RELATIVISTIC DYNAMICS AND ELECTROMAGNETIC FIELD

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Abstract

This is the concluding paper, out of a series of three, compiled under a general title *A New Approach to the Theory of Relativity*, two of them already published in the previous issue of these Proceedings. In this contribution, the equations of motion in the relativistic space are further analysed, proving that they can readily be interpreted as the general field equations. In particular, the compatibility between the General Relativity and Maxwell equations has been established, showing that both acquire an identical form in the limiting case of propagation of the system under consideration (a field of force in our case) with the speed of light. Besides, when using the formalism of the Maxwell equations, the effect of strong acceleration comes to light leading to a new interpretation of the quantum transitions of small particles and, by the same, to reconciliation of the Quantum and Relativistic Mechanics.

Another consequence of the analysis reported here is the geometric interpretation of the mass and gravitational field, allowing an easy understanding of the seemingly "strange" behaviour of some celestial bodies, such as the quasars and black holes.

I. INTRODUCTION

In two previous papers [1], [2], a relativistic* 4-space (E-4D) has been defined by the coefficients $r^k(\psi,\xi)$, related to the 4-velocity vector $(u^o,\vec{u})$ by the equation

$$u^k = c\,\frac{r^k(\psi,\xi)}{\cosh\psi} \qquad (1.1)$$

In (1.1), $r^o = 1$, and $u^o = c/\cosh\psi$ is the component of the velocity along the relativistic time-axis χ. ξ is the "arc length" along χ, related to the usual (non relativistic) time t through the differential equation:

$$d\xi \equiv d\chi = \frac{c\,dt}{\cosh\psi} \qquad (1.2)$$

and ψ is found from the relationship: $\tanh\psi = \|\vec{v}\|/c$, where the second term represents the modulus (norm) of the velocity vector $\vec{v}$ in the ordinary (3-dimensional) space, normalized with respect to the speed of light. It has been also found that the following relationship holds in E-4D {cf. [1], (4.7) and [2], (2.3b)}:

$$u^\iota u_\iota = c^2 \qquad (1.3)$$

where Einstein's rule of summation has been used and {cf.[2], (2.3a)}:

$$\sum_{\nu=1}^{3}(r^\nu)^2 = \sinh^2\psi \qquad (1.4)$$

Let Γ^ι_{jk} be the Christoffel symbols (cf. [2], APPENDIX A). Then, the following functions, closely related to the definition of the geodesics {cf. [2], (1.6) and (2.6)} have been established:

$$R^k_\xi = \frac{\partial^2 x^k}{\partial\xi^2} + \Gamma^k_{\alpha\beta} r^\alpha r^\beta \qquad (1.5)$$

Furthermore, whenever r^k vary with ψ, there also exist non-zero valued coefficients:

$$R^k_\psi = \frac{\partial r^k}{\partial\psi} \qquad (1.6)$$

An important result brought to light in [1] and drastically contrasting with a well known Einstein's proposition is that the mass of the particle or, in general, of an object, is invariant in the relativistic transformations (does not vary with its speed). To reconcile this supposition with the velocity dependent inertia of the material objects it has been found {cf. [1], (6.1a)} that the space component of a force is given by the equation

$$F_{1x} = \frac{F_1(\xi)}{\cosh\psi} \qquad (1.7a)$$

i.e., decreases, in the 3-dimensional space, with the speed $v = \|\vec{v}\|$ of the object to which the force is applied as $1/\cosh\psi = (1 - v^2/c^2)^{1/2}$. In (1.7a), $F_1(\xi)$ is the relativistic transformation-invariant force.

Likewise, the time-component of the force has been defined as {cf. [1], (6.11b)}:

$$F_{1\chi} = -\,F_1(\xi)\tanh\psi \qquad (1.7b)$$

giving the obvious relationship:

$$F^2_{1x} + F^2_{1\chi} = F^2_1(\xi) \qquad (1.8)$$

An important consequence of the foregoing statement is that the overall energy (the hamiltonian) of a particle with the mass m is invariant and equal to mc^2. When the particle is in motion, a part of its energy is transformed into kinetic and electromagnetic energies. In the limiting case of a particle acquiring (hipothetically) the speed of light, its energy becomes the radiation energy $h\nu$

(according to the famous Planck - Einstein equation). In the intermediate state ($0 < v < c$), the particle becomes partially matter and partially wave, complying with the DeBroglie *Wave Mechanics*.

Using all the previous statements and definitions, the general equation of motion has been written in the form {cf. [2]. (2.11)}:

$$\frac{dp^k}{dt} = m\left[\frac{c^2}{\cosh^2\psi} R^k_\xi + \frac{F_1(\xi)}{m}(R^k_\psi - r^k \tanh\psi)\right] \qquad (1.9)$$

where $p^k = mu^k$ is the momentum 4-vector in E-4D.

The main purpose of what follows is to relate equation (1.9) to the Maxwell equations. It will be shown that both are compatible, which is to say, can be derived one from another [the maxwellian equations from (1.9) and vice - versa].

On the other hand, it has been pointed out in the concluding section of [2] that a sufficiently strong acceleration can "neutralize" the "normal course" of the natural phenomena and the *Quantum Mechanics* laws begin to prevail. The acceleration is thus another quantity conditioning the behaviour of the matter and has to be taken into account using a rather different treatement. An iportant consequence of the present approach to the Relativistic Dynamics is that the *Equivalence Principle* [3] - the corner-stone of Einstein's *Theory of Relativity* - does not generally hold, which is to say that the relativity is no longer true for the accelerated bodies. This statement will be considered in more detail in the next sections.

Now, a few words about the coefficients R^k_ξ appearing in (1.5) and their relationship with the equation of the geodesics:

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} \qquad (1.10)$$

refered to in [1], eq.(1.6), and rewritten here for commodity. First of all, it has to be noted that ξ in (1.5) and s in (1.10) are the same variables, although related in a different way to the notion of "distance" in E-4D (ξ) and in the Minkowski space (s). In addition, both the second member in (1.5) and the first one in (1.10) represent the trajectory in a "disturbed" space. The main difference between them consists in the fact that R^k_ξ enter into the equation of motion multiplied by a convenient convergency factor, such as $c^2/\cosh^2\psi$, which becomes vanishingly small as R^k_ξ run to infinity ($v \to c$), whereas in (1.10) there is no such a fortunate circumstance and, thus, the equation of the geodesics does not apply to the objects propagating with the speed approaching *c* (in fact, it does not strictly apply anywhere). Furthermore, eq.(1.10) is homogeneous. Thus, when Einstein tried to identify the components g_{ij} of the fundamental metric tensor entering the Christoffel symbols with the driving force (gravitational field) potentials, he interpreted some terms as both the cause and the effect {cf. [3] and [2], Sec. I}. In contrast, in (1.9), the driving forces, dp^k/dt, appear in the first members of the equations, the second members being the components of the "opposing force", in agreement with the general principle of *action and reaction* presiding all the "well behaved" physical laws.

II. ELECTRIC AND MAGNETIC FIELDS IN THE RELATIVISTIC SPACE.

To apply the general equations of motion (1.9) to the electric charges driven by an electromagnetic field, the term F_1/m has to be replaced by $(c^2m/\cosh^2\psi)\frac{d\psi}{d\xi}$ {cf.[1], eq. (6.5)}. In addition, R^k_ξ can be eliminated using (1.5) and (1.6). After all these substitutions we get:

$$\frac{dp^k}{dt} = \frac{c^2m}{\cosh^2\psi}\left[\frac{\partial r^k}{\partial\xi} + \Gamma^k_{\alpha\beta}r^\alpha r^\beta + \left(\frac{\partial r^k}{\partial\psi} - r^k\tanh\psi\right)\frac{d\psi}{d\xi}\right] \qquad (2.1a)$$

where $r^k = dx^k/d\xi$ {cf. (1.1), with $u^k = dx^k/dt$}.

Frequently, when deriving (2.1a), the normalized velocity components r^k can be rendered explicitly independent of ψ by using constraints (1.4). It allows to drop the factor R^k_ψ, replace the partial derivatives $\frac{\partial r^k}{\partial\xi}$ by the total ones, $\frac{dr^k}{d\xi}$, and write (2.1a) in a slightly simpler form:

$$\frac{dp^k}{dt} = \frac{c^2m}{\cosh^2\psi}\left(\frac{dr^k}{d\xi} + \Gamma^k_{\alpha\beta}r^\alpha r^\beta - r^k\tanh\psi\frac{d\psi}{d\xi}\right) \qquad (2.1b)$$

On the other hand, it is possible to sometimes express r^k independently of ξ, in which case R^k_ξ vanish. Such is, for example, always the case in the rectilinear motion, as it has been shown in [2], Sec IV.

In eq.'s (2.1), some terms depend explicitly on r^k or their derivatives, whereas some other appear in the sum $\Gamma^k_{\alpha\beta}r^\alpha r^\beta$, with at least one of the indices α and β not equal to *k*. After separating both contributions, eq. (2.1a) can be rewritten as

$$\frac{dp^k}{dt} = \frac{c^2m}{\cosh^2\psi}\left[\frac{dr^k}{d\xi} + \left(T^\varkappa - \tanh\psi\frac{d\psi}{d\xi}\right)r^k\right] + \frac{c^2m}{\cosh^2\psi}K^k_i r^i\Big|_{i\neq k} \qquad (2.2)$$

where the index $\varkappa$ has the same meaning as k, except not being subject to the summation rule.

The ensamble of equations (2.2) with the index k running through the (ordinary space) values 1, 2, 3, can be written more concisely in the form:

$$\frac{d\vec{p}}{dt} = \frac{c^2 m}{\cosh^2\psi} (\vec{M} + \vec{r} x \vec{K}) \qquad (2.3)$$

which represents the general motion equation for a particle (charged or otherwise) driven by either electro-magnetic or mechanical force in the relativistic space.

To prove this important statement, the motion will be considered, for simplicity, to be confined to a plane defined by the orthogonal coordinates x^k, x^j. Using the well known relationship between the Christoffel symbols and the metric tensor $g_{\iota k}$ (cf. [3], [4]), the term $\Gamma^k_{\alpha\beta} r^\alpha r^\beta$ can be expanded as follows (the repetition of the indices j, k - which stand for particular values of α, β - does not imply summation):

$$\Gamma^k_{\alpha\beta} r^\alpha r^\beta = \frac{1}{2} g^{kk} \left[\frac{\partial g_{kk}}{\partial x^k} r^k + \frac{\partial g_{kk}}{\partial x^j} r^j \right] r^k +$$

$$+ \frac{1}{2} g^{kk} \left[\frac{\partial g_{kk}}{\partial x^j} r^k - \frac{\partial g_{jj}}{\partial x^k} r^j \right] r^j \qquad (2.4)$$

The first term between brackets in (2.4) can be interpreted as a contravariant vector component:

$$T^k = \frac{dg_{kk}}{d\xi} r^k \qquad (2.5)$$

Furthermore, let us define for convenience a mixed 2nd order tensor K^μ_α:

$$K^\mu_\alpha = \frac{1}{2} g^{\mu\mu} \left[\frac{\partial g_{\mu\mu}}{\partial x^\alpha} r^\mu - \frac{\partial g_{\alpha\alpha}}{\partial x^\mu} r^\alpha \right]$$

$$\mu, \alpha = j, k \qquad (2.6)$$

Then, (2.3) follows with

$$M^k = \frac{dr^k}{d\xi} + \left[T^k - \tanh\psi \frac{d\psi}{d\xi} \right] r^k \qquad (2.7)$$

and K^μ_α given by (2.6). Note that $K^\alpha_\alpha = 0$. On the other hand, in the original (orthogonal) reference system, $g_{kk} = 1/g^{kk}$. Therefore, if we consider

$$A_\alpha = \frac{1}{2} g_{\alpha\alpha} \frac{dx^\alpha}{d\xi} \; ; \; \alpha = j, k \qquad (2.8)$$

as the components of a two-dimensional vector A, equation (2.6) gives:

$$g_{kk} K^k_j = \frac{1}{2} \left[\frac{\partial g_{kk}}{\partial x^j} \frac{dx^k}{d\xi} - \frac{\partial g_{jj}}{\partial x^k} \frac{dx^j}{d\xi} \right] = (\text{curl } A)_\iota \qquad (2.9a)$$

$$g_{jj} K^j_k = \frac{1}{2} \left[\frac{\partial g_{jj}}{\partial x^k} \frac{dx^j}{d\xi} - \frac{\partial g_{kk}}{\partial x^j} \frac{dx^k}{d\xi} \right] = - (\text{curl } A)_\iota \qquad (2.9b)$$

In both equations (2.9), index ι shows the direction orthogonal to the (x^k, x^j) plane when taking as positive the sequence (i,j,k). Therefore, after generalizing upon the 3-dimensional euclidean space, the terms $S_{kj} = g_{kk} K^k_j$ define a 2nd order, covariant, antisymmetric tensor which can also be interpreted [5] as the component of an axial vector $\vec{A}$.[1]

According to the classical expression of Lorentz force [7], equation (2.3) can be used to describe the motion of the charged particle driven by the electromagnetic field. In effect, using for convenience Gauss System units, the equation in question can be rewritten as:

$$\frac{d\vec{p}}{dt} = q\vec{E} + \frac{q}{c} \vec{v} \times \vec{H} = q\vec{E} + \frac{q}{\cosh\psi} \vec{r} \times \vec{H}$$

with (2.10)

$$E_k = - \frac{c^2 m}{q \cosh^2\psi} g_{kk} M^k \qquad (2.11a)$$

$$H_\iota = \frac{c^2 m}{q \cosh\psi} g_{kk} K^k_j = \frac{c^2 m}{q \cosh\psi} (\text{curl } \vec{A})_\iota \qquad (2.11b)$$

which can be easily verified using (2.2).

Vector $\vec{A}$ is clearly the electromagnetic vector potential. E_k and H_ι are the covariant components of the electric and magnetic fields. The corresponding metric (the usual) components are derived by multiplying both equations (2.11) by g^{kk} [5].

For example, in central motion, the equations in polar coordinates giving the radial and normal acceleration {cf. [2], eq,'s (5.7)} are written as follows:

$$a_\rho = \frac{c^2}{\cosh^2\psi} \left[\frac{d^2\rho}{d\xi^2} - \rho \left(\frac{d\eta}{d\xi}\right)^2 - \frac{d\rho}{d\xi}\frac{d\psi}{d\xi} \tanh\psi \right] \qquad (2.12a)$$

$$a_\eta = \frac{c^2}{\cosh^2\psi} \left[\rho \frac{d^2\eta}{d\xi^2} + 2 \frac{d\rho}{d\xi}\frac{d\eta}{d\xi} - \rho \frac{d\eta}{d\xi}\frac{d\psi}{d\xi} \tanh\psi \right] \qquad (2.12b)$$

from which, after some trivial reorganization, the following components of the electric field are found:

$$E_\rho = \frac{mc^2}{q\cosh^2\psi}\left[\frac{d^2\rho}{d\xi^2} - \frac{d\rho}{d\xi}\frac{d\psi}{d\xi}\tanh\psi\right] \qquad (2.13a)$$

$$E_\eta = \frac{mc^2}{q\cosh^2\psi}\left[\rho\frac{d^2\eta}{d\xi^2} - \left(\rho\frac{d\eta}{d\xi}\tanh\psi - \frac{d\rho}{d\psi}\right)\frac{d\eta}{d\xi}\right] \qquad (2.13b)$$

Likewise, the only (metric) component of the magnetic field can be derived from the definition of the curl in cylindrical coordinates (cf. [5]), namely:

$$(\vec{r}\times\vec{H})_\rho = r_\eta \cdot H_z = -\frac{mc^2}{q\cosh\psi}\,\rho\left(\frac{d\eta}{d\xi}\right)^2$$

$$(\vec{r}\times\vec{H})_\eta = -r_\rho \cdot H_z = \frac{mc^2}{q\cosh\psi}\frac{d\rho}{d\xi}\frac{d\eta}{d\xi} \qquad (2.14)$$

or, using the ordinary velocity components $v_\alpha = \frac{c}{\cosh\psi} r_\alpha$ instead:

$$v_\eta H_z = -\frac{mc}{q}\,\rho\left(\frac{d\eta}{dt}\right)^2$$

$$-v_\rho H_z = \frac{mc}{q}\frac{d\rho}{dt}\frac{d\eta}{dt} \qquad (2.15)$$

Equations (2.15) lead to the vector diagram of Fig.1. Because $v_\rho = d\rho/dt$, $v_\eta = \rho\, d\eta/dt$, we get:

$$H = \frac{cm\omega_c}{q} \qquad (2.16)$$

which is the well known relationship between the magnetic field and the cyclotron frequency $\omega_c = d\eta/dt$ in Gauss System units.

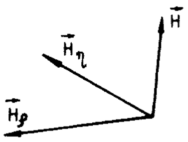

Fig.1. Vector diagram of magnetic field in cylindrical reference system.

The foregoing analysis is clearly valid for the central motion originated by a gravitational force. In particular, eq.'s (2.13) and (2.16) define what will be provisionally called "gravitational" and "inertial" fields, respectively. Both denominations and their meaning will be commented in more detail farther on. For the moment it is sufficient to say that, according to the way the equations of motion have been derived, the description of the gravitational force can be found from that of the electric and magnetic fields, respectively, by substituting **m** for **q** and conveniently re-scaling the units. The previous analysis also shows a strict analogy between the electrical and mechanical phenomena, which is a remarkable property of Nature, not sufficiently explored yet.

III. TANGENT AND CROSS-DIRECTIONAL FORCES

By analogy with the ordinary velocity $\vec{v}$, all 3-space components r^k of the (normalized) relativistic velocity r, defined in (1.1), can be assembled into the vector

$$\vec{r} = \vec{r}_\rho + \vec{\omega}\times\vec{\rho} = \vec{r}_\rho + \vec{r}_\eta \qquad (3.1)$$

Evidently, $\vec{r}$ is a vector directed along the tangent to the projection of the trajectory onto the normal 3-space. If we define the unit vectors $\vec{u}_\rho$, $\vec{u}_t$ along the radius vector $\vec{\rho}$ and the tangent to the trajectory, respectively, as depicted in Fig. 2, the several vectors entering equation (3.1) are defined as follows:

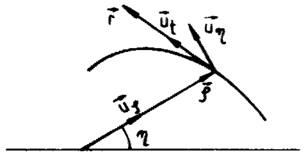

fig.2. Vector $\vec{r}$ and its radial and normal componentes.

$$\vec{r}_\rho = \frac{d\rho}{d\xi}\vec{u}_\rho$$

$$\vec{\omega} = \frac{d\eta}{d\xi}\vec{u}_\rho\times\vec{u}_\eta = \frac{d\eta}{d\xi}\vec{u}_z \qquad (3.2)$$

$$\vec{r}_\eta = \vec{\omega}\times\vec{\rho} = \rho\frac{d\eta}{d\xi}\vec{u}_\eta$$

Here $\vec{u}_\eta$, $\vec{u}_z$ are the usual unitary vectors and the triad $(\vec{u}_\rho, \vec{u}_\eta, \vec{u}_z)$ defines a mutually orthogonal, right-hand oriented basis-vector system.

Collecting the results of the previous sections the following relationships can be established:

$$\vec{r} = \frac{d\rho}{d\xi}\vec{u}_r + \rho\frac{d\eta}{d\xi}\vec{u}_\eta \qquad (3.3a)$$

$$\vec{H} = -\frac{c^2 m}{q\cosh\psi}\frac{d\eta}{d\xi}\vec{u}_z \qquad (3.3b)$$

$$\vec{r}\times\vec{H} = \frac{c^2 m}{q\cosh\psi}\left[-\rho\frac{d\eta}{d\xi}\vec{u}_\rho + \frac{d\rho}{d\xi}\vec{u}_\eta\right]\frac{d\eta}{d\xi} \qquad (3.3c)$$

On the other hand, the last term in (2.10) has the dimensions of a force and can be expressed as:

$$\vec{F}_{tr} = -\frac{mc^2}{\cosh^2\psi}\vec{r}\times\vec{\omega} = -m\,\vec{v}\times\vec{\omega}_c \qquad (3.4)$$

For a charged particle, $\vec{F}_{tr}$ is due to the magnetic field. In mechanics, the term in question accounts for the well known centrifugal force.

The remaining terms in the right-hand member in eq. (1.9) can be interpreted as components of a tangential force. Their ensamble is readily written in vector form as

$$\vec{F}_{tr} = \frac{mc^2}{\cosh\psi}\frac{d}{d\xi}\left[\frac{\vec{r}}{\cosh\psi}\right] = m\frac{d\vec{v}}{dt} \qquad (3.5)$$

System (3.4), (3.5) can again be glued together into a single vector equation:

$$\frac{d\vec{p}}{dt} = \frac{mc^2}{\cosh\psi}\left[\frac{d}{d\xi}\left(\frac{\vec{r}}{\cosh\psi}\right) - \frac{1}{\cosh\psi}\vec{r}\times\vec{\omega}\right] = m\left(\frac{d\vec{v}}{dt} - \vec{v}\times\vec{\omega}_c\right) \qquad (3.6)$$

with $\vec{r}$ given in (3.3a) and

$$\vec{v} = \frac{c}{\cosh\psi}\vec{r} = \frac{c}{\cosh\psi}\left(\frac{d\vec{\rho}}{d\xi} + \vec{\omega}\times\vec{\rho}\right) \qquad (3.7)$$

Equations (3.6 and (3.7) fully describe a planar motion in a formally invariant form under relativistic transformation.

In particular, the factor mc^2 represents the (constant) total energy of the moving body; $\vec{r}$, $\vec{\omega}$ and ξ being also invariant quantities.

IV. RELATIONSHIP BETWEEN THE RELATIVISTIC DYNAMICS AND MAXWELL EQUATIONS

IV-1. 1st pair of Maxwell equations.- Relativistic dynamic equation (3.6) has been derived under the assumption $R^k_\psi = 0$. Consequently, the equations of electric and magnetic fields derived in Sec. II should comply with the Maxwell equations of the electromagnetic field, quoted for an easy reference in APPENDIX B [cf. (B.1), (B.2) and (B.7)]. To verify that this is true we proceed as follows. First, refer to equation (A.9), APPENDIX A, which states:

$$\operatorname{div}\vec{\omega} = 0 \qquad (4.1a)$$

and, from the definition of ω and (3.3b), it is found:

$$\operatorname{div}\vec{H} = 0 \qquad (4.1b)$$

which directly gives the 2nd Maxwell equation [cf. (B.1b)]. Then, according to (A.7), APPENDIX A:

$$\vec{\omega}_c = \operatorname{curl}(\vec{\omega}_c\times\vec{\rho}) \qquad (4.2)$$

If we suppose that the driving force is electromagnetic in nature, eq. (2.10) gives:

$$\frac{d\vec{p}}{dt} = q\vec{E} + \frac{q}{c}\vec{v}\times\vec{H}$$

where, from (3.5) and (2.16),

$$\vec{E} = \frac{\vec{F}_{tr}}{q} = \frac{m}{q}\frac{d\vec{v}}{dt} \qquad (4.3a)$$

$$\vec{H} = -\frac{cm\,\vec{\omega}_c}{q} = -\frac{cm}{q}\operatorname{curl}(\vec{\omega}\times\vec{\rho}) \qquad (4.3b)$$

Furthermore, according to (3.7), and because $\operatorname{curl}\left(\frac{d\vec{\rho}}{d\xi}\right) = \frac{d}{d\xi}(\operatorname{curl}\vec{\rho}) = 0$, we have:

$$\vec{H} = -\frac{cm}{q}\operatorname{curl}\vec{v} \qquad (4.4)$$

The potential vector A can be identified as

$$\vec{A} = -\frac{cm}{q}\vec{v} \qquad (4.5)$$

then, by applying curl to both sides of (4.3a), it follows

$$\operatorname{curl}\vec{E} = \frac{m}{q}\operatorname{curl}\left(\frac{d\vec{v}}{dt}\right) = \frac{m}{q}\frac{d}{dt}(\operatorname{curl}\vec{v}) = -\frac{1}{c}\frac{d\vec{H}}{dt} \qquad (4.6)$$

In (4.6), the derivative with respect to t is supposed to be taken in a fixed point of space. If we allow vector $\vec{\rho}(x,y,z)$ to vary, the total derivative must be replaced by the partial ones, which, according to (B.1b), gives the 1st Maxwell equation, as required.

IV-2. Derivation and consequences of the 3rd Maxwell equation.- Starting with eq. (A.14) in APPENDIX A, one can write, using dt and $\vec{\omega}_c$ instead of $d\xi$ and $\vec{\omega}$, respectively:

$$\operatorname{curl}\vec{\omega}_c = \frac{\partial\omega_c}{\partial y}\vec{i} - \frac{\partial\omega_c}{\partial x}\vec{j} =$$

$$= \frac{1}{a_\rho^4}\left[y\,(yv_x - xv_y) - x\,(xv_x + yv_y)\right]\vec{i} + \frac{1}{a_\rho^4}\left[x\,(xv_y - yv_x) - y\,(xv_x + yv_y)\right]\vec{j} \qquad (4.7)$$

where $\vec{\omega}_c = \frac{d\vec{\eta}}{dt}$, $v_x = \frac{dx}{dt}$, etc. On the other hand, from (A.12) we have:

$$\omega_c = \frac{1}{a_\rho^2}(-yv_x + xv_y)$$

and because

$$xv_x + yv_y = \frac{1}{2}\frac{da_\rho^2}{dt} = a_\rho v_\rho$$

with $v_\rho = da_\rho/dt$, equation (4.7) is readily rewritten as

$$\operatorname{curl}\vec{\omega}_c = -\frac{\vec{\omega}_c\times\vec{\rho}}{a_\rho^2} - \frac{1}{a_\rho^3}v_\rho\,\vec{\rho} \qquad (4.8a)$$

Let $\vec{v}_\rho = v_\rho \vec{u}_\rho$ be a vector directed along $\vec{\rho}$-axis and v_ρ its amplitude. We have

$$\frac{\vec{\omega}_c \times \vec{\rho}}{a_\rho^2} = \frac{\vec{v} - \vec{v}_\rho}{a_\rho^2}$$

and $v_\rho \vec{\rho} = a_\rho \vec{v}_\rho$. Then, by (4.8a),

$$\text{curl } \vec{\omega}_c = - \frac{\vec{v}}{a_\rho^2} \qquad (4.8b)$$

an important equation which, when combined with (4.3b), gives:

$$\text{curl } \vec{H} = \frac{cm}{qa_\rho^2} \vec{v} \qquad (4.9)$$

To complete the 3rd Maxwell equation [cf.(B.2a), APPENDIX B], eq.'s (2.13) shall be expressed conveniently in the form:

$$E_\rho = \frac{mc^2}{q \cosh \psi} \frac{d}{d\xi} \left[\frac{r_\rho}{\cosh \psi} \right] = \frac{m}{q} \frac{\partial v_\rho}{\partial t} \qquad (4.10a)$$

$$E_\eta = \frac{mc^2}{q \cosh \psi} \frac{d}{d\xi} \left[\frac{r_\eta}{\cosh \psi} \right] = \frac{m}{q} \frac{\partial v_\eta}{\partial t} \qquad (5.10b)$$

with $v_\rho = da_\rho/dt$, $v_\eta = a_\rho d\eta/dt$. Furthermore, the partial derivatives have been used in the right-hand member of (4,10) because v_ρ and v_η are supposed to be both time and space dependent functions. Also, by applying the continuity equation to both components of $\vec{v}$ we get:

$$\frac{\partial v_\alpha}{\partial t} = - \text{div}(v_\alpha \vec{v}) =$$

$$= \vec{v}.\text{grad } v_\alpha - v_\alpha \text{ div } \vec{v} \qquad (4.11)$$

with $\alpha = \rho, \eta$.

To compute (4.11) we observe that, because v_α are refered to a given space point, their gradients should be disregarded. On the other hand,

$$\text{div } \vec{v} = \text{div } (\vec{v}_\rho + \vec{\omega}_c \times \vec{\rho}) = \text{div } (\vec{\omega}_c \times \vec{\rho})$$

By using a well known vector identity we also have:

$$\text{div } (\vec{\omega}_c \times \vec{\rho}) = \vec{\rho}.\text{curl } \vec{\omega}_c - \vec{\omega}_c.\text{curl } \vec{\rho}$$

Then, because curl $\vec{\rho} = 0$, it is found from (4.12) and (4.8b):

$$\text{div } \vec{v} = - \frac{\vec{\rho}.\vec{v}}{a_\rho^2} = - \frac{v_\rho}{a_\rho} \qquad (4.13)$$

With all that in mind, equation (4.11) can be written in vector form as

$$\frac{\partial \vec{v}}{\partial t} = \frac{v_\rho}{a_\rho} \vec{v} \qquad (4.14)$$

and, according to (4.10),

$$\vec{E} = \frac{mv_\rho}{qa_\rho} \vec{v} \qquad (4.15)$$

By taking the (partial) time derivative of $\vec{E}$ given in (4.15) we get:

$$\frac{\partial \vec{E}}{\partial t} = \frac{m}{q} \left[\vec{v} \frac{\partial}{\partial t} \left(\frac{v_\rho}{a_\rho}\right) + \frac{v_\rho}{a_\rho} \frac{\partial \vec{v}}{\partial t} \right]$$

and using (4.14),

$$\frac{\partial \vec{E}}{\partial t} = \frac{m\vec{v}}{q} \left[\frac{v_\rho}{a_\rho^2} + \frac{\partial}{\partial t} \left(\frac{v_\rho}{a_\rho}\right) \right] = \frac{m}{q} \frac{\dot{v}_\rho \vec{v}}{a_\rho} \qquad (4.16)$$

where $\dot{v}_\rho = \frac{\partial v}{\partial t}$ is interpreted as the amplitude of the radial acceleration.

Combining all the previous results, the 3rd Maxwell equation can be written as follows:

$$\frac{cm\vec{v}}{qa_\rho^2} = \frac{1}{c} \left(\frac{m}{q} \dot{v}_\rho \vec{v} + 4\pi \vec{J}\right) \qquad (4.17)$$

Accordingly, the density of the current is given by the equation

$$\vec{J} = \frac{mc^2 \vec{v}}{4\pi qa_\rho^2} \left(1 - \frac{a_\rho \dot{v}_\rho}{c^2}\right) \qquad (4.18)$$

The last term in (4.18) is usually negligible (it clearly vanishes in the classical approximation which implies $c \rightarrow \infty$). On the other hand, a violent radial acceleration (for example originated by the impact of a sufficiently energetic photon upon an electron) may cancel $\vec{J}$ altogether, with the consequence already considered in Sec. I (see also [2], Sec. V-4).

IV-3. Discussion of the 4th Maxwell equation; field induced charges.- The spatial distribution of charges induced by the motion of a (charged) particle can be derived from the 4th Maxwell equation [cf. eq. (B.2b) APPENDIX B], (4.15) and (4.13), namely:

$$\rho_q = \frac{1}{4\pi} \text{div } \vec{E} = - \frac{mv_\rho^2}{4\pi qa_\rho^2} \qquad (4.19)$$

To see that (4.19) is consistent with (4.18) it has to be noticed that ρ_q in the former represents the density of the charges induced by the electric field, which are originated, in their turn, by the motion of a particle endowed with the mass m and a (primary) charge q. The velocity of ρ_q may clearly be different from $\vec{v}$. Call it $\vec{V}_\rho$, $\vec{J}_\rho$ being the current density induced by ρ_q. To distinguish the "primary" current, due to charge q, from $\vec{J}_\rho$, the former will be relabelled $\vec{J}_q$. It will be supposed that whenever $v_\rho = 0$, V_ρ also vanishes. Then, $\vec{J}_q$ is given by the first term of (4.18), namely:

$$\vec{J}_q = \frac{mc^2 \vec{v}}{4\pi qa_\rho^2} \qquad (4.20)$$

Also, because $\vec{J} = \vec{J}_q + \vec{J}_\rho$, we find:

$$\vec{J}_\rho = - \frac{m\dot{v}_\rho \vec{v}}{4\pi qa_\rho} \qquad (4.21a)$$

To express $\vec{J}_\rho$ in a form analogous to $\vec{J}_q$ (only retaining the difference in the sign), we write:

$$\vec{J}_\rho = - \frac{mc^2 \vec{v}_\rho}{4\pi q a_\rho^2} \qquad (4.21b)$$

and, by comparing both equations (4.21), it is found that

$$\vec{v}_\rho = \frac{\dot{v}_\rho a_\rho}{c^2} \vec{v} \qquad (4.22)$$

The last equation shows that the speed of the "secondary" charges is proportional to the radial acceleration and, accordingly, with the concourse of an extreme acceleration, it becomes quite large. In any case, such situations - as it has been mentioned earlier - are beyond the scope of the present discussion. In what follows, some attention will be paid, instead, to the "classical relativistic" aspect of the motion and the (suspected) nature of the gravitational field, or the matter, in general.

V. GEOMETRIC INTERPRETATION OF THE MASS AND GRAVITATIONAL FIELD.

In [2], Sec. III, the curvature of the space has been interpreted as a distorsion of the quiescent (rectilinear) path in E-4D, when viewed by an object accelerating under the effect of a driving force, such as the one due to the gravitational field. To shed some - intuitively more comprehensible - light on this question, consider the gravitional attraction in some more detail.

As everybody knows from the very first lessons on Classsical Mechanics, two bodies of vanishingly small sizes, with the masses, say, m_1 and m_2, undergo a mutual attraction given by the inverse square law:

$$F = K \frac{m_1 m_2}{r^2} \qquad (5.1)$$

where r is the distance between the bodies in question and *K* the so-called *Universal Gravitational Constant*. The value of *K* in *SI* (International System) units is approximately

$$K = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2} \qquad (5.2a)$$

The most cursory look on (5.2a) reveals rather complex dimensions associated with the numerical value of *K*. Besides, the dimensions in question depend on the choice of a particular unit system. For example, Einstein used, instead, in his field equations {cf. [2], (1.7)} the constant:

$$\mathcal{K} = 4\pi K c^{-2} = 1.86 \times 10^{-27} cm.g^{-1} \qquad (5.2b)$$

On the other hand, as it has been previously pointed out (cf. Sec. I), the mass m_i of the particle i is proportional to its energy $\mathcal{E}_i$, according to the law $m_i = \mathcal{E}_i/c^2$. Call $p_i^o = cm_i = \mathcal{E}_i/c$ the relativistic transform-invariant quantity, with the dimensions of momentum and, thus, liable to be interpreted as the total or *scalar momentum* of the particle.[2] Then, eq. (5.1) can be written as

$$F = K c^{-2} r^{-2} p_1^o p_1^o \qquad (5.3a)$$

As it has been also pointed out in the referenced literature, eq. (5.3a) becomes very simple when using a unit system such that $K = c^2$, giving:

$$F = \frac{p_1^o p_2^o}{r^2} \qquad (5.3b)$$

It can be observed that, in such a system, the mass no longer retains its independent units and becomes expressed in the units of length. To be specific, 1 g will amount 7.407×10^{-29} cm (an extremely small quantity!).

The previous result can be thought of as a mere curiosity (and as such it has been presented in the mentioned reference). Alternatively, it can be given a definite physical meaning, in line with the Einstein's intuition about the relationship between the gravitational field and the space curvature. The essence of what is being said is illustrated in Fig. 3, where the two-dimensional relativistic space-time is pictured, the "normal" space being constrained for simplicity to the x-axis.

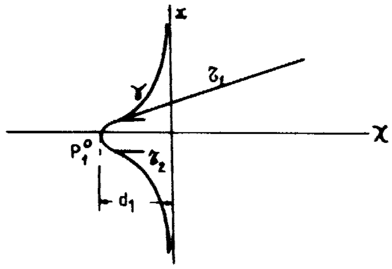

Fig.3. Space curvature originated by a mass characterized by the scalar momentum p^o.

Consistently with the foregoing discussion, a mass m_1 can be associated with the scalar momentum

$$p_1^o = \frac{\mathcal{E}_1}{c} = d_1 c$$

where, with the unit system at hand - which from now on will be called gravitational units - d_1 represents the "length" of the mass "projected" into the "backward space" (in a sense, into the past), numerically (and dimensionally) equal to m_1 [3].

In Fig. 3, x-axis marks also the border of the free space, whereas the contour γ delimits the space distorted by the gravitational field. The contour in question shows also the path followed by a beam of light (or a gamma ray) emitted by

p_1^o. τ_1 is the trajectory followed by a (secondary) particle leaving p_1^o with the initial speed larger than the *escape velocity*. If the particle is emitted with a speed smaller than the former, the particle will be "trapped" and will follow (forward and back) a trajectory such as τ_2.

The foregoing interpretation leads to the conclusion that the energy "frozen out" as a mass or the "scalar p" can be interpreted as a small region of space collapsed into the past. As such, it is responsible, for example, for the time-lag associated with the gravity, foreseen by Einstein and eventually leading to the observed red shift of the light emitted by large bodies (such as stars and, at a more overwhelming rate, by quasars [8]). Distance d_1 in Fig. 3 can thus be the measure of the *specific* gravity (depth of the "space crevice" in a given point of space). For d_1 sufficiently large, even the most energetic particles may not overcome the gravity created by the body, a situation that is currently denoted as black hole. Consequently with what is said, there can be no such a dramatic distance as the *Schwarzschild gravity radius*, and the quasars may be nearer our galaxy than it is currently believed.

The above picture can be completed with the geometric interpretation of the gravity itself. To this end, figure out a body of negligible size and mass *M*, attracting a small body of mass m from a distance r, as schematically depicted in Fig. 4. It has to be noticed, however, that r is only the apparent 3-dimensional (in general) distance between *M* and m. Fig. 4 shows that the true distance in gravitational units is, by the Pithagorean theorem $(r^2 + M^2)^{1/2}$. It means that the inverse square law (5.1) takes the form:

$$F = \frac{c^2 Mm}{r^2 + M^2} \qquad (5.4a)$$

(the negative sign showing the shortening of the distance has been omitted for simplicity). Observe that, for r = 0, $F = c^2 m/M < \infty$, which rules out the difficulties related to the attraction between two point-like bodies. Besides, the potential energy *U* (absolute value) of m turns out to be

$$U = \frac{c^2 Mm}{(r^2 + M^2)^{1/2}} \qquad (5.4b)$$

and, for $r \to 0$, $U \to U_o = c^2 m$, that is, U_o becomes finite and exactly equal to the total energy of the attracted particle.

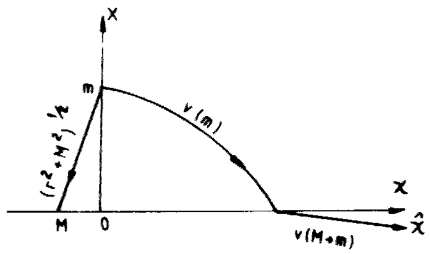

Fig.4. Graphical interpretation of the law of gravitational attraction in the relativistic space.

Again, with reference to Fig. 4, the attraction of *m* by M compells the former to "slide" down the slope $M/(r^2 + M^2)^{1/2}$ (with respect to the undistorted space) towards the latter . The observable motion of m is performed in the right-hand side of the (relativistic) space χ - x, along the trajectory shown (qualitatively) by the line v(m). After the capture of *m* by *M* (the collision is supposed perfectly inelastic), both masses "glue" together and begin to move with the speed dictated by the laws of the ordinary Physics (unless Quantum Mechanic effects take place, as suggested earlier) along the straight line v(M + m).

VII. CONLUSION

The whole philosophy underlying this series of articles concerning the Special and General Relativity is that the origin of all the forces and motion, that is, the source of the Primery Energy is the expansion of the Universe, which, in its turn, claims for a more general space for expanding into. This must be a *hyperspace*, or a sort of a field, or a superposition of such. One of the fields that shapes the hyperspace in question may be electromagnetic in nature. On the other hand, it is the matter that is responsible for the "ruggedness" of the space-time (continuum ?).

There still remain a wealth, a Universe! of questions to be answered. Nevertheless, some details are already shaping themselves into a picture quite comprehensible to the human mind. For example, the whole story may have begun as follows.

A huge needle, a gargantuan impulse of energy (a quantum of action in the briefest of times) has been injected into a certain configuration of fields (some of them are known today, some probably not). Can we think of it as if a single line of force had been suddenly (sudenly indeed!) snapped off, with the result resembling the one provoked by a stone hurled into a quiet surface of an immense lake? Somebody called this event Big Bang. Then, a (shock) wave started expanding at a constant speed c. Somebody called it the Universe we have been granted to make our living in. Then, begun the equations. Or did they begin before?

I am very grateful to all those who have helped me in this work with their encouragement and advices and, in particular, to my wife Aida, my son Wsewolod, as well as to my colleagues and fellows at the Universidad Politécnica de Madrid. Also and very especially to the Escuela Politécnica Nacional in Quito, which honoured me with the favourable reception of this series of contributions. And here are the results, may be somewhat unusual but apparently in agreement with the known facts.

APPENDIX A. COMPUTATION OF $\mathrm{curl}(\vec{\omega} \times \vec{\rho})$

The subject discused here has already been extensively treated in the literature [4], [5]. However, because of its paramount importance for the main purpose of this paper and of the curious circumstance that the result that will be derived does not fully agree with the one currently quoted, the relevant steps in the computations concerning the question at hand will be outlined.

Let $\vec{\rho}(x,y)$ be the radius-vector of the point $P(x,y)$ in the plane $x-y$, forming angle η with the x-axis. The orthogonal projections of ρ on both axes are:

$$x = a_\rho \cos \eta \;;\; y = a_\rho \sin \eta \qquad (A.1)$$

where a_ρ stands for "amplitude of $\vec{\rho}$":

$$a_\rho = \|\vec{\rho}\| = (x^2 + y^2)^{1/2} \qquad (A.2)$$

It will be assumed that $a_\rho = a_\rho(\xi)$, $\eta = \eta(\xi)$ are functions of a parameter ξ. Differentiating (A.1) we get:

$$\begin{aligned} dx &= \cos \eta \, da_\rho - a_\rho \sin \eta \, d\eta \\ dy &= \sin \eta \, da_\rho + a_\rho \cos \eta \, d\eta \end{aligned} \qquad (A.3)$$

System (A.3), when solved for da_ρ and $d\eta$, gives:

$$\begin{aligned} da_\rho &= \cos \eta \, dx + \sin \eta \, dy \\ d\eta &= \frac{1}{a_\rho}(- \sin \eta \, dx + \cos \eta \, dy) \end{aligned} \qquad (A.4)$$

Let also define a vector directed along z-axis:

$$\vec{\omega} = \omega_z \vec{k} = \frac{d\eta}{d\xi} \vec{k} = = \frac{1}{a_\rho} (- \sin \eta \frac{dx}{d\xi} + \cos \eta \frac{dy}{d\xi}) \vec{k} \qquad (A.5)$$

$(\vec{i},\vec{j},\vec{k})$ forming the basic orthogonal reference system.

Our aim is to find the value of $\mathrm{curl}(\vec{\omega} \times \vec{\rho})$. A well known vector identity gives:

$$\mathrm{curl}(\vec{\omega} \times \vec{\rho}) = \vec{\omega} \,\mathrm{div}\, \vec{\rho} - \vec{\rho} \,\mathrm{div}\, \vec{\omega} + \frac{d\vec{\omega}}{d\vec{\rho}} - \frac{d\vec{\rho}}{d\vec{\omega}} \qquad (A.6)$$

It is clear that

$$\mathrm{div}\, \vec{\rho} = 3 \qquad (A.7)$$

regardless whether one or more components of

$$\vec{\rho} = x\vec{i} + y\vec{j} + z\vec{k} \qquad (A.8)$$

are zero. Furthermore, because the only component of $\vec{\omega}$ is ω_z, independent of z, we have:

$$\mathrm{div}\, \vec{\omega} = 0 \qquad (A.9)$$

On the other hand, another general vector identity

$$\frac{d\vec{a}}{d\vec{b}} = (\vec{b}.\vec{\nabla})a = \frac{\partial \vec{a}}{\partial x} b_x + \frac{\partial \vec{a}}{\partial y} b_y + \frac{\partial \vec{a}}{\partial z} b_z \qquad (A.10)$$

gives:

$$\frac{d\vec{\rho}}{d\omega} = \omega_z \frac{\partial \vec{\rho}}{\partial z} = \vec{\omega} \qquad (A.11)$$

Using (A.5) and (A.1) we also have:

$$\omega_z = \frac{1}{a_\rho^2} (- y \frac{dx}{d\xi} + x \frac{dy}{d\xi}) = \frac{1}{a_\rho^2} (- yr_x + xr_y) \qquad (A.12)$$

with

$$r_x = \frac{dx}{d\xi} \;;\; r_y = \frac{dy}{d\xi} \qquad (A.13)$$

From (A.12) it can be infered that ω_z is a function of 4 independent variables: x, y, r_x, r_y. Taking partial derivatives of ω_z with respect to x, y, it follows:

$$\begin{aligned} \frac{\partial \omega_z}{\partial x} &= \frac{1}{a_\rho^4} \left[2xyr_x - (x^2 - y^2)\, r_y\right] \\ \frac{\partial \omega_z}{\partial y} &= \frac{1}{a_\rho^4} \left[(y^2 - x^2)\, r_x - 2xyr_y\right] \end{aligned} \qquad (A.14)$$

Also

$$\frac{d\vec{\omega}}{d\vec{\rho}} = (x \frac{\partial \omega_z}{\partial x} + y \frac{\partial \omega_z}{\partial y})\vec{k} \qquad (A.15)$$

and using (A.14) we get, after a slight rearrangement:

$$\frac{d\vec{\omega}}{d\vec{\rho}} = \frac{yr_x - xr_y}{a_\rho^2} \vec{k} = - \vec{\omega} \qquad (A.16)$$

The substitution of (A.7), (A.9), (A.11) and (A.16) into (A.6) gives:

$$\mathrm{curl}(\vec{\omega} \times \vec{\rho}) = \vec{\omega} \qquad (A.17)$$

which is the result that has been looked for (not $2\vec{\omega}$ in the second member, as it appears in the quoted literature).

APPENDIX B. BASIC RELATIONSHIPS OF ELECTRODYNAMICS.

For an easy reference, the Maxwell equations in Gauss System units are quoted and the derivation of general equations of electromagnetic field is outlined.

1st pair of Maxwell equations.-

$$\text{curl } \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \qquad \text{(B.1a)}$$

$$\text{div } \vec{H} = 0 \qquad \text{(B.1b)}$$

2nd pair of Maxwell equations.-

$$\text{curl } \vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J} \qquad \text{(B.2a)}$$

$$\text{div } \vec{E} = 4\pi\rho_q \qquad \text{(B.2b)}$$

$\vec{E}$, $\vec{H}$, $\vec{J}$ are the usual symbols (vector-valued quantities) that denote the electric and magnetic fields, and the electric current density, respectively; ρ_q is the (spatial) charge density.

On the other hand, after applying the gauge symmetry transformations to the scalar and vector potentials, φ and **A**, respectively:

$$\varphi = \varphi_o - \frac{\partial g}{\partial t} \qquad \text{(B.3a)}$$

$$\vec{A} = \vec{A}_o + \text{grad } g \qquad \text{(B.3b)}$$

the following exppressions for the electric and magnetic fields hold:

$$\vec{E} = -\frac{\partial \vec{A}_o}{\partial t} \qquad \text{(B.4a)}$$

$$\vec{H} = \text{curl } \vec{A}_o \qquad \text{(B.4b)}$$

Equations (B.4) prove that, by gauge symmetry, the potential vector $\vec{A}_o(\vec{\rho},t)$ completely determines both the electric and magnetic fields which, thereafter, can be regarded as a unique electromagnetric tensor [3], [6]. In consequence, eq.'s (B.4) can properly be considered as general equations of the electromagnetic field.

1 The tensor of the form S_{kj} and vector A_i are considered by some authors [6] as "dual" to each other.

2 In the classical literature operating in the Minkowski space [3], $p^o = \mathcal{E}/c$ is the first component of 4-vector (p^o, p).

3 Incidentally, it can be easily seen that this "equivalent length" of a mass almost exactly coincides with the shortest distance, denoted x_n in [2], eq.(4.17), at which a heavy point-like body can attract another object (an electron by a proton or a planet by a black hole). At the same time, it amounts to a half of the "gravitational radius", introduced by Schwarzschild [3], [4], [6].

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In the last few years his interest has been centered on the Theory of Relativity, that has much to do with the origin and propagation of the electromagnetic field. On this subject, he has presented two papers, published in the previous issue of these Poceedings.