psi \frac{d\psi}{d\xi} \right] r^k \quad (2.7)$$

and  $K_\alpha^\mu$  given by (2.6). Note that  $K_\alpha^\alpha = 0$ .

On the other hand, in the original

(orthogonal) reference system,  $g_{kk} = 1/g^{kk}$ . Therefore, if we consider

$$A_\alpha = \frac{1}{2} g_{\alpha\alpha} \frac{dx^\alpha}{d\xi}; \quad \alpha = j, k \quad (2.8)$$

as the components of a two-dimensional vector  $A$ , equation (2.6) gives:

In both equations (2.9), index  $i$  shows the direction orthogonal to the  $(x^k, x^j)$  plane when taking as positive the sequence  $(i, j, k)$ . Therefore, after generalizing upon the 3-dimensional euclidean space, the terms  $S_{kj} = g_{kk} K_j^k$  define a 2<sup>nd</sup> order, covariant, antisymmetric tensor which can also be interpreted [5] as the component of an axial vector  $A$ .<sup>1</sup>

According to the classical expression of Lorentz force [7], equation (2.3) can be used to describe the motion of the charged particle driven by the electromagnetic field. In effect, using for convenience Gauss System units, the equation in question can be rewritten as:

$$\frac{d\vec{p}}{dt} = q\vec{E} + \frac{q}{c} \vec{v} \times \vec{H} = q\vec{E} + \frac{q}{\cosh \psi} \vec{r} \times \vec{H} \quad (2.10)$$

with

$$E_k = \frac{c^2 m}{q \cosh^2 \psi} g_{kk} M^k \quad (2.11a)$$

$$H_i = \frac{c^2 m}{q \cosh \psi} g_{kk} K_j^k = \frac{c^2 m}{q \cosh \psi} (\text{curl } \vec{A})_i \quad (2.11b)$$

which can be easily verified using (2.2).

Vector  $\vec{A}$  is clearly the electromagnetic vector potential.  $E_k$  and  $H_i$  are the

covariant components of the electric and magnetic fields. The corresponding metric (the usual) components are derived by multiplying both equations (2.11) by  $g^{kk}$  [5].

For example, in central motion, the equations in polar coordinates giving the radial and normal acceleration {cf. [2], eq.'s (5.7)} are written as follows:

$$a_\rho = \frac{c^2}{\cosh^2 \psi} \left[ \frac{d^2 \rho}{d\xi^2} - \rho \left( \frac{d\eta}{d\xi} \right)^2 - \frac{d\rho}{d\xi} \frac{d\psi}{d\xi} \tanh \psi \right] \quad (2.12a)$$

$$a_\eta = \frac{c^2}{\cosh^2 \psi} \left[ \rho \frac{d^2 \eta}{d\xi^2} + 2 \frac{d\rho}{d\xi} \frac{d\eta}{d\xi} - \rho \frac{d\eta}{d\xi} \frac{d\psi}{d\xi} \tanh \psi \right] \quad (2.12b)$$

from which, after some trivial reorganization, the following components of the electric field are found:

$$E_\rho = \frac{mc^2}{q \cosh^2 \psi} \left[ \frac{d^2 \rho}{d\xi^2} - \frac{d\rho}{d\xi} \frac{d\psi}{d\xi} \tanh \psi \right] \quad (2.13a)$$

$$E_\eta = \frac{mc^2}{q \cosh^2 \psi} \left[ \rho \frac{d^2 \eta}{d\xi^2} - \left( \rho \frac{d\eta}{d\xi} \tanh \psi - \frac{d\rho}{d\xi} \frac{d\eta}{d\xi} \right) \right] \quad (2.13b)$$

Likewise, the only (metric) component of the magnetic field can be derived from the definition of the curl in cylindrical coordinates (cf. [5]), namely:

$$\begin{aligned} (\vec{r} \times \vec{H})_\rho &= r_\eta H_z = -\frac{mc^2}{q \cosh^2 \psi} \rho \left( \frac{d\eta}{d\xi} \right)^2 \\ (\vec{r} \times \vec{H})_\eta &= -r_\rho H_z = \frac{mc^2}{q \cosh^2 \psi} \frac{d\rho}{d\xi} \frac{d\eta}{d\xi} \end{aligned} \quad (2.14)$$

or, using the ordinary velocity components

$$v_\alpha = \frac{c}{\cosh \psi} r_\alpha \text{ instead:}$$

$$\begin{aligned} v_\eta H_z &= -\frac{mc}{q} \rho \left( \frac{d\eta}{dt} \right)^2 \\ -v_\rho H_z &= \frac{mc}{q} \frac{d\rho}{dt} \frac{d\eta}{dt} \end{aligned} \quad (2.15)$$

Equations (2.15) lead to the vector diagram of Fig.1. Because  $v_\rho = d\rho/dt$ ,  $v_\eta = \rho d\eta/dt$ , we get:

$$H_z = -\frac{cm\omega_c}{q} \quad (2.16)$$

which is the well known relationship between the magnetic field and the cyclotron frequency  $\omega_c = d\eta/dt$  in Gauss System units.

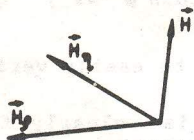


Fig.1. Vector diagram of magnetic field in cylindrical reference system.

The foregoing analysis is clearly valid for the central motion originated by a gravitational force. In particular, eq.'s (2.13) and (2.16) define what will be provisionally called "gravitational" and "inertial" fields, respectively. Both denominations and their meaning will be commented in more detail farther on. For the moment it is sufficient to say that, according to the way the equations of motion have been derived, the description of the gravitational force can be found from that of the electric and magnetic fields, respectively, by substituting  $m$  for  $q$  and conveniently re-scaling the units. The previous analysis also shows a strict analogy between the electrical and

mechanical phenomena, which is a remarkable property of Nature, not sufficiently explored yet.

### III. TANGENT AND CROSS-DIRECTIONAL FORCES

By analogy with the ordinary velocity  $v$ , all 3-space components  $r^k$  of the (normalized) relativistic velocity  $\vec{r}$ , defined in (1.1), can be assembled into the vector

$$\vec{r} = \vec{r}_\rho + \vec{\omega} \times \vec{\rho} = \vec{r}_\rho + \vec{r}_\eta \quad (3.1)$$

Evidently,  $\vec{r}$  is a vector directed along the tangent to the projection of the trajectory onto the normal 3-space. If we define the unit vectors  $\vec{u}_\rho$ ,  $\vec{u}_\eta$  along the radius vector and the tangent to the trajectory, respectively, as depicted in Fig. 2, the several vectors entering equation (3.1) are defined as follows:

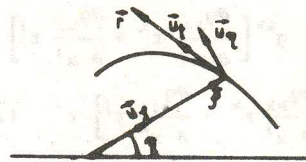


fig.2. Vector  $\vec{r}$  and its radial and normal components.

$$\begin{aligned} \vec{r}_\rho &= \frac{d\rho}{d\xi} \vec{u}_\rho \\ \vec{\omega} &= \frac{d\eta}{d\xi} \vec{u}_\rho \times \vec{u}_\eta = \frac{d\eta}{d\xi} \vec{u}_z \end{aligned} \quad (3.2)$$

$$\vec{r}_\eta = \vec{\omega} \times \vec{\rho} = \rho \frac{d\eta}{d\xi} \vec{u}_\eta$$

Here  $\vec{u}_\eta$ ,  $\vec{u}_z$  are the usual unitary vectors and the triad  $(\vec{u}_\rho, \vec{u}_\eta, \vec{u}_z)$  defines a mutually orthogonal, right-hand oriented basis-vector system.

Collecting the results of the previous sections the following relationships can be established:

$$\vec{r} = \frac{d\rho}{d\xi} \vec{u}_\rho + \rho \frac{d\eta}{d\xi} \vec{u}_\eta \quad (3.3a)$$

$$\vec{H} = -\frac{c^2 m}{q \cosh^2 \psi} \frac{d\eta}{d\xi} \vec{u}_z \quad (3.3b)$$

$$\vec{r} \times \vec{H} = \frac{c^2 m}{q \cosh^2 \psi} \left[ -\rho \frac{d\eta}{d\xi} \vec{u}_\rho + \frac{d\rho}{d\xi} \vec{u}_\eta \right] \frac{d\eta}{d\xi} \quad (3.3c)$$

On the other hand, the last term in (2.10) has the dimensions of a force and can be expressed as:

$$\vec{F}_{ir} = - \frac{mc^2}{\cosh^2 \psi} \vec{r} \times \vec{\omega} = - m \vec{v} \times \vec{\omega}_c \quad (3.4)$$

For a charged particle,  $\vec{F}_{ir}$  is due to the magnetic field. In mechanics, the term in question accounts for the well known centrifugal force.

The remaining terms in the right-hand member in eq. (1.9) can be interpreted as components of a tangential force. Their ensemble is readily written in vector form as

$$\vec{F}_{tr} = \frac{mc^2}{\cosh \psi} \frac{d}{d\xi} \left[ \frac{\vec{r}}{\cosh \psi} \right] = m \frac{d\vec{v}}{dt} \quad (3.5)$$

System (3.4), (3.5) can again be glued together into a single vector equation:

$$\frac{d\vec{p}}{dt} = \frac{mc^2}{\cosh \psi} \left[ \frac{d}{d\xi} \left( \frac{\vec{r}}{\cosh \psi} \right) - \frac{1}{\cosh \psi} \vec{r} \times \vec{\omega} \right] = m \left( \frac{d\vec{v}}{dt} - \vec{v} \times \vec{\omega}_c \right) \quad (3.6)$$

with  $\vec{r}$  given in (3.3a) and

$$\vec{v} = \frac{c}{\cosh \psi} \vec{r} = \frac{c}{\cosh \psi} \left( \frac{d\vec{\rho}}{d\xi} + \vec{\omega} \times \vec{\rho} \right) \quad (3.7)$$

Equations (3.6) and (3.7) fully describe a planar motion in a formally invariant form under relativistic transformation.

In particular, the factor  $mc^2$  represents the (constant) total energy of the moving body;  $\vec{r}$ ,  $\vec{\omega}$  and  $\xi$  being also invariant quantities.

#### IV. RELATIONSHIP BETWEEN THE RELATIVISTIC DYNAMICS AND MAXWELL EQUATIONS

IV-1. 1<sup>st</sup> pair of Maxwell equations.- Relativistic dynamic equation (3.6) has

been derived under the assumption  $R_{\psi}^k = 0$ .

Consequently, the equations of electric and magnetic fields derived in Sec. II should comply with the Maxwell equations of the electromagnetic field, quoted for an easy reference in APPENDIX B [cf. (B.1), (B.2) and (B.7)]. To verify that this is true we proceed as follows. First, refer to equation (A.9), APPENDIX A, which states:

$$\text{div } \vec{\omega} = 0 \quad (4.1a)$$

and, from the definition of  $\vec{\omega}$  and (3.3b), it is found:

$$\text{div } \vec{H} = 0 \quad (4.1b)$$

which directly gives the 2<sup>nd</sup> Maxwell equation [cf. (B.1b)]. Then, according to (A.7), APPENDIX A:

$$\vec{\omega}_c = \text{curl}(\vec{\omega}_c \times \vec{\rho}) \quad (4.2)$$

If we suppose that the driving force is electromagnetic in nature, eq. (2.10) gives:

$$\frac{d\vec{p}}{dt} = q\vec{E} + \frac{q}{c} \vec{v} \times \vec{H}$$

where, from (3.5) and (2.16),

$$\vec{E} = \frac{\vec{F}_{tr}}{q} = \frac{m}{q} \frac{d\vec{v}}{dt} \quad (4.3a)$$

$$\vec{H} = - \frac{cm \vec{\omega}_c}{q} = - \frac{cm}{q} \text{curl}(\vec{\omega}_c \times \vec{\rho}) \quad (4.3b)$$

Furthermore, according to (3.7), and because  $\text{curl} \left( \frac{d\vec{\rho}}{d\xi} \right) = \frac{d}{d\xi} (\text{curl } \vec{\rho}) = 0$ , we have:

$$\vec{H} = - \frac{cm}{q} \text{curl } \vec{v} \quad (4.4)$$

The potential vector  $\vec{A}$  can be identified as

$$\vec{A} = - \frac{cm}{q} \vec{v} \quad (4.5)$$

then, by applying curl to both sides of (4.3a), it follows

$$\text{curl } \vec{E} = \frac{m}{q} \text{curl} \left( \frac{d\vec{v}}{dt} \right) = \frac{m}{q} \frac{d}{dt} (\text{curl } \vec{v}) = - \frac{1}{c} \frac{d\vec{H}}{dt} \quad (4.6)$$

In (4.6), the derivative with respect to  $t$  is supposed to be taken at a fixed point of space. If we allow vector  $(x, y, z)$  to vary, the total derivatives must be replaced by the partial ones, which, according to (B.1b), gives the 1<sup>st</sup> Maxwell equation, as required.

IV-2. Derivation and consequences of the 3<sup>rd</sup> Maxwell equation.- Starting with eq. (A.14) in APPENDIX A, one can write, using  $dt$  and  $\vec{\omega}_c$  instead of  $d\xi$  and  $\vec{\omega}$ , respectively:

$$\begin{aligned} \text{curl } \vec{\omega}_c &= \frac{\partial \omega_c}{\partial y} \vec{i} - \frac{\partial \omega_c}{\partial x} \vec{j} = \\ &= \frac{1}{a_{\rho}^4} \left[ y (y v_x - x v_y) - x (x v_x + y v_y) \right] \vec{i} + \\ &+ \frac{1}{a_{\rho}^4} \left[ x (x v_y - y v_x) - y (x v_x + y v_y) \right] \vec{j} \end{aligned} \quad (4.7)$$

where  $\omega_c = \frac{d\eta}{dt}$ ,  $v_x = \frac{dx}{dt}$ , etc. On the other hand, from (A.12) we have:

$$\omega_c = \frac{1}{a_{\rho}^2} (-y v_x + x v_y)$$

and because

$$x v_x + y v_y = \frac{1}{2} \frac{da_{\rho}^2}{dt} = a_{\rho} v_{\rho}$$

with  $v_{\rho} = da_{\rho}/dt$ , equation (4.7) is readily rewritten as

$$\text{curl } \vec{\omega}_c = - \frac{\vec{\omega}_c \times \vec{\rho}}{a_{\rho}^2} - \frac{1}{a_{\rho}^3} v_{\rho} \vec{\rho} \quad (4.8a)$$

Let  $\vec{v}_\rho = v_\rho \vec{u}_\rho$  be a vector directed along  $\vec{\rho}$ -axis and  $v_\rho$  its amplitude. We have

$$\frac{\vec{\omega}_c \times \vec{\rho}}{a_\rho^2} = \frac{\vec{v} - \vec{v}_\rho}{a_\rho^2}$$

and  $v_\rho \vec{\rho} = a_\rho \vec{v}_\rho$ . Then, by (4.8a),

$$\text{curl } \vec{\omega}_c = - \frac{\vec{v}}{a_\rho^2} \quad (4.8b)$$

an important equation which, when combined with (4.3b), gives:

$$\text{curl } \vec{H} = \frac{cm}{qa_\rho^2} \vec{v} \quad (4.9)$$

To complete the 3<sup>rd</sup> Maxwell equation [cf. (B.2a), APPENDIX B], eq.'s (2.13) shall be expressed conveniently in the form:

$$E_\rho = \frac{mc^2}{q \cosh \psi} \frac{d}{d\xi} \left[ \frac{r_\rho}{\cosh \psi} \right] = \frac{m}{q} \frac{\partial v_\rho}{\partial t} \quad (4.10a)$$

$$E_\eta = \frac{mc^2}{q \cosh \psi} \frac{d}{d\xi} \left[ \frac{r_\eta}{\cosh \psi} \right] = \frac{m}{q} \frac{\partial v_\eta}{\partial t} \quad (4.10b)$$

with  $v_\rho = da_\rho/dt$ ,  $v_\eta = a_\rho d\eta/dt$ . Furthermore, the partial derivatives have been used in the right-hand side of (4.10) because  $v_\rho$  and  $v_\eta$  are supposed to be both time and space dependent functions. Also, by applying the continuity equation to both components of  $\vec{v}$  we get:

$$\begin{aligned} \frac{\partial v_\alpha}{\partial t} &= - \text{div}(v_\alpha \vec{v}) = \\ &= \vec{v} \cdot \text{grad } v_\alpha - v_\alpha \text{div } \vec{v} \end{aligned} \quad (4.11)$$

with  $\alpha = \rho, \eta$ .

To compute (4.11) we observe that, because  $v_\alpha$  are referred to a given space point, their gradients should be disregarded. On the other hand,

$$\text{div } \vec{v} = \text{div}(\vec{v}_\rho + \vec{\omega}_c \times \vec{\rho}) = \text{div}(\vec{\omega}_c \times \vec{\rho})$$

By using a well known vector identity we also have:

$$\text{div}(\vec{\omega}_c \times \vec{\rho}) = \vec{\rho} \cdot \text{curl } \vec{\omega}_c - \vec{\omega}_c \cdot \text{curl } \vec{\rho}$$

Then, because  $\text{curl } \vec{\rho} = 0$ , it is found from (4.12) and (4.8b):

$$\text{div } \vec{v} = - \frac{\vec{\rho} \cdot \vec{v}}{a_\rho^2} = - \frac{v_\rho}{a_\rho} \quad (4.13)$$

With all that in mind, equation (4.11) can be written in vector form as

$$\frac{\partial \vec{v}}{\partial t} = \frac{v_\rho}{a_\rho} \vec{v} \quad (4.14)$$

and, according to (4.10),

$$\vec{E} = \frac{mv_\rho}{qa_\rho} \vec{v} \quad (4.15)$$

By taking the (partial) time derivative of  $\vec{E}$  given in (4.15) we get:

$$\frac{\partial \vec{E}}{\partial t} = \frac{m}{q} \left[ \vec{v} \frac{\partial}{\partial t} \left( \frac{v_\rho}{a_\rho} \right) + \frac{v_\rho}{a_\rho} \frac{\partial \vec{v}}{\partial t} \right]$$

and using (4.14),

$$\frac{\partial \vec{E}}{\partial t} = \frac{m\vec{v}}{q} \left[ \frac{v_\rho}{a_\rho^2} + \frac{\partial}{\partial t} \left( \frac{v_\rho}{a_\rho} \right) \right] = \frac{m}{q} \frac{\dot{v}_\rho}{a_\rho} \vec{v} \quad (4.16)$$

where  $\dot{v}_\rho = \frac{\partial v_\rho}{\partial t}$  is interpreted as the amplitude of the radial acceleration.

Combining all the previous results, the 3<sup>rd</sup> Maxwell equation can be written as follows:

$$\frac{cm\vec{v}}{qa_\rho^2} = \frac{1}{c} \left( \frac{m}{qa_\rho} \dot{v}_\rho \vec{v} + 4\pi\vec{J} \right) \quad (4.17)$$

Accordingly, the density of the current is given by the equation

$$\vec{J} = \frac{mc^2 \vec{v}}{4\pi qa_\rho^2} \left( 1 - \frac{a_\rho v_\rho}{c^2} \right) \quad (4.18)$$

The last term in (4.18) is usually negligible (it clearly vanishes in the classical approximation which implies  $c \rightarrow \infty$ ). On the other hand, a violent radial acceleration (for example originated by the impact of a sufficiently energetic photon upon an electron) may cancel  $\vec{J}$  altogether, with the consequence already considered in Sec. I (see also [2], Sec. V-4).

**IV-3. Discussion of the 4<sup>th</sup> Maxwell equation; field induced charges.**— The spatial distribution of charges induced by the motion of a (charged) particle can be derived from the 4<sup>th</sup> Maxwell equation [cf. eq. (B.2b) APPENDIX B], (4.15) and (4.13), namely:

$$\rho_q = \frac{1}{4\pi} \text{div } \vec{E} = - \frac{mv_\rho^2}{4\pi qa_\rho^2} \quad (4.19)$$

To see that (4.19) is consistent with (4.18) it has to be noticed that  $\rho_q$  in the former represents the density of the charges induced by the electric field, which are originated, in their turn, by the motion of a particle endowed with the mass  $m$  and a (primary) charge  $q$ . The velocity of  $\rho_q$  may clearly be different from  $\vec{v}$ . Call it  $\vec{v}_q, \vec{J}_q$  being the current density induced by  $\rho_q$ . To distinguish the "primary" current, due to charge  $q$ , from  $\vec{J}_q$ , the former will be relabelled  $\vec{J}_a$ . It will be supposed that whenever  $v_\rho = 0$ ,  $\dot{v}_\rho$  also vanishes. Then,  $\vec{J}_q$  is given by the first term of (4.18), namely:

$$\vec{J}_q = \frac{mc^2 \vec{v}}{4\pi qa_\rho^2} \quad (4.20)$$

Also, because  $\vec{J} = \vec{J}_a + \vec{J}_q$ , we find:

$$\vec{J}_\rho = - \frac{m\dot{v}_\rho \vec{v}}{4\pi qa_\rho} \quad (4.21a)$$

To express  $\vec{J}_\rho$  in a form analogous to  $\vec{J}_q$  (only retaining the difference in the sign), we write:

$$\vec{J}_\rho = - \frac{mc^2 \vec{v}_\rho}{4\pi qa^2} \quad (4.21b)$$

and, by comparing both equations (4.21), it is found that

$$\vec{v}_\rho = \frac{\dot{\rho} a}{c^2} \vec{v} \quad (4.22)$$

The last equation shows that the speed of the "secondary" charges is proportional to the radial acceleration and, accordingly, with the concourse of an extreme acceleration, it becomes quite large. In any case, such situations - as it has been mentioned earlier - are beyond the scope of the present discussion. In what follows, some attention will be paid, instead, to the "classical relativistic" aspect of the motion and the (suspected) nature of the gravitational field, or the matter, in general.

#### V. GEOMETRIC INTERPRETATION OF THE MASS AND GRAVITATIONAL FIELD.

In [2], Sec. III, the curvature of the space has been interpreted as a distortion of the quiescent (rectilinear) path in E-4D, when viewed by an object accelerating under the effect of a driving force, such as the one due to the gravitational field. To shed some - intuitively more comprehensible - light on this question, consider the gravitational attraction in some more detail.

As everybody knows from the very first lessons on Classical Mechanics, two bodies of vanishing small sizes, with the masses, say,  $m_1$  and  $m_2$ , undergo a mutual attraction given by the inverse square law:

$$F = K \frac{m_1 m_2}{r^2} \quad (5.1)$$

where  $r$  is the distance between the bodies in question and  $K$  the so-called Universal Gravitational Constant. The value of  $K$  in SI (International System) units is approximately

$$K = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad (5.2a)$$

The most cursory look on (5.2a) reveals rather complex dimensions associated with the numerical value of  $K$ . Besides, the dimensions in question depend on the choice of a particular unit system. For example, Einstein used, instead, in his field equations (cf. [2], (1.7)) the constant:

$$\mathcal{K} = 4\pi Kc^{-2} = 1.86 \times 10^{-27} \text{ cm.g}^{-1} \quad (5.2b)$$

On the other hand, as it has been previously pointed out (cf. Sec. I), the mass  $m_i$  of the particle  $i$  is proportional to its energy  $\mathcal{E}_i$ , according to the law

$m_i = \mathcal{E}_i/c^2$ . Call  $p_i^0 = cm_i = \mathcal{E}_i/c$  the relativistic transform-invariant quantity, with the dimensions of momentum and, thus, liable to be interpreted as the total or scalar momentum of the particle.<sup>2</sup> Then, eq. (5.1) can be written as

$$F = K c^{-2} r^{-2} p_1^0 p_2^0 \quad (5.3a)$$

As it has been also pointed out in the referenced literature, eq. (5.3a) becomes very simple when using a unit system such that  $K = c^2$ , giving:

$$F = \frac{p_1^0 p_2^0}{r^2} \quad (5.3b)$$

It can be observed that, in such a system, the mass no longer retains its independent units and becomes expressed in the units of length. To be specific, 1 g will amount  $7.407 \times 10^{-28}$  cm (an extremely small quantity!).

The previous result can be thought of as a mere curiosity (and as such it has been presented in the mentioned reference). Alternatively, it can be given a definite physical meaning, in line with the Einstein's intuition about the relationship between the gravitational field and the space curvature. The essence of what is being said is illustrated in Fig. 3, where the two-dimensional relativistic space-time is pictured, the "normal" space being constrained for simplicity to the x-axis.

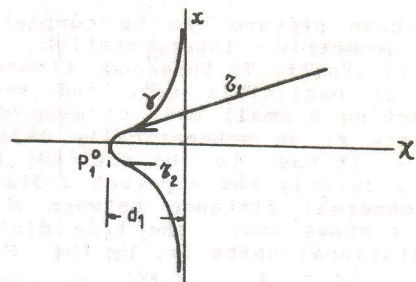


Fig.3. Space curvature originated by a mass characterized by the scalar momentum  $P^0$ .

Consistently with the foregoing discussion, a mass  $m_1$  can be associated with the scalar momentum

$$p_1^0 = \frac{\mathcal{E}_1}{c} = d_1 c$$

where, with the unit system at hand - which from now on will be called gravitational units -  $d_1$  represents the "length" of the mass "projected" into the "backward space" (in a sense, into the past), numerically (and dimensionally) equal to  $m_1$ <sup>3</sup>.

In Fig. 3, x-axis marks also the border of the free space, whereas the contour  $\gamma$  delimits the space distorted by the gravitational field. The contour in question shows also the path followed by a beam of light (or a gamma ray) emitted by

$p_1^0$ .  $\tau_1$  is the trajectory followed by a (secondary) particle leaving  $p_1^0$  with the initial speed larger than the escape velocity. If the particle is emitted with a speed smaller than the former, the particle will be "trapped" and will follow (forward and back) a trajectory such as  $\tau_2$ .

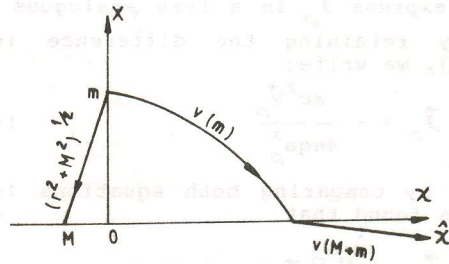


Fig.4. Graphical interpretation of the law of gravitational attraction in the relativistic space.

The foregoing interpretation leads to the conclusion that the energy "frozen out" as a mass or the "scalar p" can be interpreted as a small region of space collapsed into the past. As such, it is responsible, for example, for the time-lag associated with the gravity, foreseen by Einstein and eventually leading to the observed red shift of the light emitted by large bodies (such as stars and, at a more overwhelming rate, by quasars [8]). Distance  $d_1$  in Fig. 3 can thus be the measure of the specific gravity (depth of the "space crevice" in a given point of space). For  $d_1$  sufficiently large, even the most energetic particles may not overcome the gravity created by the body, a situation that is currently denoted as **black hole**. Consequently with what is said, there can be no such a dramatic distance as the Schwarzschild gravity radius, and the quasars may be nearer our galaxy than it is currently believed.

The above picture can be completed with the geometric interpretation of the gravity itself. To this end, figure out a body of negligible size and mass  $M$ , attracting a small body of mass  $m$  from a distance  $r$ , as schematically depicted in Fig. 4. It has to be noticed, however, that  $r$  is only the apparent 3-dimensional (in general) distance between  $M$  and  $m$ . Fig. 4 shows that the true distance in gravitational units is, by the Pithagorean theorem  $(r^2 + M^2)^{1/2}$ . It means that the inverse square law (5.1) takes the form:

$$F = \frac{c^2 Mm}{r^2 + M^2} \quad (5.4a)$$

(the negative sign showing the shortening of the distance has been omitted for simplicity). Observe that, for  $r = 0$ ,  $F = c^2 m/M < \infty$ , which rules out the difficulties related to the attraction between two point-like bodies. Besides, the potential energy  $U$  (absolute value) of  $m$  turns out to be

$$U = \frac{c^2 Mm}{(r^2 + M^2)^{1/2}} \quad (5.4b)$$

and, for  $r \rightarrow 0$ ,  $U \rightarrow U_0 = c^2 m$ , that is,  $U_0$  becomes finite and exactly equal to the total energy of the attracted particle.

Again, with reference to Fig. 4, the attraction of  $m$  by  $M$  compels the former

to "slide" down the slope  $M/(r^2 + M^2)^{1/2}$  (with respect to the undisturbed space) towards the latter. The observable motion of  $m$  is performed in the right-hand side of the (relativistic) space  $\chi - x$ , along the trajectory shown (qualitatively) by the line  $v(m)$ . After the capture of  $m$  by  $M$  (the collision is supposed perfectly inelastic), both masses "glue" together and begin to move with the speed dictated by the laws of the ordinary Physics (unless Quantum Mechanic effects take place, as suggested earlier) along the straight line  $v(M+m)$ .

## VII. CONCLUSION

The whole philosophy underlying this series of articles concerning the Special and General Relativity is that the origin of all the forces and motion, that is, the source of the **Primary Energy** is the expansion of the Universe, which, in its turn, claims for a more general space for expanding into. This must be a **hyperspace**, or a sort of a field, or a superposition of such. One of the fields that shapes the hyperspace in question may be electromagnetic in nature. On the other hand, it is the matter that is responsible for the "ruggedness" of the space-time (continuum ?).

There still remain a wealth, a Universe! of questions to be answered. Nevertheless, some details are already shaping themselves into a picture quite comprehensible to the human mind. For example, the whole story may have begun as follows.

A huge needle, a gargantuan impulse of energy (a quantum of action in the briefest of times) has been injected into a certain configuration of fields (some of them are known today, some probably not). Can we think of it as if a single line of force had been suddenly (suddenly indeed! snapped off, with the result resembling the one provoked by a stone hurled into a quiet surface of an immense lake? Somebody called this event **Big Bang**. Then, a (shock) wave started expanding at a constant speed  $c$ . Somebody called it the **Universe** we have been granted to make our living in. Then, begun the equations. Or did they begin before?



I am very grateful to all those who have helped me in this work with their encouragement and advices and, in particular, to my wife Aida, my son Wsewolod, as well as to my colleagues and fellows at the Universidad Politécnica de Madrid. Also and very especially to the Escuela Politécnica Nacional in Quito, which honoured me with the favourable reception of this series of contributions. And here are the results, may be somewhat unusual but apparently in agreement with the known facts.

#### APPENDIX A. COMPUTATION OF $\text{curl}(\vec{\omega} \times \vec{\rho})$

The subject discussed here has already been extensively treated in the literature [4], [5]. However, because of its paramount importance for the main purpose of this paper and of the curious circumstance that the result that will be derived does not fully agree with the one currently quoted, the relevant steps in the computations concerning the question at hand will be outlined.

Let  $\vec{\rho}(x,y)$  be the radius-vector of the point  $P(x,y)$  in the plane  $x-y$ , forming angle  $\eta$  with the  $x$ -axis. The orthogonal projections of  $\vec{\rho}$  on both axes are:

$$x = a_\rho \cos \eta ; y = a_\rho \sin \eta \quad (\text{A.1})$$

where  $a_\rho$  stands for "amplitude of  $\vec{\rho}$ ":

$$a_\rho = \|\vec{\rho}\| = (x^2 + y^2)^{1/2} \quad (\text{A.2})$$

It will be assumed that  $a_\rho = a_\rho(\xi)$ ,  $\eta = \eta(\xi)$  are functions of a parameter  $\xi$ . Differentiating (A.1) we get:

$$\begin{aligned} dx &= \cos \eta da_\rho - a_\rho \sin \eta d\eta \\ dy &= \sin \eta da_\rho + a_\rho \cos \eta d\eta \end{aligned} \quad (\text{A.3})$$

System (A.3), when solved for  $da_\rho$  and  $d\eta$ , gives:

$$\begin{aligned} da_\rho &= \cos \eta dx + \sin \eta dy \\ d\eta &= \frac{1}{a_\rho} (-\sin \eta dx + \cos \eta dy) \end{aligned} \quad (\text{A.4})$$

Let also define a vector directed along  $z$ -axis:

$$\begin{aligned} \vec{\omega} &= \omega_z \vec{k} = \frac{d\eta}{d\xi} \vec{k} = \\ &= \frac{1}{a_\rho} (-\sin \eta \frac{dx}{d\xi} + \cos \eta \frac{dy}{d\xi}) \vec{k} \end{aligned} \quad (\text{A.5})$$

$(\vec{i}, \vec{j}, \vec{k})$  forming the basic orthogonal reference system.

Our aim is to find the value of  $\text{curl}(\vec{\omega} \times \vec{\rho})$ . A well known vector identity gives:

$$\text{curl}(\vec{\omega} \times \vec{\rho}) = \vec{\omega} \text{div} \vec{\rho} - \vec{\rho} \text{div} \vec{\omega} + \frac{d\vec{\omega}}{d\rho} - \frac{d\vec{\rho}}{d\omega} \quad (\text{A.6})$$

It is clear that

$$\text{div} \vec{\rho} = 3 \quad (\text{A.7})$$

regardless whether one or more components of

$$\vec{\rho} = x\vec{i} + y\vec{j} + z\vec{k} \quad (\text{A.8})$$

are zero. Furthermore, because the only component of  $\omega$  is  $\omega_z$ , independent of  $z$ , we have:

$$\text{div} \vec{\omega} = 0 \quad (\text{A.9})$$

On the other hand, another general vector identity

$$\frac{d\vec{a}}{db} = (\vec{b} \cdot \nabla) \vec{a} = \frac{\partial \vec{a}}{\partial x} b_x + \frac{\partial \vec{a}}{\partial y} b_y + \frac{\partial \vec{a}}{\partial z} b_z \quad (\text{A.10})$$

gives:

$$\frac{d\vec{\rho}}{d\omega} = \omega_z \frac{\partial \vec{\rho}}{\partial z} = \vec{\omega} \quad (\text{A.11})$$

Using (A.5) and (A.1) we also have:

$$\omega_z = \frac{1}{a_\rho} (-y \frac{dx}{d\xi} + x \frac{dy}{d\xi}) = \frac{1}{a_\rho} (-y r_x + x r_y) \quad (\text{A.12})$$

with

$$r_x = \frac{dx}{d\xi} ; r_y = \frac{dy}{d\xi} \quad (\text{A.13})$$

From (A.12) it can be inferred that  $\omega_z$  is a function of 4 independent variables:  $x$ ,  $y$ ,  $r_x$ ,  $r_y$ . Taking partial derivatives of  $\omega_z$  with respect to  $x$ ,  $y$ , it follows:

$$\frac{\partial \omega_z}{\partial x} = \frac{1}{a_\rho} \left[ 2xyr_x - (x^2 - y^2) r_y \right] \quad (\text{A.14})$$

$$\frac{\partial \omega_z}{\partial y} = \frac{1}{a_\rho} \left[ (y^2 - x^2) r_x - 2xyr_y \right]$$

Also

$$\frac{d\vec{\omega}}{d\rho} = (x \frac{\partial \omega_z}{\partial x} + y \frac{\partial \omega_z}{\partial y}) \vec{k} \quad (\text{A.15})$$

and using (A.14) we get, after a slight rearrangement:

$$\frac{d\vec{\omega}}{d\rho} = \frac{yr_x - xr_y}{a_\rho^2} \vec{k} = -\vec{\omega} \quad (\text{A.16})$$

The substitution of (A.7), (A.9), (A.11) and (A.16) into (A.6) gives:

$$\text{curl}(\vec{\omega} \times \vec{\rho}) = \vec{\omega} \quad (\text{A.17})$$

which is the result that has been looked for (not  $2\vec{\omega}$  in the second member, as it appears in the quoted literature).

#### APPENDIX B. BASIC RELATIONSHIPS OF ELECTRODYNAMICS.

For an easy reference, the Maxwell equations in Gauss System units are quoted and the derivation of general equations of electromagnetic field is outlined.

1<sup>st</sup> pair of Maxwell equations. -

$$\text{curl } \vec{E} = - \frac{1}{c} \frac{\partial \vec{H}}{\partial t} \quad (\text{B.1a})$$

$$\text{div } \vec{H} = 0 \quad (\text{B.1b})$$

2<sup>nd</sup> pair of Maxwell equations. -

$$\text{curl } \vec{H} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{J} \quad (\text{B.2a})$$

$$\text{div } \vec{E} = 4\pi \rho_q \quad (\text{B.2b})$$

$\vec{E}$ ,  $\vec{H}$ ,  $\vec{J}$  are the usual symbols (vector-valued quantities) that denote the electric and magnetic fields, and the electric current density, respectively;  $\rho_q$  is the (spatial) charge density.

On the other hand, after applying the gauge symmetry transformations to the scalar and vector potentials,  $\varphi$  and  $\vec{A}$ , respectively:

$$\varphi = \varphi_0 - \frac{\partial g}{\partial t} \quad (\text{B.3a})$$

$$\vec{A} = \vec{A}_0 + \text{grad } g \quad (\text{B.3b})$$

the following expressions for the electric and magnetic fields hold:

$$\vec{E} = - \frac{\partial \vec{A}_0}{\partial t} \quad (\text{B.4a})$$

$$\vec{H} = \text{curl } \vec{A}_0 \quad (\text{B.4b})$$

Equations (B.4) prove that, by gauge symmetry, the potential vector  $\vec{A}_0(\varphi, t)$  completely determines both the electric and magnetic fields which, therefore, can be regarded as a unique electromagnetic tensor [3], [6]. In consequence, eq.'s (B.4) can properly be considered as general equations of the electromagnetic field.

<sup>1</sup> The tensor of the form  $S_{kj}$  and vector  $A_i$  are considered by some authors [6] as "dual" to each other.

<sup>2</sup> In the classical literature operating in the Minkowski space [3],  $p^0 = \mathcal{E}/c$  is the first component of 4-vector  $(p^0, \mathbf{p})$ .

<sup>3</sup> Incidentally, it can be easily seen that this "equivalent length" of a mass almost exactly coincides with the shortest distance, denoted  $x_n$  in [2], eq.(4.17), at which a heavy point-like body can attract another object (an electron by a proton or a planet by a black hole). At the same time, it amounts to a half of the "gravitational radius", introduced by Schwarzschild [3], [4], [6].

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Wsewolod Warzanskyj Poliscuk was born in Ukraine in 1925. He received the degree of Ingeniero from Escuela Especial de Ingenieros de Telecomunicación in Madrid, Spain, in 1958, and the Doctor Ingeniero degree in 1963.

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In the last few years his interest has been centered on the Theory of Relativity, that has much to do with the origin and propagation of the electromagnetic field. On this subject, he has presented two papers, published in the previous issue of these Proceedings.