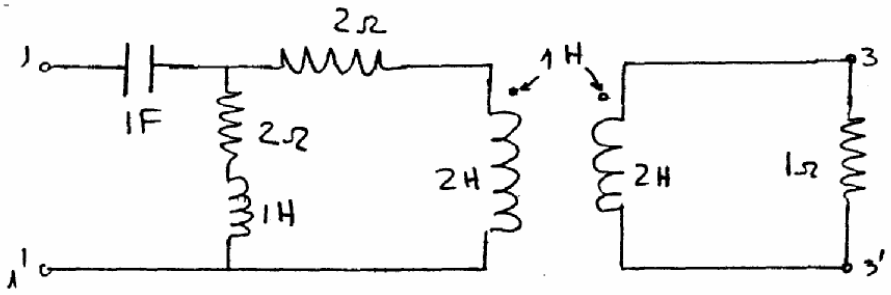
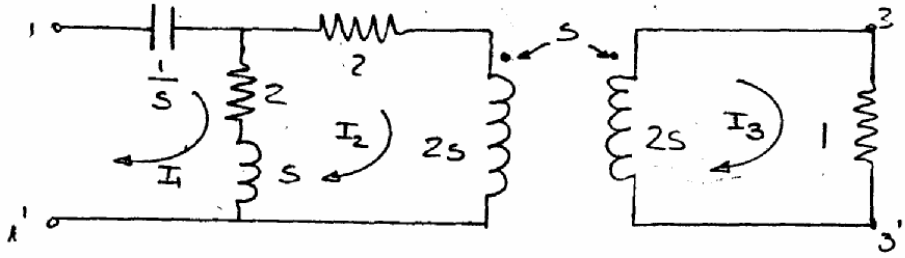
PROBLEMA: CALCULAR Z_{11}, Y_{11}, Z_{31}, Y_{31}, T_V y T_I.



EN EL DOMINIO DE LA FRECUENCIA:



SOLUCION

$$\Delta z_m = \begin{bmatrix} 2+s+\frac{1}{s} & -2-s & 0 \\ -2-s & 4+3s & -s \\ 0 & -s & 1+2s \end{bmatrix}$$

$$\Delta z_m = \frac{3s^4+12s^3+19s^2+15s+4}{s}$$

$$\Delta_{11} = 5s^2+11s+4$$

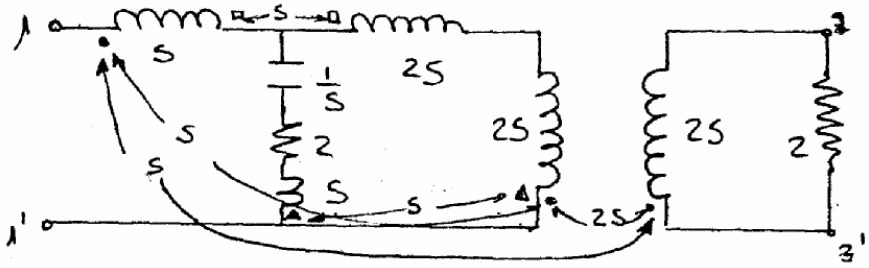
$$\Delta_{13} = s^2+2s$$

$$z_{11} = \frac{1}{y_{11}} = \frac{3s^4+12s^3+19s^2+15s+4}{5s^3+11s^2+4s}$$

$$y_{31} = T_V = \frac{s^3+2s^2}{3s^4+12s^3+19s^2+15s+4}$$

$$z_{31} = T_I = \frac{s^2+2s}{5s^2+11s+4}$$

PROBLEMA: CALCULAR $Z_{11}, y_{11}, y_{31}, Z_{31}, T_V$ y T_I



SOLUCION: POR EL ANALISIS DE MALLAS

$$\Delta Z_{11} = \begin{bmatrix} 2s+\frac{1}{s}+2 & -2s-\frac{1}{s}-2 & s \\ -2s-\frac{1}{s}-2 & 3s+2+\frac{1}{s} & -2s \\ s & -2s & 2+2s \end{bmatrix}$$

$$\Delta Z = s^3+6s^2+5s+2$$

DE DONDE

$$Z_{11} = \frac{1}{y_{11}} = \frac{s^4+6s^3+5s^2+2s}{2s^3+10s^2+6s+2}$$

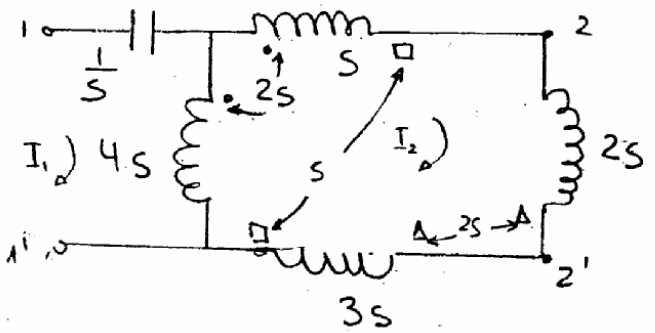
$$y_{31} = \frac{s^2+2s+1}{s^3+6s^2+5s+2}$$

$$Z_{31} = \frac{s(s^2+2s+1)}{s^3+5s^2+3s+1}$$

$$T_I = \frac{s(s^2+2s+1)}{2s^3+10s^2+6s+2}$$

$$T_V = 2y_{31}$$

PROBLEMA : DETERMINAR $Z_{11}, Y_{11}, Z_{21}, Y_{21}, T_V$ y T_I



SOLUCION : LAS ECUACIONES DE MALLA SON:

$$\left(4s + \frac{1}{s}\right) I_1 - (4s - 2s)\, I_2 = E_1$$

$$-(4s - 2s) I_1 + (10s - 4s + 2s - 4s)\, I_2 = 0$$

$$\Delta_Z = \begin{bmatrix} 4s + \frac{1}{s} & -2s \\ -2s & 4s \end{bmatrix}$$

$\Delta_Z = 12s^2 + 4$

$\Delta_{11} = 4s$

$\Delta_{12} = 2s$

$$y_{11} = \frac{\Delta_{11}}{\Delta} = \frac{1}{Z_{11}} = \frac{s}{3s^2 + 1}$$

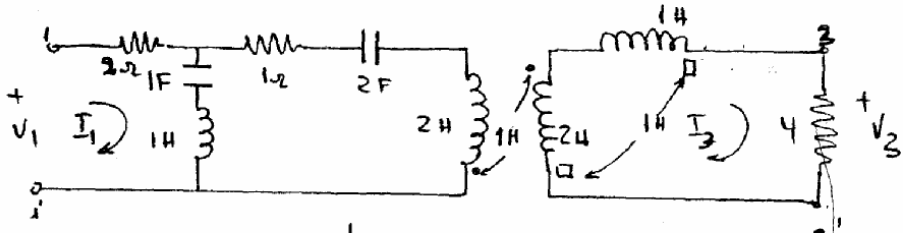
$$y_{21} = \frac{\Delta_{12}}{\Delta} = \frac{s}{6s^2 + 2}$$

$$T_V = \frac{E_2}{E_1} = \frac{(2s - 2s) I_2}{E_1} = 0$$

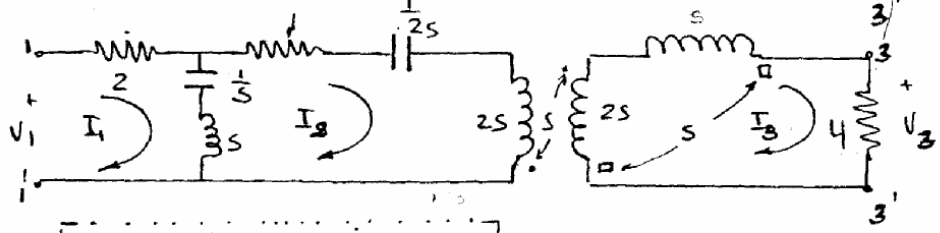
$$Z_{21} = \frac{E_2}{I_1} = 0$$

$$T_I = \frac{I_2}{I_1} = \frac{\Delta_{12}}{\Delta_{11}} = \frac{1}{2}$$

PROBLEMA: CALCULAR $Z_{11}, Y_{11}, Z_{31}, Y_{31}, T_V$. T_I



SOLUCION:



$$\Delta_Z = \begin{bmatrix} 2+s+\frac{1}{s} & -s-\frac{1}{s} & 0 \\ -s-\frac{1}{s} & 1+3s+\frac{1}{s}+\frac{1}{2s} & s \\ 0 & s & 3s+4-2s \end{bmatrix}$$

$$\Delta_Z = \frac{2s^5+26s^4+63s^3+44s^2+33s+4}{2s^2}$$

$$Z_{11} = \frac{1}{Y_{11}} = \frac{2s^5+26s^4+63s^3+44s^2+33s+4}{s(4s^3+26s^2+11s+12)}$$

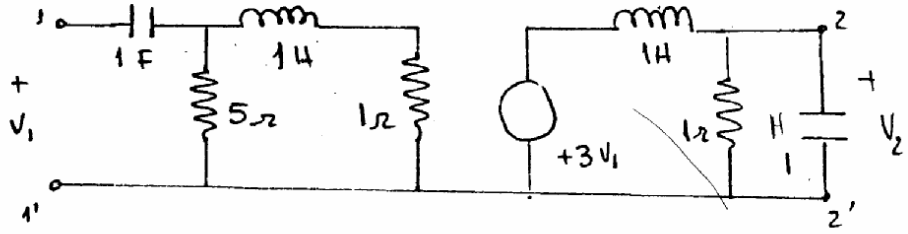
$$Y_{31} = \frac{2s^2(-s^2-1)}{2s^5+26s^4+63s^3+44s^2+33s+4}$$

$$T_V = Z_L Y_{31} = \frac{-8s^4-8s^2}{2s^5+26s^4+63s^3+44s^2+33s+4}$$

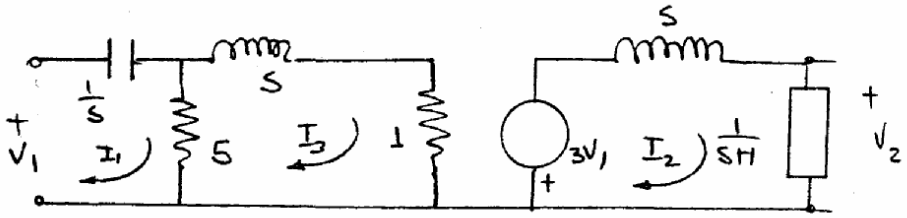
$$Z_{31} = \frac{T_V}{Y_{11}} = \frac{-8s^3-8s}{4s^3+26s^2+11s+12}$$

$$T_I = \frac{2s(-s^2-1)}{4s^3+26s^2+11s+12}$$

PROBLEMA: HALLAR $Z_{21}(s)$ PARA LA RED ACTIVA DE LA FIGURA



SOLUCION: EN EL DOMINIO DE LA FRECUENCIA



LAS ECUACIONES DE MALLA SON:

$$V_1 = \left(\frac{1}{s} + 5\right) I_1 - 5 I_3$$

$$0 = -5 I_1 + (6+s) I_3$$

$$-3V_1 = \left(s + \frac{1}{s+1}\right) I_2$$

$$V_2 = I_2 \left(\frac{1}{s+1}\right)$$

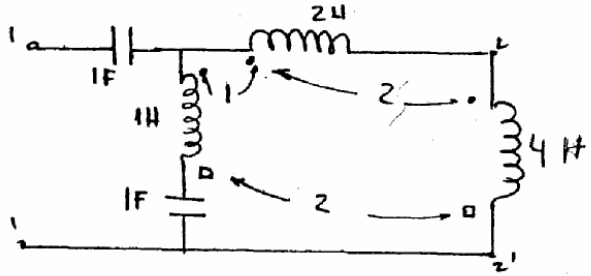
REDUCIENDO LAS ECUACIONES

$$-\frac{1}{3}\left(s + \frac{1}{s+1}\right)(s+1)\, V_2 = -5\left(\frac{5 I_1}{6+s}\right) + \left(\frac{1}{s} + 5\right) I_1$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{5 + \frac{1}{s} - \frac{25}{6+s}}{-\frac{1}{3}\left(s + \frac{1}{s+1}\right)(s+1)}$$

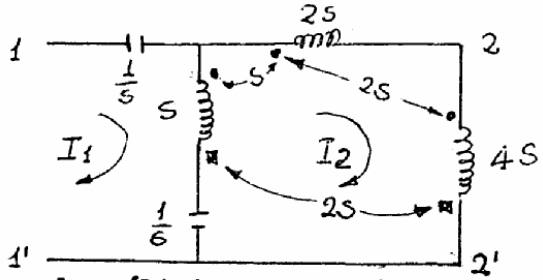
$$Z_{21} = \frac{-3(5s^2 + 6s + 6)}{s\,(6+s)(s^2+s+1)}$$

PROBLEMA: HALLAR $T_V = V_2/V_1$



SOLUCION:

En el dominio de la frecuencia:



Mediante el análisis matricial de mallas tenemos:

$$Z_M = \begin{bmatrix} \frac{2+s^2}{s} & \frac{2s^2-I}{s} \\ \frac{2s^2-I}{s} & \frac{5s^2+I}{s} \end{bmatrix}$$

$$\Delta Z_M = \left| \frac{(2+s^2)}{s}\frac{(5s^2+I)}{s} - \frac{(2s^2-I)^2}{s^2} \right|$$

$$\Delta Z_M = \frac{s^4 + I5s^2 + I}{s^2}$$

$$T_V = \frac{V_2}{V_I} = Z_L\, Y_{2I} \quad , \qquad Y_{2I} = \frac{\Delta_{I2}}{\Delta Z_M}$$

$$Y_{2I} = \frac{-(2s^2 - I)/s}{(s^4 + I5s^2 + I)/s^2}$$

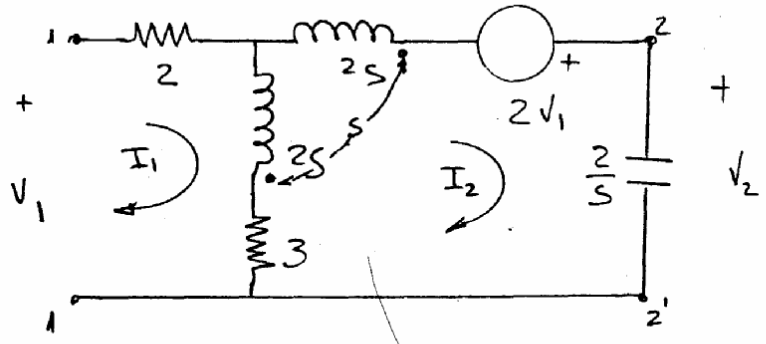
$$Y_{2I} = \frac{s(I - 2s^2)}{s^4 + I5s^2 + I}$$

$$Z_L = \cancel{4s} + \cancel{4s} - \cancel{4s} = 4s$$

$$T_V = \frac{(4s)s(I - 2s^2)}{s^4 + I5s^2 + I}$$

$$T_V = \frac{4s^2(I - 2s^2)}{s^4 + I5s^2 + I}$$

PROBLEMA : DETERMINAR T_V y T_I



SOLUCION : DADO QUE LA RED ES ACTIVA NO PODEMOS APLICAR LOS CONCEPTOS DE IMPEDANCIA DE ENTRADA Y TRANSFERENCIA. LA EC. MATRICIAL DE MALLAS:

$$\begin{bmatrix} 5+2s & -3-s \\ -3-s & 2s+3+\frac{2}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 2V_1 \end{bmatrix}$$

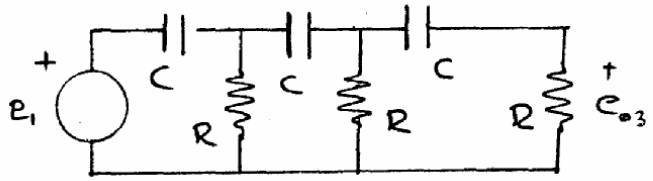
$$\Delta z = \frac{3s^3+10s^2+10s+10}{s}$$

$$V_2 = \frac{2}{s} I_2 = \frac{\frac{2}{s}\begin{vmatrix} 5+2s & V_1 \\ -3-s & 2V_1 \end{vmatrix}}{\Delta z} =$$

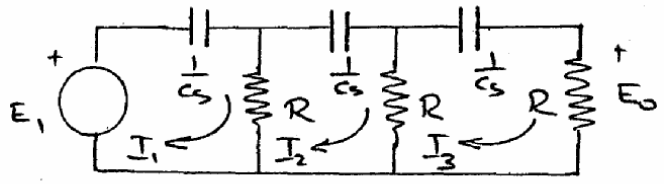
$$T_V = \frac{10s+26}{3s^3+10s^2+10s+10}$$

$$T_I = \frac{I_2}{I_1} = \frac{\begin{vmatrix} 5+2s & V_1 \\ -3-s & 2V_1 \end{vmatrix}}{\begin{vmatrix} V_1 & -3-s \\ 2V_1 & \frac{2s^2+3s+2}{s} \end{vmatrix}} \qquad T_I = \frac{5s^2+13s}{4s^2+9s+2}$$

PROBLEMA: HALLAR E_o/E_1



SOLUCION: DEL DOMINIO DEL TIEMPO A LA FRECUENCIA



LA EC. MATRICIAL DE MALLAS ES

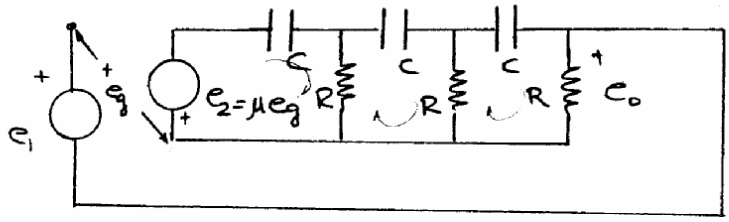
$$\begin{bmatrix} R+\frac{1}{Cs} & -R & 0 \\ -R & 2R+\frac{1}{sC} & -R \\ 0 & -R & 2R+\frac{1}{sC} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad \begin{bmatrix} E_1 \\ 0 \\ 0 \end{bmatrix}$$

$$I_3 = \frac{\begin{vmatrix} R+\frac{1}{Cs} & -R & E_1 \\ -R & 2R+\frac{1}{sC} & 0 \\ 0 & -R & 0 \end{vmatrix}}{\begin{vmatrix} R+\frac{1}{Cs} & -R & 0 \\ -R & 2R+\frac{1}{sC} & -R \\ 0 & -R & 2R+\frac{1}{sC} \end{vmatrix}} = \frac{E_1 R^2 s^3 C^3}{R^3 s^3 C^3 + 6R^2C^2s^2 + 5RCs + 1}$$

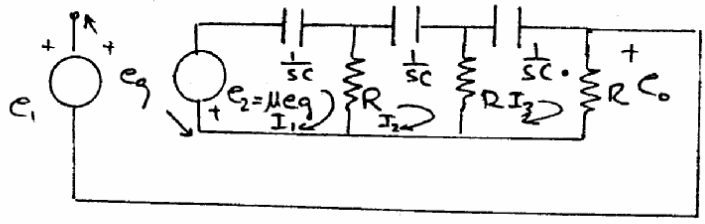
$$E_o = R I_3$$

$$\frac{E_o}{E_1} = \frac{(RCs)^3}{(RCs)^3 + 6(RCs)^2 + 5RCs + 1}$$

PROBLEMA : HALLAR E_0 / E_1



SOLUCION : IGUAL QUE ANTES EN EL DOMINIO DE LA FRECUENCIA



$$e_0 = R I_3$$

$$e_1 + e_0 - e_g = 0$$

$$\begin{bmatrix} R + \frac{1}{Cs} & -R & 0 \\ -R & 2R + \frac{1}{Cs} & -R \\ 0 & -R & 2R + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad \begin{bmatrix} -e_2 \\ 0 \\ 0 \end{bmatrix}$$

$$I_3 = -e_2 \frac{(RCs)^3}{(RCs)^3 + 6(RCs)^2 + 5RCs + 1}$$

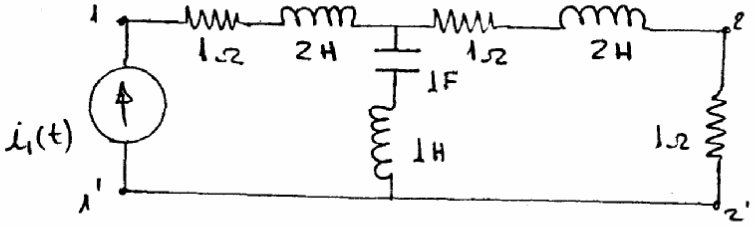
$$T = \frac{(RCs)^3}{(RCs)^3 + 6(RCs)^2 + 5RCs + 1}$$

$$\frac{RI_3}{-Re_2} = T \quad \therefore \quad \frac{e_0}{-Re_2} = T \quad \therefore \quad \frac{e_0}{-R\mu e_g} = T \quad \therefore \quad \frac{e_0}{-R\mu(e_1 + e_0)} = T$$

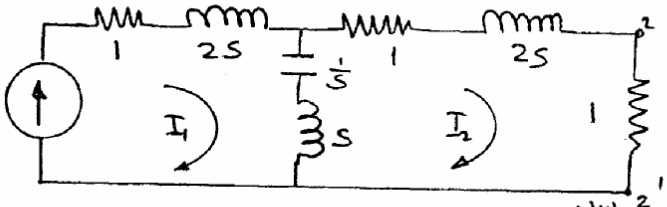
$$\frac{e_0}{e_1 + e_0} = -RT\mu \quad \therefore \quad e_0 = -RT\mu e_1 - RT\mu e_0 \quad \therefore \quad \frac{e_0}{e_1} = \frac{-RT\mu}{1 + RT\mu}$$

$$\frac{E_0}{E_1} = \frac{-\mu (RCs)^3}{(1+\mu)(RCs)^3 + 6(RCs)^2 + 5RCs + 1}$$

PROBLEMA: HALLAR LA FUNCION E_2/I_1. UBICAR LOS POLOS Y CEROS EN EL PLANO S. ¿CUAL SERA LA RESPUESTA FORZADA DE LA RED SI $i_1(t) = 100\, e^{-2t} \cos(t + 45°)$

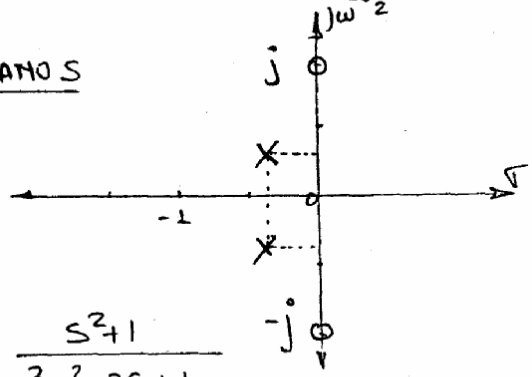


SOLUCION: EN EL DOMINIO DE LA FRECUENCIA



$$\Delta_{Zm} = \begin{bmatrix} 1+3s+\frac{1}{s} & -s-\frac{1}{s} \\ -s-\frac{1}{s} & 2+3s+\frac{1}{s} \end{bmatrix}$$

PLANO S



$$Z_{21} = Z_L \frac{\Delta_{12}}{\Delta_{11}} = \frac{s+\frac{1}{s}}{2+3s+\frac{1}{s}} = \frac{s^2+1}{3s^2+2s+1}$$

CEROS: $s^2+1=0 \quad \therefore \quad s_{z1,2} = \pm j$

POLOS: $3s^2+2s+1 \quad \therefore \quad s_{p1,p2} = -\frac{1}{3} \pm \frac{\sqrt{2}}{3} j$

LA FRECUENCIA COMPLEJA DE LA EXCITACION $S = -2+j$

$$Z_{21}(-2+j) = 0.486 \angle 14°$$

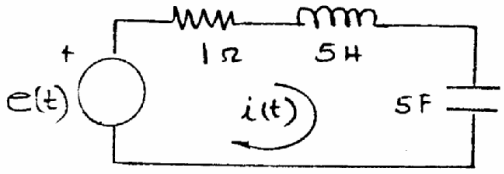
$$V_2 = Z_{21} \cdot I_1 = 0.486 \angle 14° \cdot 100 \angle 45°$$

$$V_2 = 48.6 \angle 59°$$

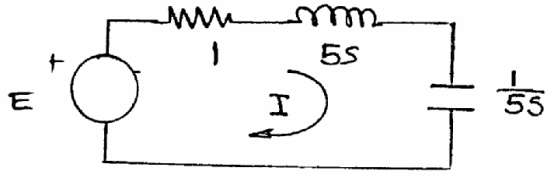
EN EL TIEMPO

$$v_2(t) = 48.6\, e^{-2t} \cos(t + 59°)$$

PROBLEMA: EN EL CIRCUITO, EL INTERRUPTOR SE CIERRA A $t=0$. DETERMINAR LAS FRECUENCIAS COMPLEJAS QUE APARECERAN EN LA SOLUCION $i(t)$ PARA $t>0$. INDIQUE SI SON FRECUENCIAS DE LA RESPUESTA LIBRE O FORZADA.



SOLUCION: EN EL DOMINIO DE LA FRECUENCIA



LA IMPEDANCIA DE EXCITACION

$$Z_{11} = 1 + 5S + \frac{1}{5S} = \frac{25S^2 + 5S + 1}{5S}$$

LA FRECUENCIA COMPLEJA $S = -2 + 3j$

$$Z_{11}(-2+3j) = \frac{25(-2+3j)^2 + 5(-2+3j) + 1}{5(-2+3j)}$$

PROBLEMA : CALCULAR LAS PARTES DE LA SIGUIENTE FUNCION

$$T(s) = \frac{s^2 - 2s + 1}{s^3 + s^2 + s + 1}$$

SOLUCION :

$$P_R\, T(s) = \frac{1}{2}\left[T(s) + T(-s)\right] = \frac{1}{2}\left[\frac{s^2-2s+1}{s^3+s^2+s+1} + \frac{s^2+2s+1}{-s^3+s^2-s+1}\right] = -\frac{3s^4+4s^2+1}{s^6+s^4-s^2-1}$$

$$I_R\, T(s) = \frac{1}{2}\left[T(s) - T(-s)\right] = \frac{1}{2}\left[\frac{s^2-2s+1}{s^3+s^2+s+1} - \frac{s^2+2s+1}{-s^3+s^2-s+1}\right] = \frac{s^5+4s^3+3s}{s^6+s^4-s^2-1}$$

$$Re\, T(j\omega) = -\left.\frac{3s^4+4s^2+1}{s^6+s^4-s^2-1}\right|_{s=j\omega} = \frac{3\omega^4-4\omega^2+1}{\omega^6-\omega^4-\omega^2+1}$$

$$j\, Im\, T(j\omega) = \left.\frac{s^5+4s^3+3s}{s^6+s^4-s^2-1}\right|_{s=j\omega} = \frac{j\omega^5-4j\omega^3+3j\omega}{-\omega^6+\omega^4+\omega^2-1}$$

$$Im\, T(j\omega) = \frac{\omega^5-4\omega^3+3\omega}{-\omega^6+\omega^4+\omega^2-1}$$

$$\left|T(j\omega)\right| = \left.\sqrt{T(s)\,.\,T(-s)}\right|_{s=j\omega} = \left.\sqrt{\frac{(s^2-2s+1)(s^2+2s+1)}{(s^3+s^2+s+1)(-s^3+s^2-s+1)}}\right|_{s=j\omega}$$

$$\left|T(j\omega)\right| = \left.\sqrt{\frac{s^4-2s^2+1}{s^6+s^4-s^2-1}}\right|_{s=j\omega} = \sqrt{\frac{\omega^4+2\omega^2+1}{-\omega^6+\omega^4+\omega^2-1}}$$

$$\beta(\omega) = tg^{-1} = \frac{\omega^5-4\omega^3+3\omega}{3\omega^4-4\omega^2+1}$$

PROBLEMA: CALCULAR LAS PARTES DE LA FUNCION DE RED

$$T(s) = \frac{s^3 - 5s^2 + 7s + 13}{s^3 + 4s^2 + 8s}$$

SOLUCION:

$$P_R T(s) = \frac{1}{2}\left[\frac{s^3 - 5s^2 + 7s + 13}{s^3 + 4s^2 + 8s} + \frac{-s^3 - 5s^2 - 7s + 13}{-s^3 + 4s^2 - 8s}\right]$$

$$P_R T(s) = \frac{1}{2}\left[\frac{-2s^6 - 50s^4 - 8s^2}{-s^6 - 64s^2}\right] = \frac{s^6 + 25s^4 + 4s^2}{s^6 + 64s^2} = \frac{s^4 + 25s^2 + 4}{s^4 + 64}$$

$$I_R T(s) = \frac{1}{2}\left[\frac{s^3 - 5s^2 + 7s + 13}{s^3 + 4s^2 + 8s} - \frac{-s^3 - 5s^2 - 7s + 13}{-s^3 + 4s^2 - 8s}\right]$$

$$I_R T(s) = \frac{1}{2}\left[\frac{18s^5 + 110s^3 - 208s}{-s^6 - 64s^2}\right] = \frac{-9s^5 - 55s^3 + 104s}{s^6 + 64s^2}$$

$$\mathrm{Re}\, T(j\omega) = \frac{\omega^4 - 25\omega^2 + 4}{\omega^4 + 64}$$

$$\mathrm{Im}\, T(j\omega) = \frac{-9\omega^5 + 55\omega^3 + 104\omega}{-\omega^6 - 64\omega^2}$$

$$|T(j\omega)| = \left.\sqrt{\frac{(s^3 - 5s^2 + 7s + 13)(-s^3 - 5s^2 - 7s + 13)}{(s^3 + 4s^2 + 8s)(-s^3 + 4s^2 - 8s)}}\right|_{s = j\omega}$$

$$|T(j\omega)| = \sqrt{\frac{\omega^6 + 11\omega^4 + 179\omega^2 + 169}{\omega^6 + 64\omega^2}}$$

$$\beta(\omega) = \mathrm{tg}^{-1}\, \frac{-9\omega^5 + 55\omega^3 + 104\omega}{-\omega^6 + 25\omega^4 - 4\omega^2}$$

PROBLEMA : CALCULAR LAS PARTES DE LA FUNCION DE RED

$$T(s) = \frac{s^3 + 3s + 2}{s^4 + 2s^3 + 3s + 1}$$

SOLUCION :

$$P_rT(S) = \frac{I}{2}\left|T(S) + T(-S)\right|$$

$$P_rT(S) = \frac{I}{2}\left[\frac{S^3 + 3S + 2}{S^4 + 2S^3 + 3S + I} + \frac{-S^3 - 3S + 2}{S^4 - 2S^3 - 3S + I}\right]$$

$$P_rT(S) = \frac{I}{2}\left[\frac{-4S^6 - I4S^4 - I8S^2 + 4}{S^8 - 4S^6 - IOS^4 - 9S^2 + I}\right]$$

$$P_rT(S) = \frac{2S^6 + 7S^4 + 9S^2 - 2}{-S^8 + 4S^6 + IOS^4 + 9S^2 - I}$$

$$I_rT(S) = \frac{I}{2}\left|T(S) - T(-S)\right|$$

$$I_rT(S) = \frac{I}{2}\left[\frac{S^3 + 3S + 2}{S^4 + 2S^3 + 3S + I} - \frac{-S^3 - 3S + 2}{S^4 - 2S^3 - 3S + I}\right]$$

$$I_rT(S) = \frac{I}{2}\left[\frac{2S^7 + 6S^5 - 6S^3 - 6S}{S^8 - 4S^6 - IOS^4 - 9S^2 + I}\right]$$

$$I_rT(S) = \frac{-S^7 - 3S^5 + 3S^3 + 3S}{-S^8 + 4S^6 + IOS^4 + 9S^2 - I}$$

$$R_eT(jW) = P_rT(S)\Big|_{S = jW}$$

$$R_eT(jW) = \frac{2(jW)^6 + 7(jW)^4 + 9(jW)^2 - 2}{-(jW)^8 + 4(jW)^6 + IO(jW)^4 + 9(jW)^2 - I}$$

$$ReT(jW) = \frac{-2W^6 + 7W^4 - 9W^2 - 2}{-W^8 - 4W^6 + IOW^4 - 9W^2 - I}$$

$$R_eT(jW) = \frac{2W^6 - 7W^4 + 9W^2 + 2}{W^8 + 4W^6 - IOW^4 + 9W^2 + I}$$

$$I_m T(jW) = \frac{I}{j} I_r T(S) \Big|_{S = jW}$$

$$I_m T(jW) = \frac{I}{j}\left[\frac{-(jW)^7 - 3(jW)^5 + 3(jW)^3 + 3(jW)}{-(jW)^8 + 4(jW)^6 + IO(jW)^4 + 9(jW)^2 - I}\right]$$

$$I_m T(jW) = \frac{-W^7 - 3W^5 - 3W^3 + 3W}{-W^8 - 4W^6 + IOW^4 - 9W^2 - I}$$

$$I_m T(jW) = \frac{W^7 + 3W^5 + 3W^3 - 3W}{W^8 + 4W^6 - IOW^4 + 9W^2 + I}$$

$$\left|T(jW)\right| = \sqrt{T(S) . T(-S)} \Big|_{S = jW}$$

$$\left|T(jW)\right| = \left|\sqrt{\frac{S^3 + 3S + 2}{S^4 + 2S^3 + 3S + I} \cdot \frac{-S^3 - 3S + 2}{S^4 - 2S^3 - 3S + I}}\right|_{S = jW}$$

$$\left|T(jW)\right| = \left|\sqrt{\frac{-S^6 - 6S^4 - 9S^2 + 4}{S^8 - 4S^6 - IOS^4 - 9S^2 + I}}\right|_{S = jW}$$

$$\left|T(jW)\right| = \sqrt{\frac{W^6 - 6W^4 + 9W^2 + 4}{W^8 + 4W^6 - IOW^4 + 9W^2 + I}}$$

$$\beta(W) = tg^{-I} . \frac{I_m T(jW)}{ReT(jW)}$$

$$\beta(W) = tg^{-I} \frac{W^7 + 3W^5 + 3W^3 - 3W}{2W^6 - 7W^4 + 9W^2 + 2}$$

$$\alpha(W) = \ln\left|T(jW)\right|$$

$$\alpha(W) = \ln \sqrt{\frac{W^6 - 6W^4 + 9W^2 + 4}{W^8 + 4W^6 - IOW^4 + 9W^2 + I}}$$

$$\gamma(W) = \ln\sqrt{\frac{W^6 - 6W^4 + 9W^2 + 4}{W^8 + 4W^6 - IOW^4 + 9W^2 + I}} + j\,tg^{-I}\left[\frac{W^7 + 3W^5 + 3W^3 - 3W}{2W^6 - 7W^4 + 9W^2 + 2}\right]$$

PROBLEMA: CONSTRUIR $T(s)$ A PARTIR DE LA FUNCION:

$$R_e\, T(j\omega) = \frac{1}{1+\omega^6}$$

SOLUCION:

$$R_e\, T(j\omega)\Big|_{\omega^2=-s^2} = P_R\, T(s) \qquad \therefore \quad P_R\, T(s) = \frac{1}{1-s^6}$$

$$Q(s)\cdot Q(-s) = 1-s^6 = (1-s^2)(1+s^2+s^4) = (1-s)(1+s+s^2)(1-s+s^2)(1+s)$$

$$Q(s) = (1+s)(1+s+s^2)$$

$$T(s) = \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{(1+s)(1+s+s^2)}$$

$$P_R\, T(s) = P_R\left[\frac{P(s)\cdot Q(-s)}{Q(s)\cdot Q(-s)}\right] = \frac{1}{1-s^6}$$

$$P_R\left[(a_3 s^3 + a_2 s^2 + a_1 s + a_0)(-s^3+2s^2-2s+1)\right] = 1$$

$$\begin{cases} 2a_2 - a_1 = 0 \\ a_2 - 2a_1 + 2a_0 = 0 \end{cases}$$

$$a_0 = 1 \qquad a_1 = \frac{4}{3} \qquad a_2 = \frac{2}{3} \qquad a_3 = 0$$

$$T(s) = \frac{2s^2+4s+3}{3(s^3+2s^2+2s+1)}$$

PROBLEMA: CONSTRUIR T(s) A PARTIR DE LA FUNCION

$$\mathcal{R}e\, T(j\omega) = \frac{1+\omega^4}{(1+\omega^2)^2}$$

SOLUCION:

$$\mathcal{P}_R\, T(s) = \frac{1+s^4}{(1-s^2)^2} = \frac{1+s^4}{(1+s)^2(1-s)^2} = \mathcal{P}_R \frac{P(s)Q(-s)}{Q(s)Q(-s)}$$

$$Q(s) = s^2+2s+1$$

$$Q(-s) = s^2-2s+1$$

$$\mathcal{P}_R\left[P(s)Q(-s)\right] = \mathcal{P}_R\left[(a_0+a_1 s+a_2 s^2)(s^2-2s+1)\right] = 1+s^4$$

$$a_0 = 1$$

$$a_2 = 1$$

$$a_1 = 1$$

$$T(s) = \frac{s^2+s+1}{s^2+2s+1}$$

PROBLEMA: CONSTRUIR T(S) A PARTIR DE LA FUNCION:

$$\mathrm{Re}\, T(j\omega) = \frac{1}{1-\omega^2+\omega^4}$$

SOLUCION:

I) $P_r T_{(s)} = \dfrac{1}{1 + S^2 + S^4}$

2) $N_{(s^2)} = Q_{(s)}\, Q_{(-s)} = 1 + S^2 + S^4$

$$N_{(s^2)} = (1 + S + S^2)(1 - S + S^2)$$
$$Q_{(s)} = (1 + S + S^2)$$

3) $T_{(s)} = \dfrac{a_o + a_I S + a_2 S^2}{1 + S + S^2}$

$$M_{(s^2)} = 1 = P_r\left[(a_o + a_I S + a_2 S^2)(1 - S + S^2)\right]$$

$$1 = a_o + (a_o - a_I + a_2)S^2 + a_2' S^4$$

de donde:

$a_o = 1$; $a_I = 1$; $a_2 = 0$

Luego:

$$T_{(S)} = \frac{1 + S}{S^2 + S + 1}$$

PROBLEMA : DADA $\quad Re\ Z(j\omega) = \dfrac{3\omega^2+8}{\omega^2+4}$

CALCULAR $Z(s)$ CON PARTE IMAGINARIA MÍNIMA

SOLUCION :

$$P_R T(s) = Re\ Z(j\omega)\Big|_{j\omega=s}$$

$$P_R T(s) = \frac{-3s^2+8}{-s^2+4}$$

$$P_R Z(s) = \frac{M(s^2)}{N(s^2)}$$

$$N(s^2) = Q(s).Q(-s)$$

$$N(s^2) = 4-s^2 = (2+s)(2-s)$$

$$Q(s) = 2+s$$

LUEGO $\quad Z(s) = \dfrac{P(s)}{Q(s)} = \dfrac{a_1 s + a_0}{s+2}$

Y $\quad -3s^2+8 = P_R\left[(a_1 s + a_0)(2-s)\right]$

$$\therefore \quad a_0 = 4$$
$$a_1 = 3$$

LA FUNCION $Z(s)$ CON PARTE IMAGINARIA MINIMA ES:

$$Z(s) = \frac{3s+4}{s+2}$$

PROBLEMA: CONSTRUIR T(s) A PARTIR DE

$$\operatorname{Im} T(j\omega) = \frac{\omega^5 - 6\omega^3 + 5\omega}{1+\omega^6}$$

SOLUCION:

I) $$I_r T_{(S)} = \frac{S^5 + 6S^3 + 5S}{I - S^6}$$

2) $$N_{(S^2)} = Q_{(S)} Q_{(-S)} = I - S^6$$

$$N_{(S^2)} = (I + S^3)(I - S^3)$$

$$Q_{(S)} = (I + S^3)$$

3) $$T_{(S)} = \frac{a_o + a_I S + a_2 S^2 + a_3 S^3 + a_4 S^4}{I + S^3}$$

$$S^5 + 6S^3 + 5S = I_r \left[(a_o + a_I S + a_2 S^2 + a_3 S^3 + a_4 S^4)(I - S^3) \right]$$

$$S^5 + 6S^3 + 5S = a_I S + (a_3 - a_o) S^3 - a_2 S^5 - a_4 S^7$$

De donde:

$$a_4 = 0 \; ; \; a_3 = a_o + 6 \; ; \; a_2 = -I \; ; \; a_I = 5$$

luego:

$$T_{(S)} = \frac{a_o + 5S - S^2 + (a_o + 6)S^3}{I + S^3}$$

entonces para que no exista término independiente: $a_o + 6 = 0$

luego: $a_o = -6$

finalmente tenemos:

$$T_{(S)} = \frac{-6 + 5S - S^2}{I + S^3}$$

PROBLEMA : CONSTRUIR T(S) A PARTIR DE

$$I_m\, T(j\omega) = \frac{\omega(1-\omega^2)}{(1+\omega^2)^2}$$

SOLUCION :

$$I_R\, T(s) = \frac{s+s^3}{(1-s^2)^2}$$

$$s\,M(s^2) = s+s^3$$

$$N(s^2) = (1-s^2)^2 = (1+s)^2(1-s)^2$$

$$Q(s) = (1+s)^2$$

$$T(s) = \frac{a_0+a_1 s+a_2 s^2+a_3 s^3}{s^2+2s+1}$$

$$s+s^3 = I_R\left[(a_0+a_1 s+a_2 s^2+a_3 s^3)(s^2-2s+1)\right]$$

$$a_3 = 0$$

$$a_1 - 2a_0 = 1 \quad \therefore \quad a_1 = 1+2a_0$$

$$a_1 - 2a_2 = 1 \qquad a_2 = a_0$$

$$T(s) = \frac{a_0+(1+2a_0)s+a_0 s^2}{s^2+2s+1}$$

ELIMINANDO EL TERMINO INDEPENDIENTE

$$a_0 = 0$$

$$T(s) = \frac{s}{s^2+2s+1}$$

PROBLEMA: CONSTRUIR T(S) A PARTIR DE

$$I_m\, T(j\omega) = \frac{-\omega^3}{1-\omega^2+\omega^4}$$

SOLUCION:

$$1)\ I_r T(S) = \frac{S^3}{I + S^2 + S^4}$$

$$2)\ N(S^2) = Q(S)Q(-S) = I + S^2 + S^4$$

$$N(S^2) = (I + S + S^2)(I - S + S^2)$$

$$Q(S) = (I + S + S^2)$$

$$3)\ T(S) = \frac{a_o + a_I S + a_2 S^2 + a_3 S^3}{I + S + S^2}$$

$$S^3 = I_r\left[(a_o + a_I S + a_2 S^2 + a_3 S^3)(I - S + S^2)\right]$$

$$S^3 = (a_I - a_o)S + (a_I - a_2 + a_3)S^3 + (a_3)S^5$$

De donde:

$$a_3 = 0 \quad ; \quad a_2 = a_o - I \quad ; \quad a_I = a_o$$

luego tenemos:

$$T(S) = \frac{a_o + a_o S + (a_o - I)S^2}{I + S + S^2}$$

Para que no exista término independiente a_o debe ser: $a_o = I$

finalmente:

$$T(S) = \frac{I + S}{I + S + S^2}$$

PROBLEMA : DADA $\operatorname{Im} Z(j\omega) = \dfrac{\omega^3 + 6\omega}{\omega^2 + 4}$

CALCULAR $Z(s)$ CON PARTE REAL MINIMA

SOLUCION :

$$I_R\, Z(s) = \frac{-s^3 + 6s}{-s^2 + 4}$$

$$Q(s) \cdot Q(-s) = -s^2 + 4 = (s+2)(-s+2)$$

$$Z(s) = \frac{a_2 s^2 + a_1 s + a_0}{(s+2)}$$

$$-s^3 + 6s = I_e \left[(a_2 s^2 + a_1 s + a_0)(-s+2) \right] = -a_2 s^3 + (2a_1 + a_0)s$$

$$a_2 = 1$$
$$2a_1 - a_0 = 6$$

$$Z(s) = \frac{s^2 + a_1 s + 2(a_1 - 3)}{s+2} = \frac{s + (a_1 - 2)s + 2(a_1 - 3)}{s+2}$$

Y PARA QUE NO EXISTA PARTE INDEPENDIENTE

$$a_1 = 2$$

$$Z(s) = \frac{s^2 + 2s - 2}{s+2}$$

PROBLEMA :

DADA $\left|T(j\omega)\right|^2 = \dfrac{1-\omega^2+\omega^4}{1+\omega^4}$

HALLAR T(s). DE FASE MINIMA

SOLUCION :

$$\left|T(j\omega)\right|^2_{j\omega=s} = \frac{1+s^2+s^4}{1+s^4} = \frac{(s^2+s+1)(s^2-s+1)}{(1+\sqrt{2}s+s^2)(1-\sqrt{2}s+s^2)}$$

$$1+s^2+s^4 = (as^2+bs+c)(as^2-bs+c)$$

$1 = a^2 \quad \therefore \quad a = 1$

$1 = c^2 \quad \therefore \quad c = 1$

$1 = 2ac - b^2 \quad \therefore \quad b = 1$

$P(s) = s^2+s+1$

$P(-s) = s^2-s+1$

$Q(s) = 1+\sqrt{2}s+s^2$

$Q(-s) = 1-\sqrt{2}s+s^2$

$$T(s) = \frac{s^2+s+1}{1+\sqrt{2}s+s^2}$$

PROBLEMA: DADA $\left|T(j\omega)\right|^2 = \dfrac{1}{1+\omega^6}$

SOLUCION:

$$\left|T(j\omega)\right|^2\Big|_{j\omega = s} = T(s)\cdot T(-s) = \frac{1}{1-s^6}$$

$$T(s)\cdot T(-s) = \frac{1}{(1+s^3)(1-s^3}$$

$$T(s)\cdot T(-s) = \frac{1}{(1+s)(1-s+s^2)(1-s)(1+s+s^2)}$$

DONDE:

$$Q(s) = (1+s)(1+s+s^2)$$

$$Q(-s) = (1-s)(1-s+s^2)$$

$$P(s) = 1$$

$$P(-s) = 1$$

LUEGO $T(s) = \dfrac{1}{(1+s)(1+s+s^2)}$

PROBLEMA: DADA $\left|T_{(j\omega)}\right|^2 = \dfrac{1+\omega^2}{1-\omega^2+\omega^4}$

HALLAR $T(s)$.

SOLUCION:

I) $\left|T_{(jW)}\right|^2 = T_{(S)} \cdot T_{(-S)}$

$$T_{(S)} \cdot T_{(-S)} = \frac{I - S^2}{I + S^2 + S^4} = \frac{(I + S)(I - S)}{(I + S + S^2)(I - S + S^2)}$$

De aquí obtenemos las soluciones:

$$T_{I(S)} = \frac{I + S}{I + S + S^2} \quad y \quad T_{2(S)} = \frac{I - S}{I + S + S^2}$$

La $T_{I(S)}$ es la fase mínima

La $T_{2(S)}$ puede ser obtenido de $T_{I(S)}$ multiplicando $T_{I(S)}$ por:

$$T_{tp(S)} = \frac{I - S}{I + S}$$

La solución general es:

$$T_{(S)} = \frac{I + S}{I + S + S^2} \cdot T_{tp(S)}$$

donde $T_{tp(S)}$ es una función de todo paso arbitraria

$$T_{(S)} = \frac{\cancel{I + S}}{I + S + S^2} \cdot \frac{I - S}{\cancel{I + S}}$$

$$T_{(S)} = \frac{I - S}{I + S + S^2}$$

PROBLEMA : DADA $\left|T(j\omega)\right|^2 = 8\left|\dfrac{\omega^6+\omega^4+4\omega^2+4}{\omega^4-8\omega^2+16}\right|$

CALCULAR T(s) MINIMA

SOLUCION :

$$T(s)\,.\,T(-s)\Big|_{s=j\omega} = \left|T(j\omega)\right|^2 = 8\,\frac{-s^6+s^4-4s^2+4}{s^4+8s^2+16}$$

$$T(s)\,.\,T(-s) = 8\,\frac{-s^2(s^4+4)+s^4+4)}{(s^2+4)^2} = 8\,\frac{(s^4+4)(-s^2+1)}{(s^2+4)^2}$$

$$T(s)\,.\,T(-s) = \frac{\sqrt{8}\,(s^2+2s+2)(s+1)}{s^2+4}\,.\,\frac{\sqrt{8}\,(s^2-2s+2)(-s+1)}{s^2+4}$$

LUEGO

$$T(s) = \sqrt{8}\,\frac{(s^2+2s+2)(s+1)}{s^2+4}$$

$$T(s) = \sqrt{8}\,\frac{s^3+3s^2+4s+2}{s^2+4}$$

PROBLEMA: ENCONTRAR T(S) MINIMA DADA:

$$\beta(\omega) = tg^{-1}(-\omega^3)$$

SOLUCION:

$$tg\,\beta(\omega) = -\omega^3$$

$$G(j\omega) = \frac{1 + j\,tg\,\beta(\omega)}{1 - j\,tg\,\beta(\omega)}$$

$$G(j\omega) = \frac{1 - j\omega^3}{1 + j\omega^3}$$

$$G(s) = \frac{1 + s^3}{1 - s^3} = \frac{(1+s)(1-s+s^2)}{(1-s)(1+s+s^2)}$$

PUEDE SER:

$Q_1(s) = 1 \pm s$ $\qquad P_1(s) = 1 - s + s^2$

$Q_2(s) = 1 + s + s^2$ $\qquad P_2(s) = 1 + s$

$Q_3(s) = (1-s)(1+s+s^2)$ $\qquad P_3(s) = 1$

LAS SOLUCIONES SERAN:

$$T_1(s) = \frac{1 - s + s^2}{1 + s}$$

$$T_2(s) = \frac{1 + s}{1 + s + s^2}$$

$$T_3(s) = \frac{1}{(1-s)(1+s+s^2)}$$

$T_2(s)$ MINIMA

PROBLEMA : ENCONTRAR T(S) DADA :

$$\beta(\omega) = tg^{-1} \frac{\omega(\omega^2-1)}{1+\omega^4}$$

SOLUCION :

$$tg\,\beta(\omega) = \frac{\omega(\omega^2-1)}{1+\omega^4}$$

$$G(j\omega) = \frac{1 + j\,tg\,\beta(\omega)}{1 - j\,tg\,\beta(\omega)}$$

$$G(j\omega) = \frac{1+\omega^4 + j\omega^3 - j\omega}{1+\omega^4 - j\omega^3 + j\omega}$$

$$G(s) = \frac{1 - s - s^3 + s^4}{1 + s + s^3 + s^4}$$

$$G(s) = \frac{(s-1)^2(s^2+s+1)}{(s+1)^2(s^2-s+1)}$$

PUEDE SER :

$$Q_1(s) = (s+1)^2 \qquad P_1(s) = s^2+s+1$$

$$Q_2(s) = (s^2-s+1) \qquad P_2(s) = (s-1)^2$$

$$Q_3(s) = (s+1)^2(s^2-s+1) \qquad P_3(s) = 1$$

LAS SOLUCIONES SERAN :

$$T_1(s) = \frac{s^2+s+1}{(s+1)^2}$$

$$T_2(s) = \frac{(s-1)^2}{s^2-s+1}$$

$$T_3(s) = \frac{1}{(s+1)^2(s^2-s+1)}$$

T_3 FUNCION DE FASE MINIMA

PROBLEMA: ENCONTRAR T(S) DE FASE MINIMA DADA

$$\beta(\omega) = tg^{-1} \frac{\omega}{\omega^2 - 1}$$

SOLUCION:

$$tg\,\beta(\omega) = \frac{\omega}{\omega^2 - 1}$$

$$G(j\omega) = \frac{1 + j\,tg\,\beta(\omega)}{1 - j\,tg\,\beta(\omega)}$$

$$G(j\omega) = \frac{\omega^2 - 1 + j\omega}{\omega^2 - 1 - j\omega}$$

$$G(s) = \frac{1 - s + s^2}{1 + s + s^2}$$

ENTONCES: $Q(s) = 1 + s + s^2$

$P(s) = 1$

LA SOLUCION ES: $T(s) = \dfrac{1}{1 + s + s^2}$

PROBLEMA: DIBUJAR LA MAGNITUD Y LA FASE DE LA FUNCION

$$T(s) = \frac{s^2 - 2s + 1}{s^3 + s^2 + s + 1}$$

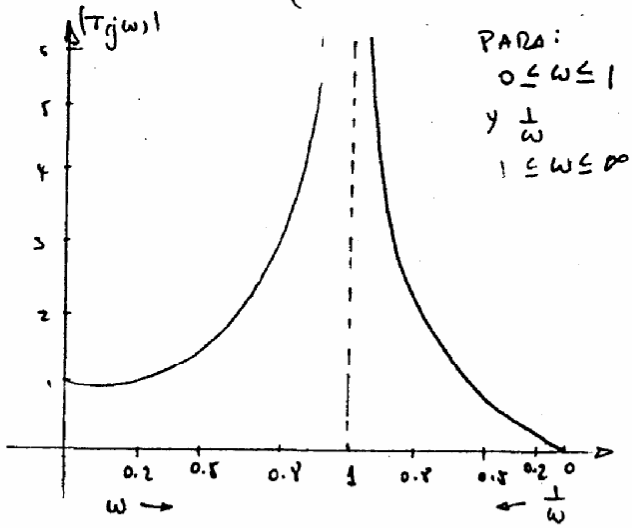
SOLUCION:

$$T(j\omega) = \frac{(1-\omega^2) + j\omega(-2)}{(1-\omega^2) + j\omega(1-\omega^2)}$$

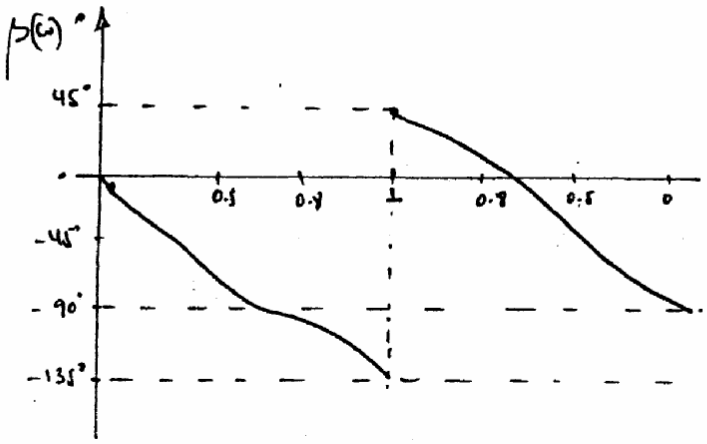
$$|T(j\omega)| = \sqrt{\frac{(1-\omega^2)^2 + 4\omega^2}{(1-\omega^2)^2 + \omega^2(1-\omega^2)^2}} = \frac{\sqrt{1+\omega^2}}{1-\omega^2}$$

$$\beta(\omega) = tg^{-1}\frac{-2\omega}{1-\omega^2} - tg^{-1}\frac{\omega(1-\omega^2)}{(1-\omega^2)}$$

ω	$\lvert H(j\omega)\rvert$	$\beta(\omega)$
0	1	0°
0.1	1.01	-16.9°
0.2	1.06	-34°
0.5	1.49	-79.6°
0.8	3.57	-116°
1.0-	∞	-135°
1.0+	∞	+45°
1.25	2.84	+26°
2.0	0.75	-10.4°
5.0	0.21	-56°
10.0	0.10	-73.1°
∞	0	-90°



PARA:
$0 \le \omega \le 1$
y $\frac{1}{\omega}$
$1 \le \omega \le \infty$



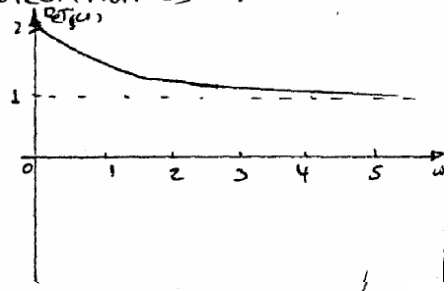
PROBLEMA: GRAFICAR EN FUNCION DE ω LAS PARTES: $Re\,T(j\omega)$, $Im\,T(j\omega)$, $|T(j\omega)|$ y $\beta(\omega)$ DE LA FUNCION DE RED:

$$T(s) = \frac{s^2+2s+2}{s+1}$$

SOLUCION: RESOLVIENDO SUS PARTES ENCONTRAMOS QUE:

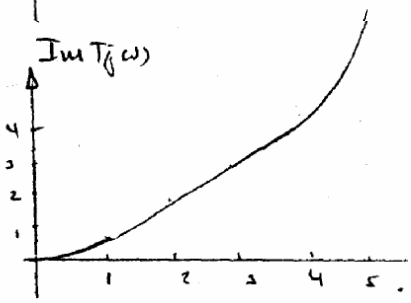
$$Re\,T(j\omega) = \frac{\omega^2+2}{\omega^2+1}$$

ω	$Re\,T(j\omega)$
0	2
0.5	1.8
1	3/2
2	6/5
5	27/26
∞	1



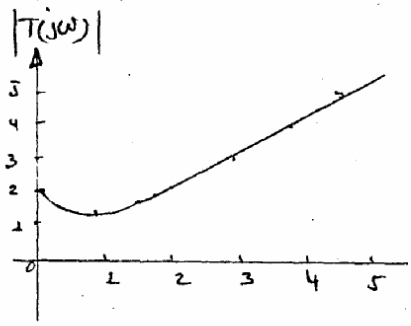
$$Im\,T(j\omega) = \frac{\omega^3}{\omega^2+1}$$

ω	$Im\,T(j\omega)$
0	0
1	1/2
2	8/5
3	27/10
5	125/26
10	1000/101
⋮	
∞	∞



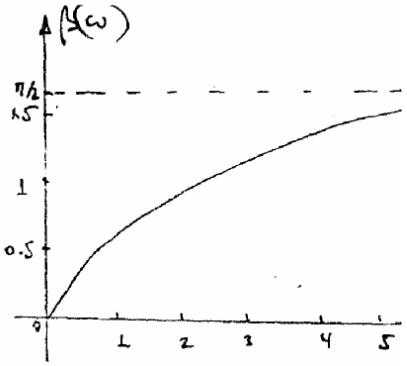
$$|T(j\omega)| = \sqrt{\frac{\omega^4+4}{\omega^2+1}}$$

ω	$\lvert T(j\omega)\rvert$
0	2
1	1.58
2	2
3	2.9
5	4.9
⋮	⋮
∞	∞



$$\beta(\omega) = tg^{-1}\frac{\omega^3}{\omega^2+2}$$

ω	$\beta(\omega)$
0	0
1	0.32
2	0.92
3	1.35
⋮	
∞	$\pi/2$



ROBLEMA: GRAFICAR $Re\,T(j\omega)$, $Im\,T(j\omega)$, $|T(j\omega)|$ y $\beta(\omega)$ DE LA FUNCION DE RED

$$T(s) = \frac{s^2 - 2s + 1}{s^3 + s^2 + s + 1}$$

OLUCION:

$$P_R T(s) = \frac{1}{2}\left[T(s) + T(-s)\right] = \frac{1}{2}\left[\frac{s^2 - 2s + 1}{s^3 + s^2 + s + 1} + \frac{s^2 + 2s + 1}{-s^3 + s^2 - s + 1}\right]$$

$$P_R T(s) = \frac{1}{2}\left[\frac{6s^4 + 8s^2 + 2}{-s^6 - s^4 + s^2 + 1}\right]$$

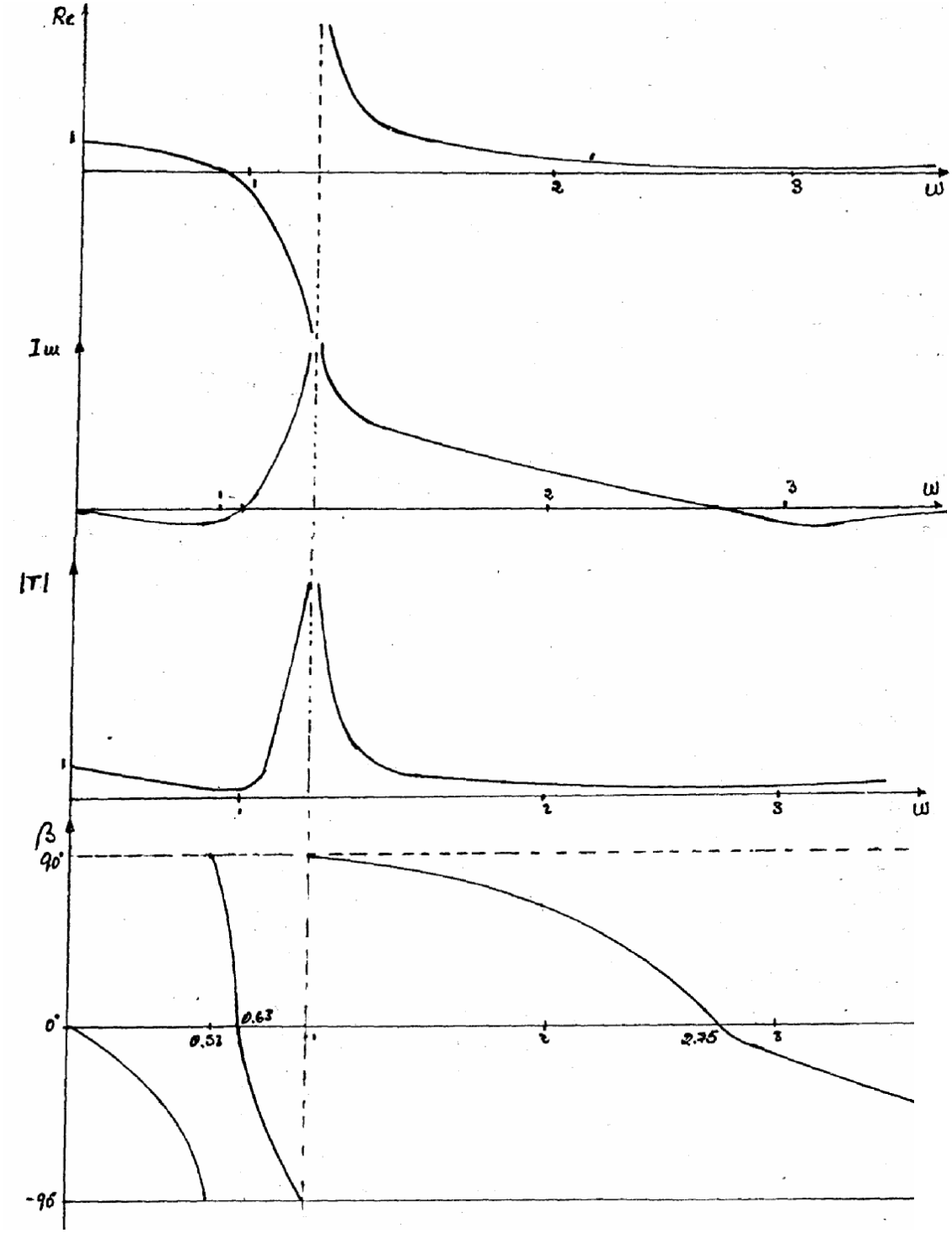
$$Re\,T(j\omega) = P_R T(s)\Big|_{s=j\omega} = \left[\frac{6\omega^4 - 8\omega^2 + 2}{\omega^6 - \omega^4 - \omega^2 + 1}\right]\frac{1}{2} = \frac{3\omega^4 - 4\omega^2 + 1}{\omega^6 - \omega^4 - \omega^2 + 1}$$

$$I_R T(s) = \frac{1}{2}\left[T(s) - T(-s)\right] = \frac{1}{2}\left[\frac{-2s^5 - 8s^3 - 6s}{-s^6 - s^4 + s^2 + 1}\right]$$

$$j\,Im\,T(j\omega) = I_R T(s)\Big|_{s=j\omega} = \frac{1}{2}\left[\frac{-2j\omega^5 + 8j\omega^3 - 6j\omega}{\omega^6 - \omega^4 - \omega^2 + 1}\right] = \frac{-\omega^5 + 8\omega^3 - 3\omega}{\omega^6 - \omega^4 - \omega^2 + 1}$$

$$|T(j\omega)| = \sqrt{Re\,T(j\omega)^2 + Im\,T(j\omega)^2} \qquad \beta = tg^{-1}\frac{Im\,T(j\omega)}{Re\,T(j\omega)}$$

ω	Re	Im	|T|	β
0	1	0	1	0
0.1	0.9	-0.29	1.01	-16.9
0.01	0.997	-0.029	1.0001	-1.71
0.55	0.102	-0.583	0.592	-80.1
0.57	0.028	-0.48	0.488	-86.6
0.58	-0.01	-0.416	0.416	88.67
0.59	-0.05	-0.346	0.35	81.72
0.6	-0.09	-0.268	0.284	71.12
0.625	-0.203	-0.03	0.205	9.35
0.63	-0.226	0.021	0.227	-5.52
0.64	-0.274	0.142	0.309	-27.32
0.9	-4.16	38.89	39.117	-83.89
0.99	-49.24	4898.8	4899.12	-89.42
1.000	600.74	800998.9	800999.2	89.99
1.5	1.41	2.93	3.25	64.25
2	0.73	0.67	0.93	38.23
0.7	0.4002	0.0179	0.4006	2.562
0.8	0.372	-0.0188	0.3726	-1.81
0.9	0.347	-0.038	0.349	-5.94
5	0.118	-0.143	0.185	-50.3
10	0.029	-0.092	0.097	-72.16

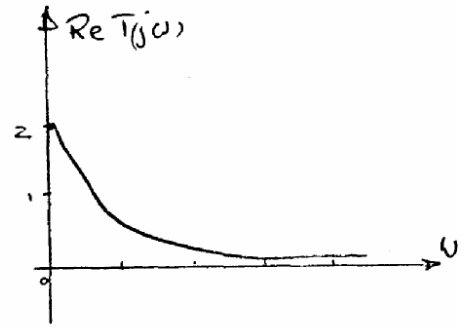

Re
1
1
2
3
ω
Im
1
2
3
ω
|T|
1
1
2
3
ω
β
90°
0°
0.63
0.51
1
2
2.75
3
-90

PROBLEMA: GRAFICAR $Re\,T(j\omega)$, $Im\,T(j\omega)$, $|T(j\omega)|$ y $\beta(\omega)$ DE LA FUNCION DE RED:

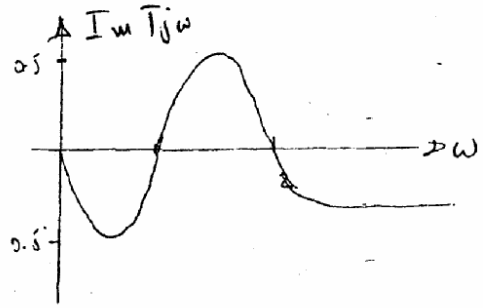
$$T(s) = \frac{s^3+3s+2}{s^4+2s^3+3s+1}$$

SOLUCION:

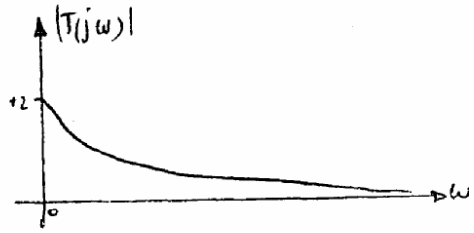
$$Re\,T(j\omega) = \frac{2\omega^6-7\omega^4+9\omega^2+2}{\omega^8+4\omega^6-10\omega^4+9\omega^2+1}$$



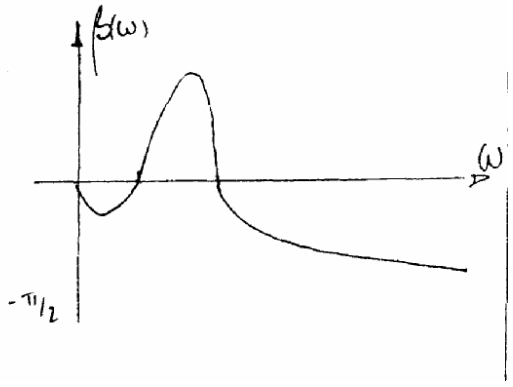
$$Im\,T(j\omega) = \frac{-\omega^7+3\omega^5+3\omega^3-3\omega}{\omega^8+4\omega^6-10\omega^4+9\omega^2+1}$$



$$|T(j\omega)| = \sqrt{\frac{\omega^6-6\omega^4+9\omega^2+4}{\omega^8+4\omega^6-10\omega^4+9\omega^2+1}}$$



$$\beta(\omega) = \frac{-\omega^7+3\omega^5+3\omega^3-3\omega}{2\omega^6-7\omega^4+9\omega^2+2}$$



PROBLEMA: GRAFICAR $Re\,T(j\omega)$, $Im\,T(j\omega)$, $|T(j\omega)|$ y $\beta(\omega)$ DE LA FUNCION DE RED

$$T(s) = \frac{s^3 - 5s^2 + 7s + 13}{s^3 + 4s^2 + 8s}$$

SOLUCION:

$$Re\,T(s) = \frac{1}{2}\left[T(s) + T(-s)\right] = \frac{1}{2}\left[\left[\frac{s^3 - 5s^2 + 7s + 13}{s^3 + 4s^2 + 8s}\right] + \left[\frac{-s^3 - 5s^2 - 7s + 13}{-s^3 + 4s^2 - 8s}\right]\right]$$

$$Re\,T(s) = \frac{1}{2}\left[\frac{-2s^6 + 70s^4 - 8s^2}{-s^6 - 64s^2}\right]$$

$$Im\,T(s) = \frac{1}{2}\left[T(s) - T(-s)\right] = \frac{1}{2}\left[\frac{18s^5 + 110s^3 - 208s}{-s^6 - 64s^2}\right]$$

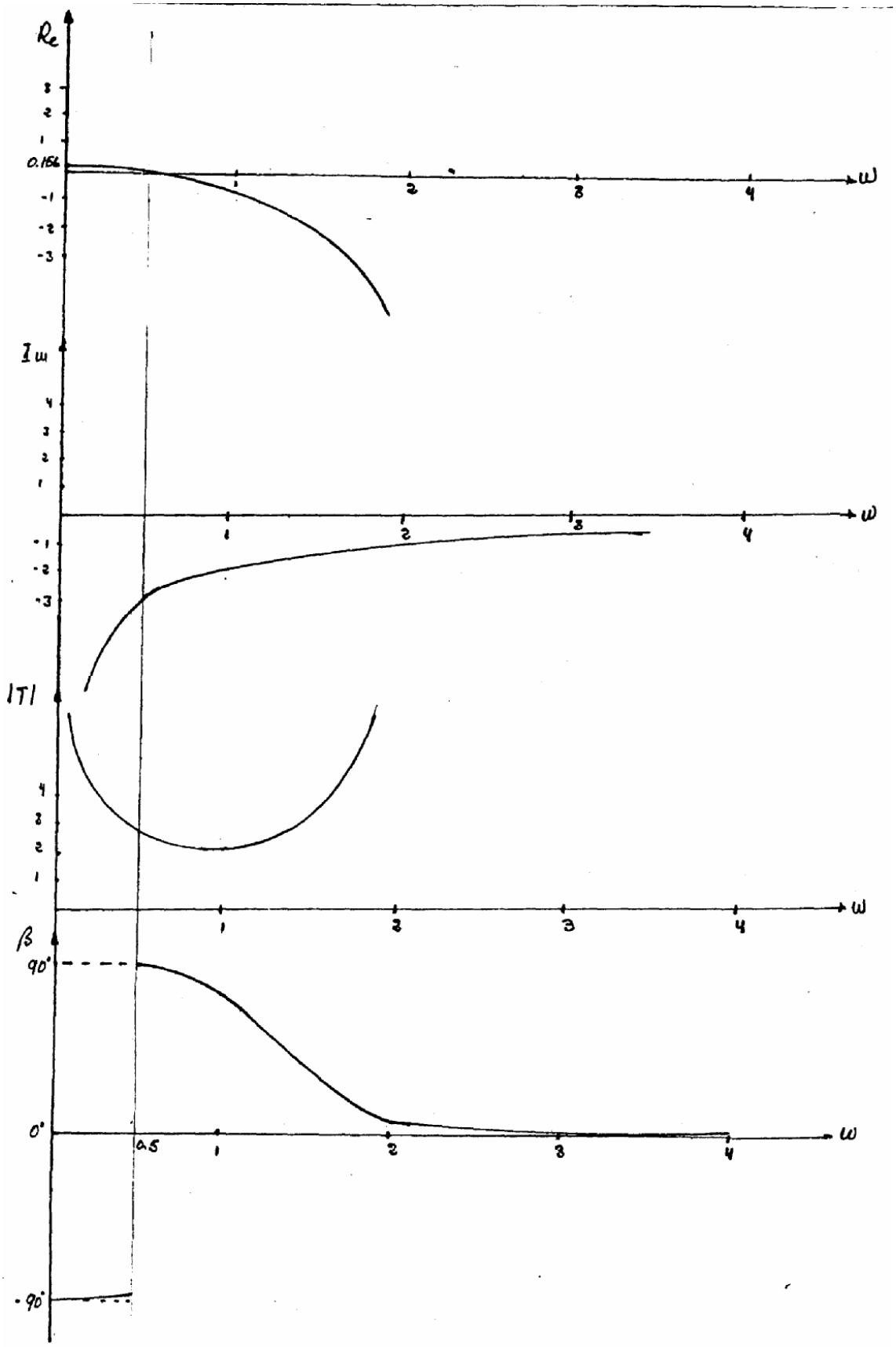
$$Re\,T(j\omega) = Re\,T(s)\Big|_{s=j\omega} = \left[\frac{-2(j\omega)^6 - 70(j\omega)^4 - 8(j\omega)^2}{-(j\omega)^6 - 64(j\omega)^2}\right] = \frac{1}{2}\left[\frac{2\omega^6 - 70\omega^4 + 8\omega^2}{\omega^6 + 64\omega^2}\right]$$

$$j\,Im\,T(j\omega) = Im\,T(s)\Big|_{s=j\omega} = \left[\frac{18j\omega^5 - 110j\omega^3 - 208j\omega}{\omega^6 + 64\omega^2}\right] = \frac{1}{2}\left[\frac{18\omega^5 - 110\omega^3 - 208\omega}{\omega^6 + 64\omega^2}\right]$$

$$Re\,T(j\omega) = \frac{\omega^4 - 35\omega^2 + 1}{\omega^4 + 64} \qquad Im\,T(j\omega) = \frac{9\omega^4 - 55\omega^2 - 104}{\omega^5 + 64\omega}$$

$$|T(j\omega)| = \sqrt{Re\,T(j\omega)^2 + Im\,T(j\omega)^2} \qquad \beta = tg^{-1}\frac{Im\,T(j\omega)}{Re\,T(j\omega)}$$

ω	Re	Im	$\lvert T\rvert$	β
0	0.015	$-\infty$	∞	$-90°$
0.1	0.0156	-14.92	14.929	-89.94
0.2	0.0155	-7.593	7.593	-89.88
0.25	0.0153	-6.152	6.151	-89.87
0.3	0.013	-5.205	5.205	-89.83
0.4	0.0081	-4.05	4.053	-89.80
0.5	-0.0041	-3.39	3.39	-89.88
0.7	-0.12	-2.71	2.712	+89.05
0.8	-0.507	-2.525	2.529	87.26
1	-1.53	-2.307	2.362	77.59
1.2	-5.68	-2.19	2.68	65.05
1.5	-27.78	-2.111	6.06	20.03
2	-176.4	-1.968	27.86	4.05
3	-722.8	-1.356	176.4	0.44
5	-3476.7	-0.426	722.8	0.03
10	-3476.7	-0.05	3476.7	0.009


Re
0.156
Im
|T|
β
90°
0°
-90°
0.5
ω

PROBLEMA: Encontrar el Diagrama de Nyquist de la funci de red dada.

$$T(s) = \frac{s^2+2s+2}{s+1}$$

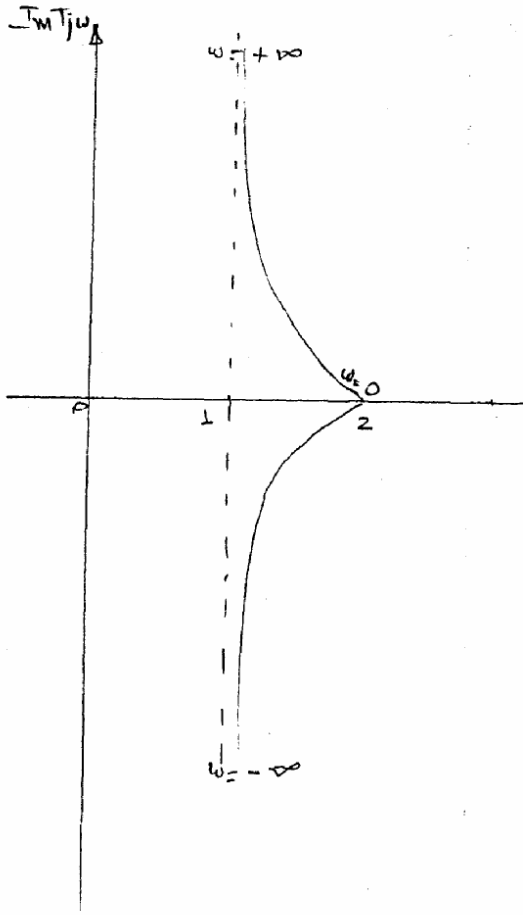
SOLUCION: LA Parte real e imaginaria ya fueron encontra

$$Re\ T(j\omega) = \frac{\omega^2+2}{\omega^2+1}$$

$$Im\ T(j\omega) = \frac{\omega^3}{\omega^2+1}$$

La variación de las parte respecto a ω se tabulan:

ω	$Re\ T(j\omega)$	$Im\ T(j\omega)$
0	2.0	0
1	1.5	0.5
2	1.2	1.6
5	1.04	4.8
10	1.01	9.9
100	1.00	999.9
∞	1.00	∞



PROBLEMA: ENCONTRAR EL DIAGRAMA DE NYQUIST DE LA FUNCION DADA

$$T(s) = \frac{s^2+4s+20}{s^3+3s^2+4s+2}$$

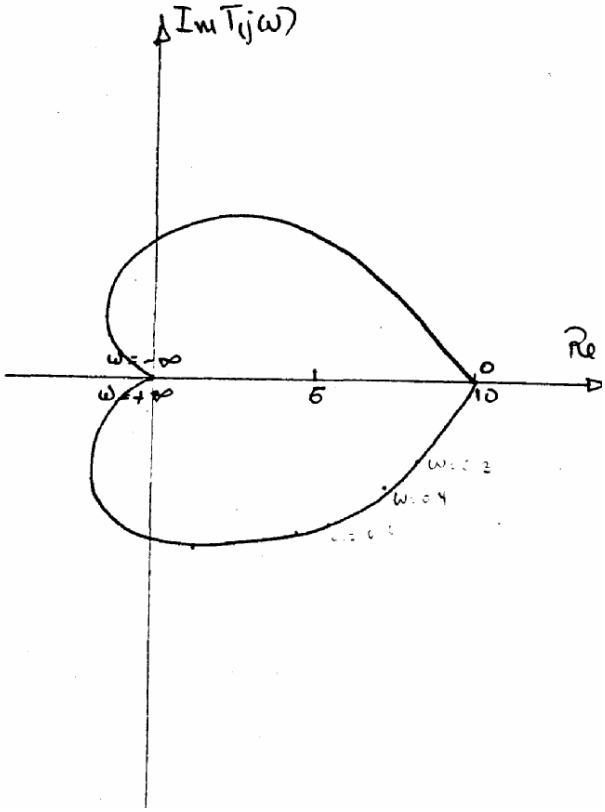
SOLUCION:

MEDIANTE LA DESCOMPOSICION DE ESTA FUNCION EN SUS PARTES PAR E IMPAR Y LUEGO HACIENDO $s = j\omega$ SE OBTIENE LAS PARTES: REAL E IMAGINARIA

$$Re\ T(j\omega) = \frac{-\omega^4 - 46\omega^2 + 40}{\omega^6 + \omega^4 + 4\omega^2 + 4}$$

$$Im\ T(j\omega) = \frac{-\omega^5 + 12\omega^3 - 72\omega}{\omega^6 + \omega^4 + 4\omega^2 + 4}$$

ω	$Re\ T(j\omega)$	$Im\ T(j\omega)$
0	10	0
0.3	8.2	-4.9
0.4	7.0	-6.0
0.6	4.2	-7.25
1	-0.7	-6.1
2	-1.6	-0.8
3	-0.54	-0.16
4	-0.22	-0.12
5	-0.11	-0.12
∞	0	0



PROBLEMA : DIBUJAR EL DIAGRAMA DE BODE DE LA FUNCION

$$T(s) = \frac{s^3 - 5s^2 + 7s + 13}{s^3 + 4s^2 + 8s}$$

SOLUCION : Factoramos la expresión y luego hacemos el término independiente de cada factor igual a la unidad, se tiene para S = jW :

$$T(s) = \frac{(s+1)(s^2 - 6s + 13)}{s(s^2 + 4s + 8)}$$

$$T(s) = \frac{13(s+1)\left(\frac{s^2}{13} - \frac{6s}{13} + 1\right)}{8s\left(\frac{s^2}{8} + \frac{4s}{8} + 1\right)}$$

$$T(j\omega) = \frac{\frac{13}{8}(j\omega+1)\left(\left(\frac{j\omega}{\sqrt{13}}\right)^2 - \frac{6}{13}j\omega + 1\right)}{(j\omega)\left[\left(\frac{j\omega}{\sqrt{8}}\right)^2 + \frac{j\omega}{2} + 1\right]}$$

$$T(j\omega) = \frac{\frac{13}{8}(j\omega+1)\left(-\left(\frac{\omega}{\sqrt{13}}\right)^2 - \frac{6}{13}j\omega + 1\right)}{(j\omega)\left(-\left(\frac{\omega}{\sqrt{8}}\right)^2 + \frac{j\omega}{2} + 1\right)}$$

Analizando término por término, determinamos las frecuencias de esquinas y la pendiente:

I) La constante(I3/8): 20 log (I3/8) = 4.2I7 db recta horizontal

2) Cero real: W_n = I ; pendiente = 20 db/dc

3) Cerô múltiple: W_n = 3.6 ; pendiente = 40 db/dc

4) Polo en el origen:(jW): W_n = I ; pendiente = - 20 db/dc

5) Polo cuadrático simple: W_n = 2.83 ; pendiente = - 40 db/dc

Para la representación en papel semilogarítmico

Frecuencia inferior = I

Frecuencia superior = 3.6

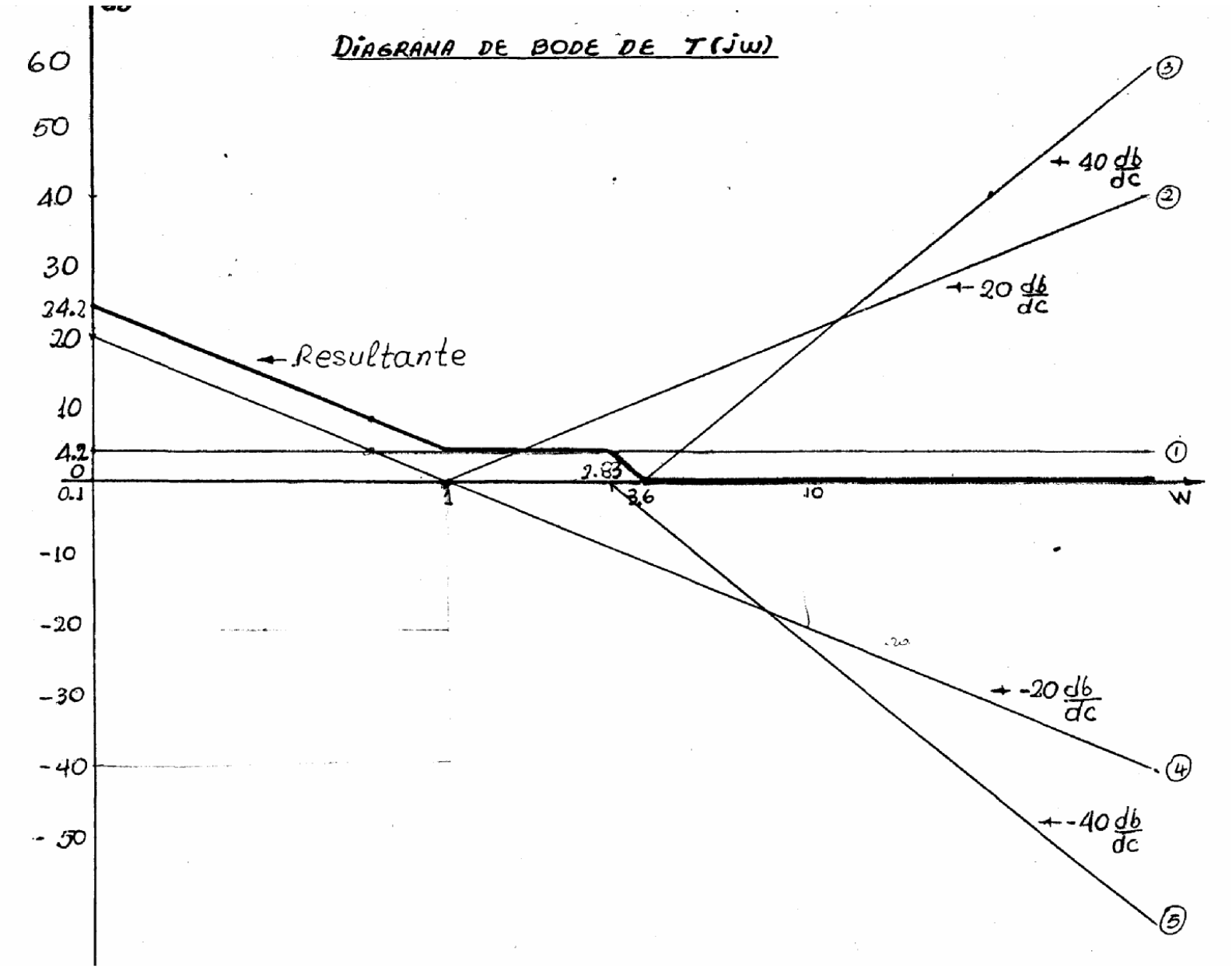


DIAGRAMA DE BODE DE T(jw)
60
50
40
30
24.2
20
10
4.2
0
0.1
-10
-20
-30
-40
-50
1
2.83
3.6
10
W
Resultante
40 db/dc
20 db/dc
-20 db/dc
-40 db/dc
1
2
3
4
5

PROBLEMA: DIBUJAR EL DIAGRAMA DE BODE DE LA FUNCION

$$T(s) = \frac{s^2+2s+1}{s^3+s^2+s+1}$$

SOLUCION: FACTORANDO:

$$T(s) = \frac{(s+1)^2}{(s+1)(s^2+1)} = \frac{(s+1)}{(s^2+1)}$$

$$T(j\omega) = T(s)\Big|_{s=j\omega} = \frac{(j\omega+1)}{(1-\omega^2)}$$

a) CERO SIMPLE REAL

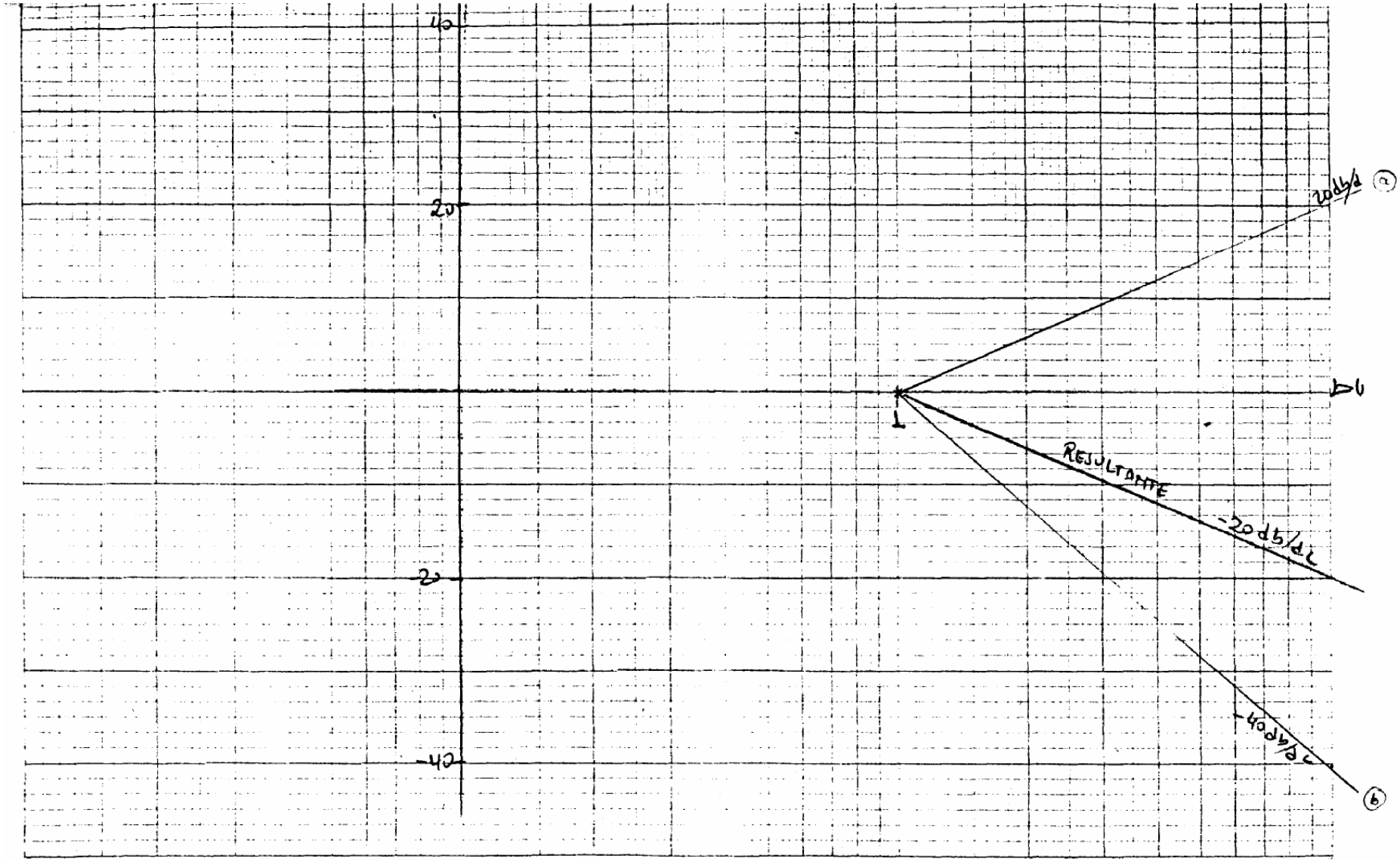
FREC DE ESQUINA: $\omega_u = 1$

PENDIENTE: 20 db/dc

b) POLO CUADRATICO

FREC. DE ESQUINA: $\omega_u = 1$

PENDIENTE: - 40 db/dc.


40
20
-20
-40
20db/d
a
b
1
RESULTANTE
-20db/dc
-40db/dc
b

PROBLEMA: DIBUJAR EL DIAGRAMA DE BODE DE LA FUNCIO

$$T(s) = \frac{s^3+3s+2}{s^4+2s^3+3s+1}$$

SOLUCION:

$$T(s) = \frac{(s+0.596)(s^2-0.596s+3.355)}{(s+0.3155)(s+2.436)(s^2-0.751s+1.299)}$$

$$T(s) = \frac{0.596 \times 3.355\left(\frac{s}{0.596}+1\right)\left(\frac{s^2}{3.355}-\frac{s}{5.629}+1\right)}{0.3155 \times 2.436 \times 1.299\left(\frac{s}{0.3155}+1\right)\left(\frac{s}{2.436}+1\right)\left(\frac{s^2}{1.299}-\frac{s}{1.728}\right.}$$

$$T(j\omega) = 2\,\frac{\left(\frac{j\omega}{0.596}+1\right)\left[-\left(\frac{\omega}{\sqrt{3.335}}\right)^2-\frac{j\omega}{5.629}+1\right]}{\left(\frac{j\omega}{0.3155}+1\right)\left(\frac{j\omega}{2.436}+1\right)\left[-\left(\frac{\omega}{\sqrt{1.299}}\right)^2-\frac{j\omega}{1.7285}+1\right]}$$

1) LA CONSTANTE 2 : $20 \log 2 = 6$ db RECTA HORIZONTAL

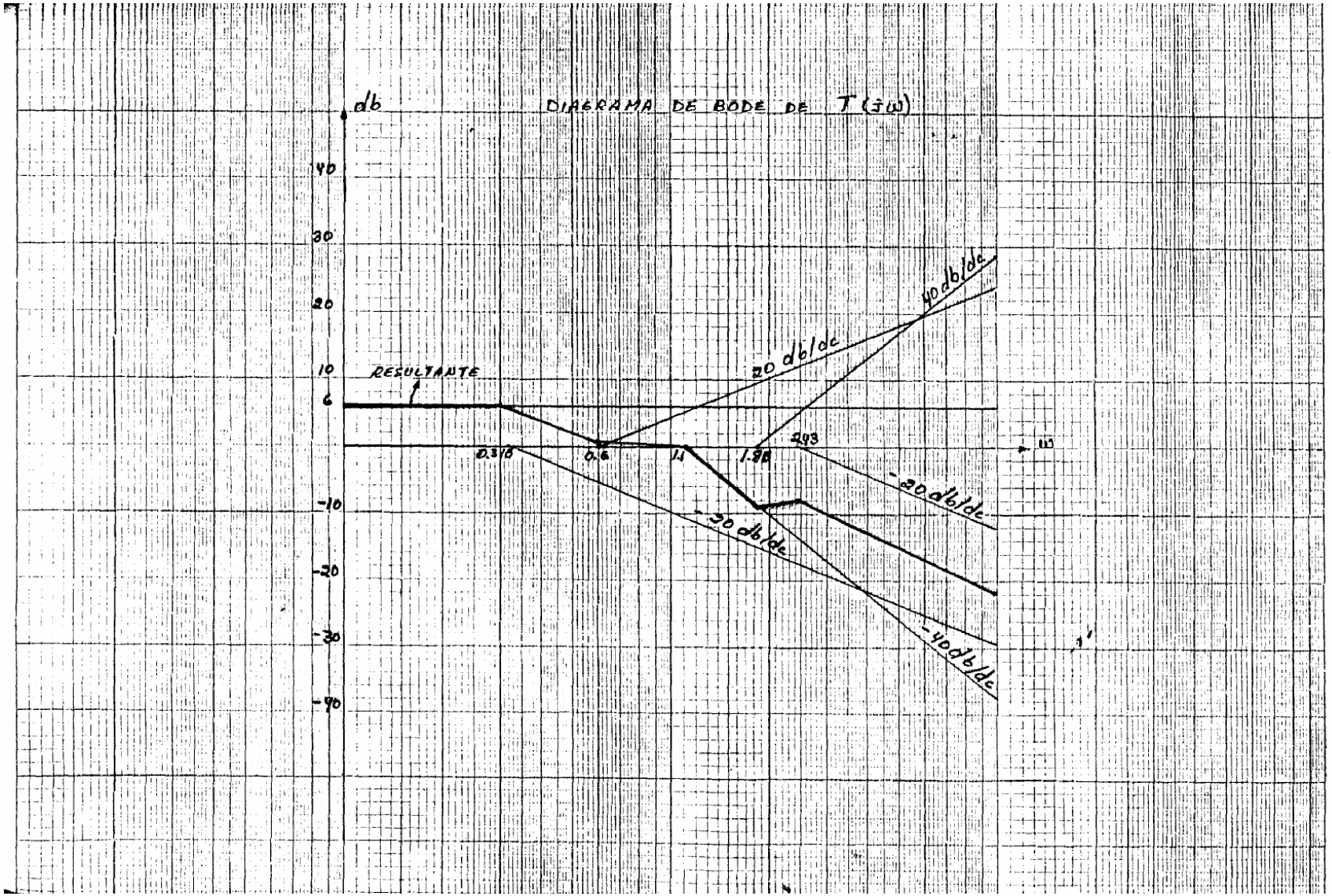
2) CERO REAL : $Wn = 0.596$ PEND = 20 db/dc

3) CERO CUADRATICO : $Wn = 1.831$ PEND = 40 db/dc

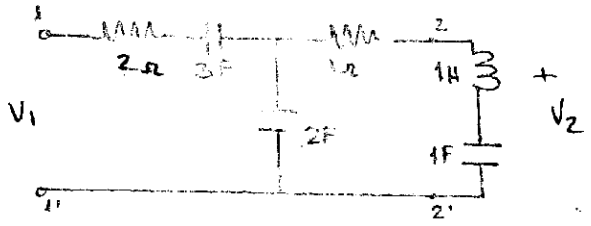
4) POLO REAL : $Wn = 0.3155$ PEND = - 20 db/dc

5) POLO REAL : $Wn = 2.436$ PEND = - 20 db/dc

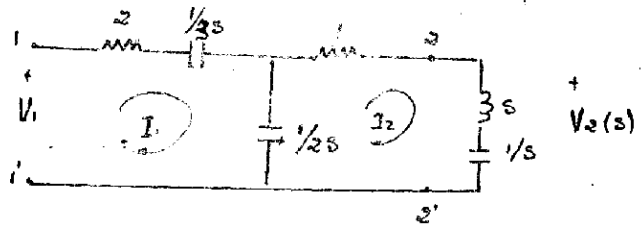
6) POLO CUADRATICO : $Wn = 1.139$ PEND = - 40 db/dc


db
DIAGRAMA DE BODE DE T (jω)
40
30
20
10
6
-10
-20
-30
-40
RESULTANTE
40 db/dc
20 db/dc
-20 db/dc
-20 db/dc
-40 db/dc
0.316
0.6
1.1
1.98
2.93
ω

PROBLEMA: DIBUJAR LA MAGNITUD EN DB y LA FASE EN GRADOS DE LA FUNCION $T(s) = V_2/V_1$ DEL SIGUIENTE CIRCUITO



SOLUCION:



$$Z = \begin{bmatrix} 2 + 1/3s + 1/2s & -1/2s \\ -1/2s & 1 + s + 1/s + 1/2s \end{bmatrix}$$

$$\Delta Z = \left(2 + \frac{1}{3s} + \frac{1}{2s}\right)\left(1 + s + \frac{1}{s} + \frac{1}{2s}\right) - \frac{1}{4s^2} = \frac{(12s+5)(2s^2+2s+3)}{12s^2} - \frac{1}{4s^2}$$

$$\Delta Z = \frac{(12s+5)(2s^2+2s+3) - 3}{12s^2}$$

$$Z_L = s + \frac{1}{s} = \frac{s^2+1}{s}$$

$$T(s) = \frac{V_2}{V_1} = \frac{Z_L I_2}{V_1} = Z_L Y_{21}$$

$$Y_{21} = \frac{\Delta_{12}}{\Delta Z} = \frac{6s}{(12s+5)(2s^2+2s+3) - 3}$$

$$T(s) = \frac{s^2+1}{s} \cdot \frac{6s}{(12s+5)(2s^2+2s+3) - 3}$$

$$T(s) = \frac{6(s^2+1)}{12s^3 + 17s^2 + 23s + 6}$$

$$T(s) = \frac{s(s^2+1)}{(s+0.3196)(12s^2+13.16s+18.79)}$$

$$T(s) = 0.16\,\frac{s(s^2+1)}{\left(\frac{s}{0.3196}+1\right)\left(\frac{s^2}{1.56}+\frac{s}{1.43}+1\right)}$$

1) LA CONSTANTE 0.16 : $20 \log 0.16 = -15.91$ db

2) CERO EN EL ORIGEN $(j\omega)$: $W_n = 1$ PEND = 20 db/dc

3) CERO CUADRATICO : $W_n = 1$ PEND = 40 db/dc

4) POLO REAL : $W_n = 0.3196$ PEND = -20 db/dc

5) POLO CUADRATICO : $W_n = 1.248$ PEND = -40 db/dc

$$P_R T(s) = \frac{1}{2}\left[T(s) + T(-s)\right] = \frac{3}{2}\left[\frac{s^2+1}{13s^3+17s^2+23s+6} + \frac{s^2+1}{-13s^3+17s^2-23s+6}\right]$$

$$P_R T(s) = \frac{3}{2}\left[\frac{34s^4+46s^2+12}{(13s^3+17s^2+23s+6)(-13s^3+17s^2-23s+6)}\right]$$

$$I_R T(s) = \frac{1}{2}\left[T(s) - T(-s)\right] = \frac{3}{2}\left[\frac{s^2+1}{13s^3+17s^2+23s+6} - \frac{s^2+1}{-13s^3+17s^2-23s+6}\right]$$

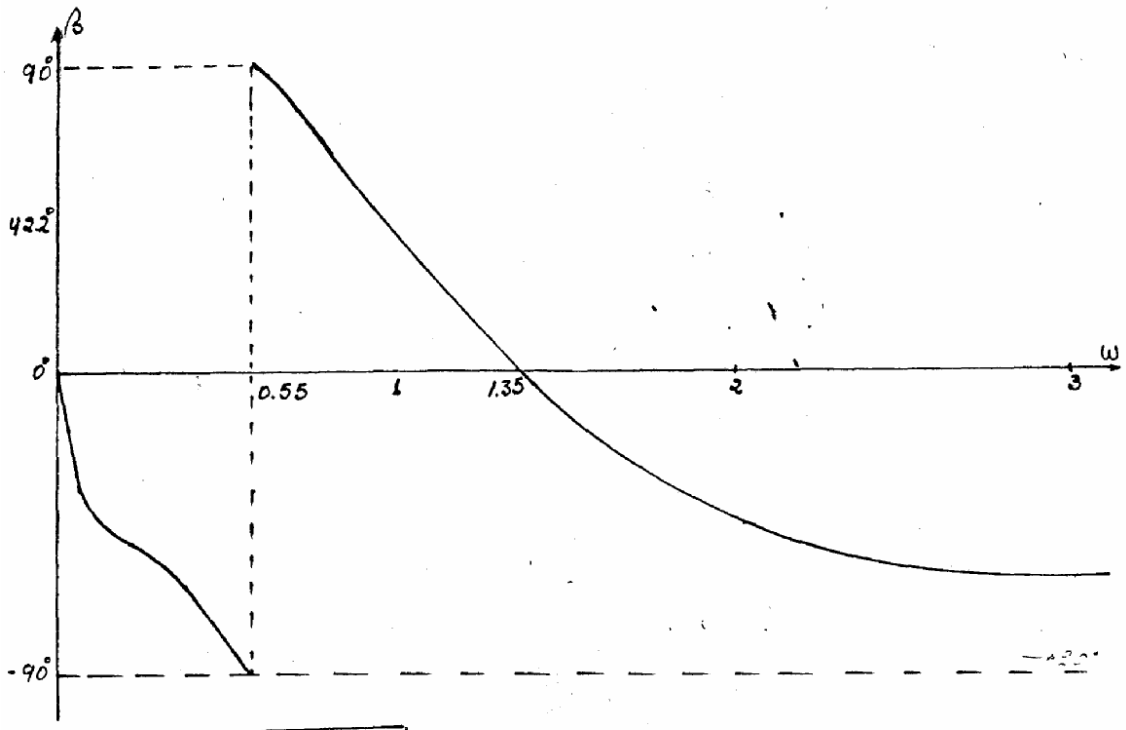
$$I_R T(s) = \frac{3}{2}\left[\frac{-26s^5-72s^3-46s}{(13s^3+17s^2+23s+6)(-13s^3+17s^2-23s+6)}\right]$$

$$tg\,\beta(\omega_3) = \frac{1}{j}\left(\frac{I_R T(s)\big|_{s=j\omega}}{P_R T(s)\big|_{s=j\omega}}\right)$$

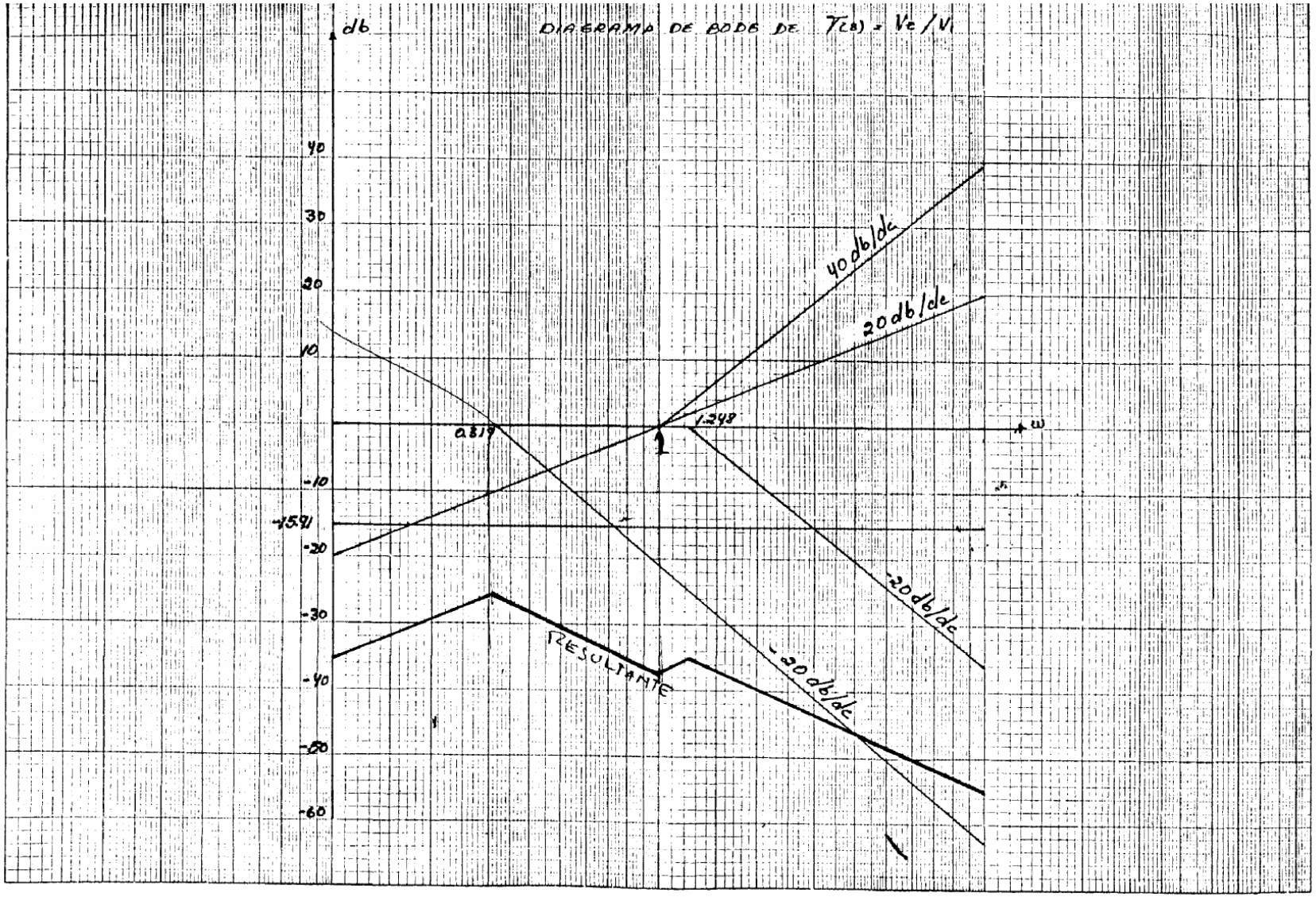
$$tg\,\beta = \frac{1}{j}\left(\frac{-26s^5-72s^3-46s}{34s^4+46s^2+12}\right) = \frac{1}{j}\left(\frac{-26(j\omega^5)-72(-j\omega^3)-46j\omega}{34\omega^4-46\omega^2+12}\right)$$

$$tg\,\beta = \frac{-13\omega^5+36\omega^3-23\omega}{17\omega^4-23\omega^2+6}$$

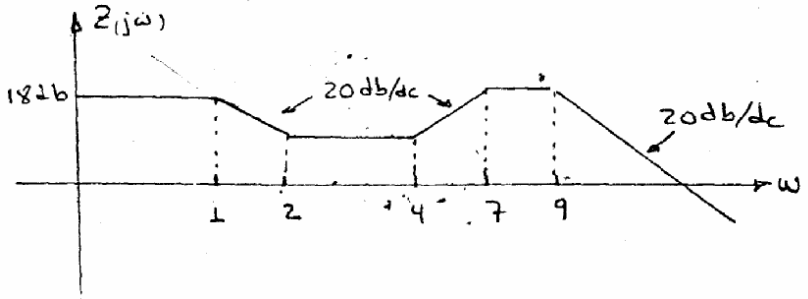
$$\beta = tg^{-1}\left(\frac{-13\omega^5+36\omega^3-23\omega}{17\omega^4-23\omega^2+6}\right)$$



ω	β
0	0
0.1	-21.419
0.3	-55.68
0.5	-79.95
0.6	89.37
0.7	78.68
0.8	67.43
0.9	55.30
0.9999	42.28
1.05	35.53
1.1	28.76
1.2	15.58
1.25	9.27
1.3	3.37
1.35	-2.14
1.4	-7.24
1.45	-11.92
2	-43.09
4	-70.22
8	-80.50
10	-82.44


db
DIAGRAMA DE BODE DE T(s) = Vc/Vi
40
30
20
10
-10
-15.91
-20
-30
-40
-50
-60
0.819
1.248
1
ω
40 db/dc
20 db/dc
-20 db/dc
-20 db/dc
RESULTANTE

PROBLEMA : LA FIGURA MUESTRA EL DIAGRAMA DE BODE CORRESPONDIENTE A UNA IMPEDANCIA DE TRANSFERENCIA. DETERMINAR LA EXPRESION ANALITICA DE DICHA IMPEDANCIA.



SOLUCION :

OBSERVANDO LAS FRECUENCIAS DE ESQUINA, ASI COMO LAS PENDIENTES SE OBTIENE :

$$T(s) = K \frac{(s+2)(s+4)}{(s+1)(s+7)(s+9)}$$

PARA EL CALCULO DE LA CONSTANTE :

$$T(0) = K \frac{8}{63}$$

PERO $\quad 20 \log|T(0)| = 18$

$$\therefore \; T(0) = 8$$

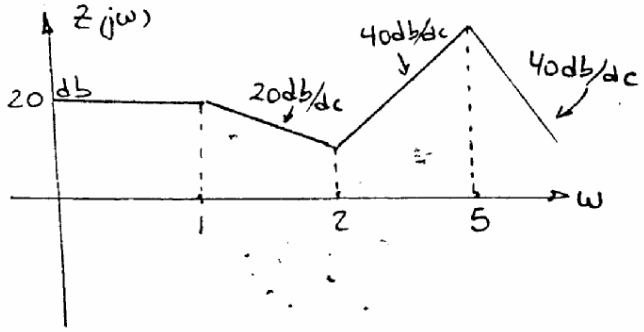
y $\quad 8 = K \frac{8}{63}$

$$\therefore \; K = 63$$

LUEGO

$$T(s) = 63 \frac{(s+2)(s+4)}{(s+1)(s+7)(s+9)}$$

PROBLEMA: LA FIGURA MUESTRA EL DIAGRAMA DE BODE CORRESPONDIENTE A UNA IMPEDANCIA. DETERMINAR LA EXPRESIO ANALITICA DE DICHA IMPEDANCIA.



SOLUCION: OBSERVANDO LAS FRECUENCIAS DE ESQUINA ASI COMO LAS PENDIENTES

$$T(s) = K \frac{(s+2)^3}{(s+1)(s+5)^4}$$

$$T(0) = K \frac{8}{625}$$

K 8/625

$$20\log|T(0)| = 20$$

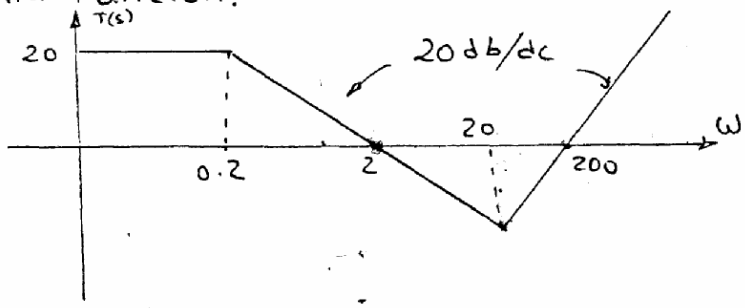
$$T(0) = 10$$

$$10 = K \frac{8}{625}$$

$$K = 781$$

$$T(s) = 781 \frac{(s+2)^3}{(s+1)(s+5)^4}$$

PROBLEMA: LA FIGURA MUESTRA EL DIAGRAMA DE BODE DE UNA FUNCION DE RED. DETERMINAR LA EXPRESION ANALITICA DE DICHA FUNCION.



SOLUCION: OBSERVANDO LAS FRECUENCIAS DE ESQUINA ASI COMO LAS PENDIENTES

$$T(s) = K \frac{(S+20)^2}{(S+0.2)}$$

$$T(0) = K \frac{400}{0.2}$$

$$20 \log T(0) = 20$$

$$T(0) = 10$$

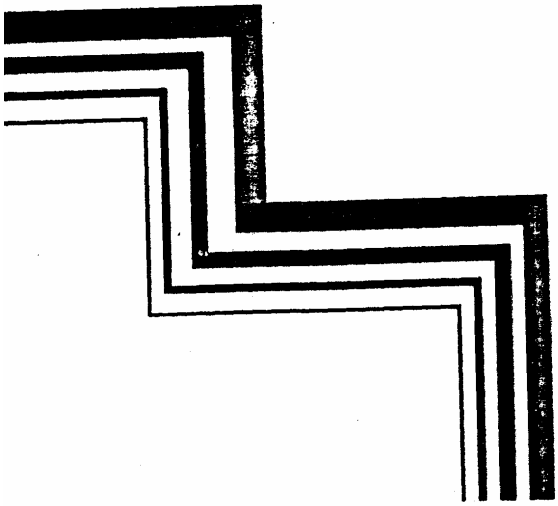
$$10 = K \frac{400}{0.2}$$

$$K = \frac{1}{200}$$

$$T(s) = \frac{1}{200} \frac{(S+20)^2}{(S+0.2)}$$

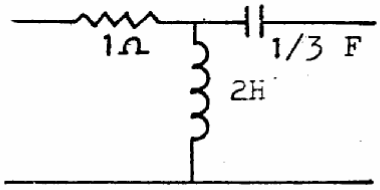
CAPITULO II

CUADRIPOLOS

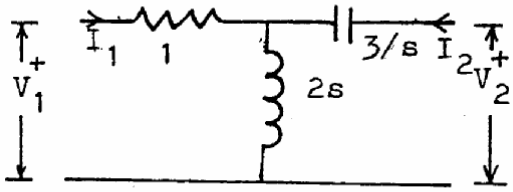


1.-Hallar la matriz $[Z]$ de los siguientes circuitos:

a)



Pasando al dominio de la frecuencia tenemos:



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad V_1 = (1 + 2s)\, I_1 \qquad \frac{V_1}{I_1} = 1 + 2s$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad V_1 = (\ 2s\ )\, I_2 \qquad \frac{V_1}{I_2} = 2s$$

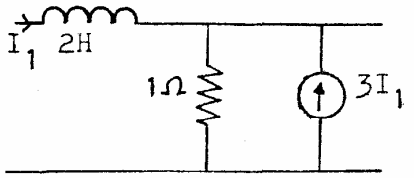
$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad V_2 = (\ 2s\ )\, I_1 \qquad \frac{V_2}{I_1} = 2s$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \qquad V_2 = (2s + 3/s) I_2 \qquad \frac{V_2}{I_2} = 2s + 3/s$$

Luego:

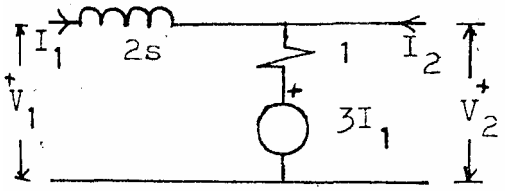
$$[Z] = \begin{bmatrix} 2s + 1 & 2s \\ 2s & 2s + 3/s \end{bmatrix}$$

b).-Hallar la matriz [Z]



SOLUCION:

Pasando al dominio de la frecuencia,y transformando la fuente de corriente en fuente de tensión se tiene:



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad V_1 = I_1(2s + 1) + 3I_1 \qquad \frac{V_1}{I_1} = 2s + 4$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad V_1 = (1)\, I_2 \qquad \frac{V_1}{I_2} = 1$$

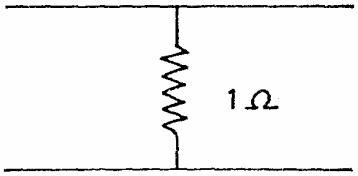
$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad V_2 = (1)I_1 + 3I_1 \qquad \frac{V_2}{I_1} = 4$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \qquad V_2 = (1)\, I_2 \qquad \frac{V_2}{I_2} = 1$$

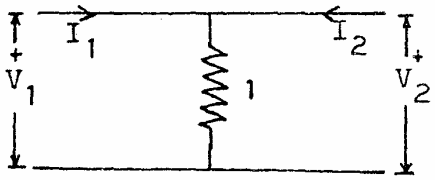
Luego:

$$[Z] = \begin{bmatrix} 2s + 4 & 1 \\ 4 & 1 \end{bmatrix}$$

c).- Hallar la matriz $[Z]$



SOLUCION: En el dominio de la frecuencia tenemos:



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad V_1 = (1)\ I_1 \qquad \frac{V_1}{I_1} = 1$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad V_1 = (1)\ I_2 \qquad \frac{V_1}{I_2} = 1$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad V_2 = (1)\ I_1 \qquad \frac{V_2}{I_1} = 1$$

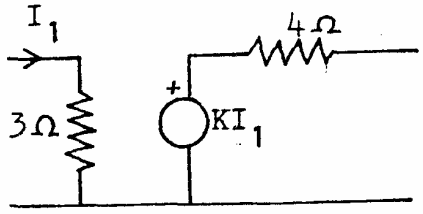
$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \qquad V_2 = (1)\ I_2 \qquad \frac{V_2}{I_2} = 1$$

Luego:

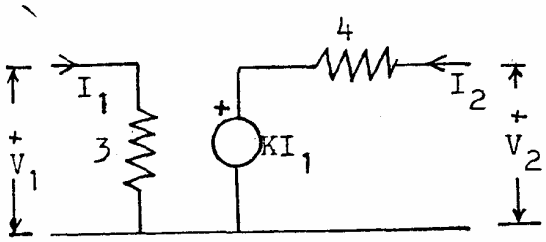
$$[Z] = \begin{bmatrix} 1 & 1 \\ 1 & \end{bmatrix}$$

Como se puede ver, que a partir de un circuito simétrico obtenemos una matriz simétrica.

d).- Hallar la matriz $[Z]$



SOLUCION: En el dominio de la frecuencia tenemos:



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad V_1 = (3)\ I_1 \qquad \frac{V_1}{I_1} = 3$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad V_1 = (0)\ I_2 \qquad \frac{V_1}{I_2} = 0$$

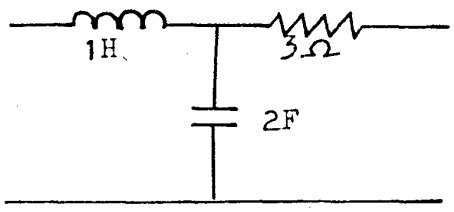
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad V_2 = (K)\ I_1 \qquad \frac{V_2}{I_1} = K$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad V_2 = (4)\ I_2 \qquad \frac{V_2}{I_2} = 4$$

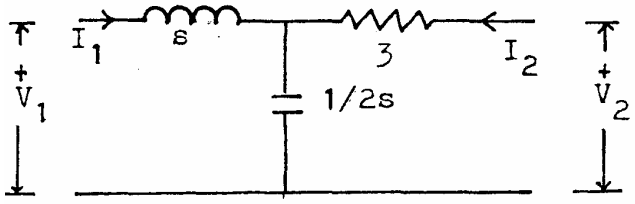
Luego:

$$[Z] = \begin{bmatrix} 3 & 0 \\ K & 4 \end{bmatrix}$$

e).-Hallar la matriz [Z]



SOLUCION: Transformando al dominio de la frecuencia:



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad V_1 = (s + 1/2s)I_1 \qquad \frac{V_1}{I_1} = \frac{2s^2 + 1}{2s}$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad V_1 = (1/2s)I_2 \qquad \frac{V_1}{I_2} = \frac{1}{2s}$$

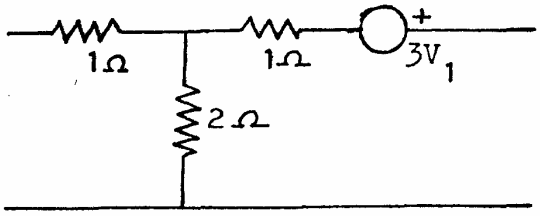
$$Z_{21} = Z_{12} = \frac{1}{2s}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \qquad V_2 = (3 + 1/2s)I_2 \qquad \frac{V_2}{I_2} = \frac{6s + 1}{2s}$$

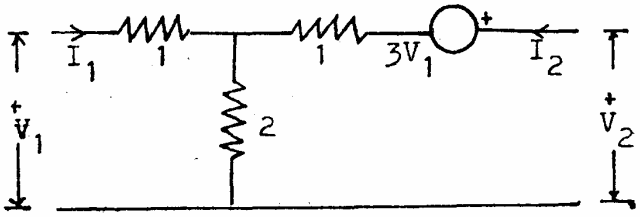
Luego:

$$[Z] = \begin{bmatrix} \frac{2s^2 + 1}{2s} & \frac{1}{2s} \\ \frac{1}{2s} & \frac{6s + 1}{2s} \end{bmatrix}$$

f).-Hallar la matriz [Z]



SOLUCION: Transformando al dominio de la frecuencia:



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad V_1 = (1+2)I_1 \qquad \frac{V_1}{I_1} = 3$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad V_1 = (2)I_2 \qquad \frac{V_1}{I_2} = 2$$

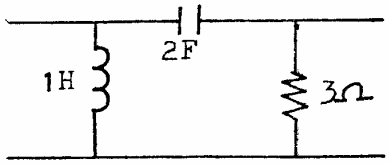
$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad \begin{array}{ll} V_2 = 3V_1 + 2I_1 & (a) \\ V_1 = 3I_1 & (b) \end{array} \qquad (b) \text{ en } (a) \quad Z_{21} = 11$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \qquad \begin{array}{ll} V_2 = 3V_1 + (1)I_2 + V_1 & (a') \\ V_1 = 2I_2 & (b') \end{array} \qquad Z_{22} = 9$$

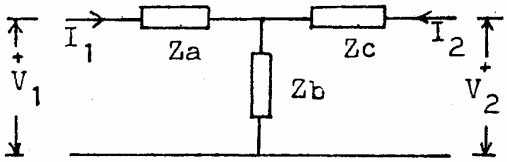
Luego:

$$[Z] = \begin{bmatrix} 3 & 2 \\ 11 & 9 \end{bmatrix}$$

g).- Hallar la matriz [Z]



SOLUCION: Pasamos al dominio de la frecuencia y utilizamos la transformación delta -estrella :



donde: $Za = s/(2s^2 + 6s+1)$

$Zb = 6s^2/(2s^2+6s+1)$

$Zc = 3/(2s^2 + 6s+1)$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad V_1 = (Za + Zb)I1 \qquad Z_{11} = \frac{6s^2 + s}{2s^2+6s+1}$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad V_1 = (Zb)\, I_2 \qquad Z_{12} = Zb$$

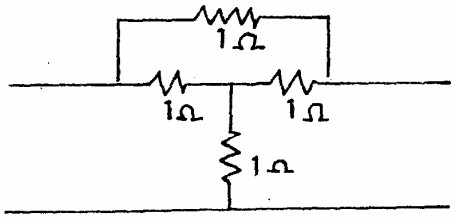
$$Z_{21} = Z_{12} = Zb$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \qquad V_2 = (Zb + Zc)I_2 \qquad Z_{22} = \frac{6s^2 + 3}{2s^2+6s+1}$$

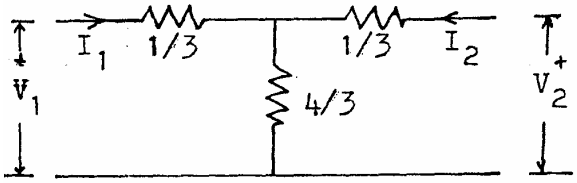
Luego:

$$[Z] = \frac{1}{2s^2+6s+1}\begin{bmatrix} 6s^2 + 1 & 6s^2 \\ 6s^2 & 6s^2 + 3 \end{bmatrix}$$

h).- Hallar la matriz $[Z]$



SOLUCION; Pasando al dominio de la frecuencia , y utilizando la respectiva transformación $\Delta - Y$ tenemos: -



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad V_1 = (1/3 + 4/3)\ I_1 \qquad Z_{11} = 5/3$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad V_1 = (4/3)I_2 \qquad Z_{12} = 4/3$$

$$Z_{21} = Z_{12} = 4/3$$

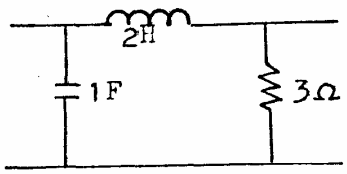
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad V_2 = (1/3 + 4/3)I_2 \qquad Z_{22} = 5/3$$

Luego:

$$[Z] = \begin{bmatrix} 5/3 & 4/3 \\ 4/3 & 5/3 \end{bmatrix}$$

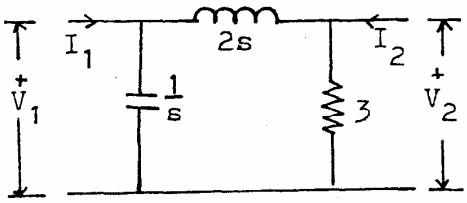
2.-Hallar la matriz $[Y]$ de los siguientes circuitos:

a).-



SOLUCION:

Poniendo en el dominio de la frecuencia tenemos:



$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad V_1 = \left(\frac{2s(1/s)}{2s + 1/s}\right)I_1 \qquad Y_{11} = \frac{2s^2 + 1}{2s}$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad V_2 + (2s)I_1 = 0 \qquad Y_{12} = -\frac{1}{2s}$$

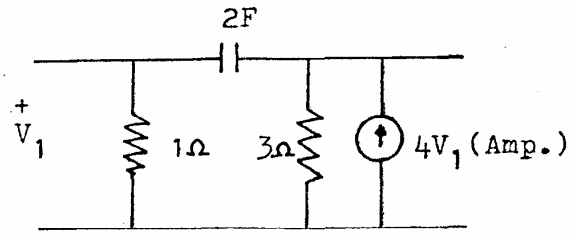
$$Y_{21} = Y_{12} = -\frac{1}{2s}$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} \qquad V_2 = \left(\frac{3(2s)}{3 + 2s}\right)I_2 \qquad Y_{22} = \frac{2s + 3}{6s}$$

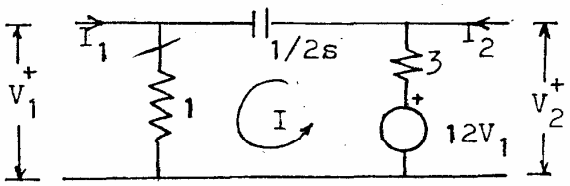
Luego:

$$[Y] = \begin{bmatrix} \frac{2s^2 + 1}{2s} & -\frac{1}{2s} \\ -\frac{1}{2s} & \frac{2s + 3}{6s} \end{bmatrix}$$

b).-Hallar la matriz $[Y]$



SOLUCION : Transformemos la fuente de corriente en fuente de tensión y pasemos al dominio de la frecuencia.



$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$ $\quad V_1 = \left(\frac{1/2s(1)}{1 + 1/2s}\right) I_1$ $\quad Y_{11} = 1 + 2s$

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$ $\quad I_1(1/2s) + V_2 = 0$ $\quad Y_{12} = -2s$

$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$ planteamos el sistema: (Ec. de mallas)

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 4+\frac{1}{2s} & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 12V_1 \\ -12V_1 \end{bmatrix}$$

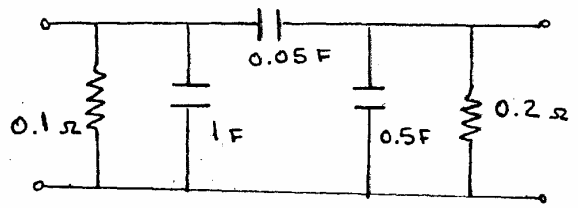
de donde: $Y_{21} = -4-2s$

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$ Con el sistema anterior ,cambiando unicamente la fuente respecto de I_2 que ahora es V_2 y con $V_1=0$,al resolver tenemos: $Y_{22} = \frac{6s+1}{3}$

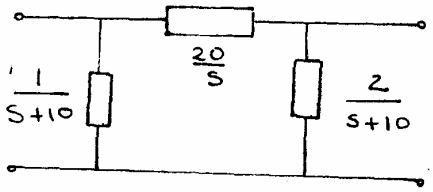
Luego:

$$[Y] = \begin{bmatrix} 2s + 1 & -2s \\ -2s - 4 & \frac{6s+1}{3} \end{bmatrix}$$

c).

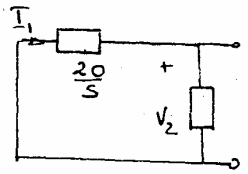


SOLUCION : REDUCIENDO LA RED A UNA DELTA



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = (10+s) + \frac{s}{20} = \frac{21s+200}{20} = 1.05\,s + 10$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{s}{20}$$



$$\frac{20}{s} I_1 + V_2 = 0$$

$$\frac{20}{s} I_1 = -V_2$$

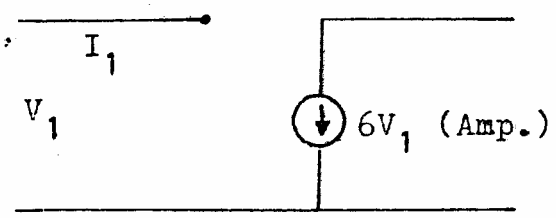
$$\frac{I_1}{V_2} = -\frac{s}{20}$$

$$y_{21} = y_{12}$$

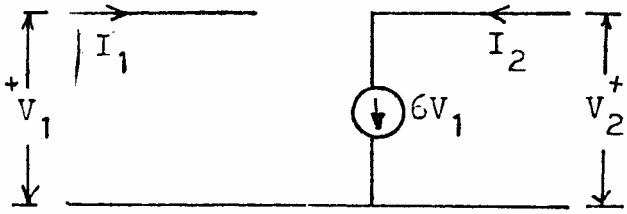
$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{s}{20} + \frac{s+10}{2} = \frac{22s+200}{40}$$

$$[y] = \begin{bmatrix} 1.05s+10 & -0.05s \\ -0.05s & 0.55s+5 \end{bmatrix}$$

d).- Hallar la matriz Y



SOLUCION:



$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad Y_{11} = 0$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad Y_{12} = 0$$

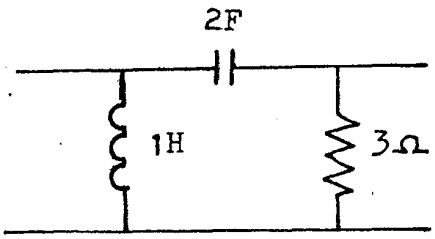
$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \qquad I_2 = 6V_1 \qquad Y_{21} = 6$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} \qquad Y_{22} = 0$$

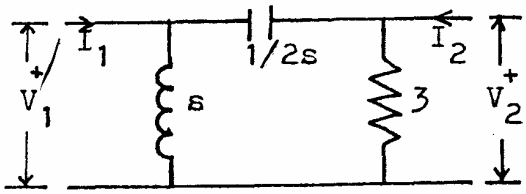
Luego:

$$[Y] = \begin{bmatrix} 0 & 0 \\ 6 & 0 \end{bmatrix}$$

e).- Hallar la matriz [Y]



SOLUCION : Pongamos en el dominio de la frecuencia:



$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad V_1 = \frac{s(1/2s)}{s + 1/2s} I_1 \qquad Y_{11} = \frac{2s^2 + 1}{s}$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad V_2 = -(1/2s)I_1 \qquad Y_{12} = -2s$$

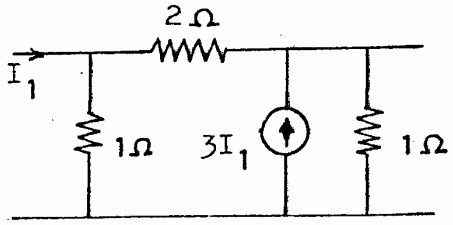
$$Y_{12} = Y_{21}$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} \qquad V_2 = \frac{3(1/2s)}{3 + 1/2s} \qquad Y_{22} = \frac{6s + 1}{3}$$

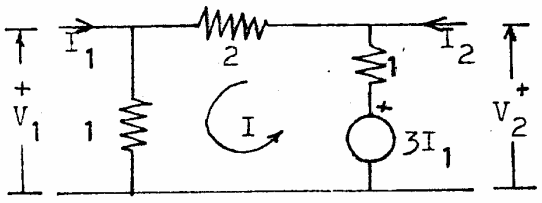
Luego:

$$[Y] = \begin{bmatrix} \frac{2s^2 + 1}{s} & -2s \\ -2s & \frac{6s + 1}{3} \end{bmatrix}$$

f).- Hallar la matriz $[Y]$



SOLUCION: Transformemos la fuente de corriente en fuente de tensión y pasemos al dominio fe la frecuencia.



$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad V_1 = \left(\frac{2(1)}{2+1}\right) I_1 \qquad Y_{11} = \frac{3}{2}$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad I_1(2) + V_2 = 0 \qquad Y_{12} = -\frac{1}{2}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

Para esto planteo el sistema de ecuaciones correspondientes a las tres mallas:

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 3I_1 \\ V_2 - 3I_1 \end{bmatrix}$$

Resolviendo en base a I_2 con $V_2=0$ obtenemos : $Y_{21} = -5$

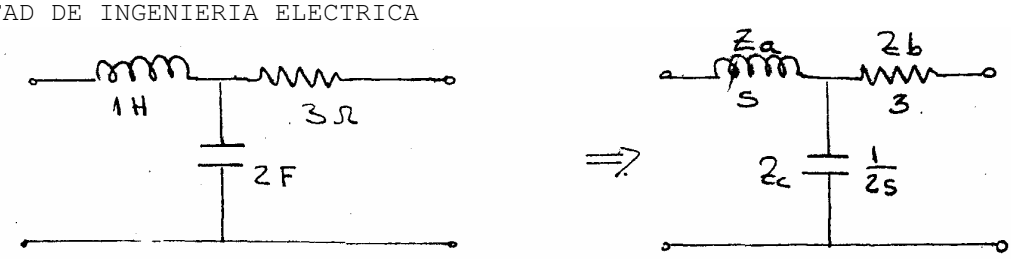
$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

Resolviendo con respecto a I_2 y $V_1=0$ obtenemos : $Y_{22} = 3$

Luego:

$$[Y] = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -5 & 3 \end{bmatrix}$$

g)



SOLUCION:

DE LAS TABLAS TENEMOS:

$$\Delta z = Za.Zb + Zc(Za + Zb)$$

$$\Delta z = s.3 + \frac{1}{2s}(s+3) = \frac{6s^2+s+3}{2s}$$

$$y_{11} = \frac{Zb+Zc}{\Delta z} = \frac{3+\frac{1}{2s}}{\frac{6s^2+s+3}{2s}} = \frac{6s+1}{6s^2+s+3}$$

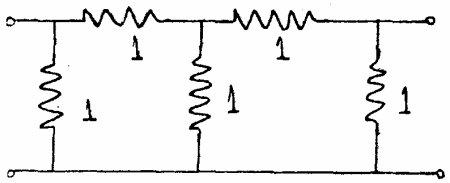
$$y_{12} = y_{21} = \frac{-Zc}{\Delta z} = \frac{-\frac{1}{2s}}{\frac{6s^2+s+3}{2s}} = \frac{-1}{6s^2+s+3}$$

$$y_{22} = \frac{Za+Zc}{\Delta z} = \frac{s+\frac{1}{2s}}{\frac{6s^2+s+3}{2s}} = \frac{2s^2+1}{6s^2+s+3}$$

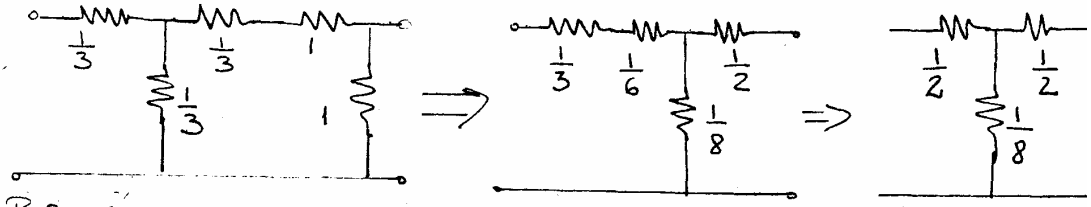
LA MATRIZ RESULTA:

$$[y] = \begin{bmatrix} \frac{6s+1}{6s^2+s+3} & \frac{-1}{6s^2+s+3} \\ \frac{-1}{6s^2+s+3} & \frac{2s^2+1}{6s^2+s+3} \end{bmatrix}$$

h).



SOLUCION: a) PRIMER METODO: TRANSFORMANDO LA RED ASI:



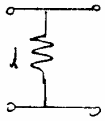
POR TABLAS:

$$\Delta Z = \frac{1}{4} + \frac{1}{8}\left(\frac{1}{2} + \frac{1}{2}\right) = \frac{3}{8}$$

$$y_{11} = \frac{5}{3} \qquad y_{12} = y_{21} = -\frac{1}{3} \qquad y_{22} = \frac{5}{3}$$

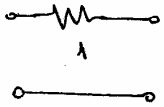
b) SEGUNDO METODO: UTILIZANDO EL CONCEPTO DE REDES EN CASCADA:

RED PARALELO



$$[a] = \begin{bmatrix} 1 & 0 \\ \phi & 1 \end{bmatrix}$$

RED EN SERIE



$$[a] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

TENIENDO EN CUENTA CINCO CUADRIPOLOS EN CASCADA ES:

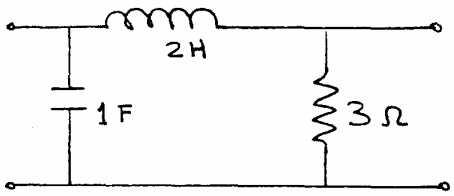
$$[a] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$[a] = \begin{bmatrix} 5 & 3 \\ 8 & 5 \end{bmatrix} \qquad \Delta a = 1 \text{ pero, } [y] = \begin{bmatrix} \frac{D}{B} & -\frac{\Delta a}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$$

LA MATRIZ [Y] SERA:

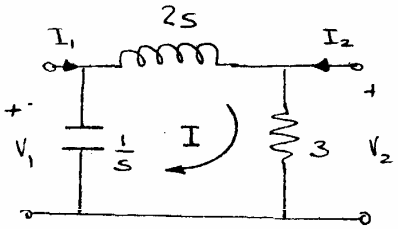
$$[y] = \begin{bmatrix} \frac{5}{3} & -\frac{1}{3} \\ \frac{-1}{3} & \frac{5}{3} \end{bmatrix}$$

3.- Hallar la matriz |g| del siguiente circuito.



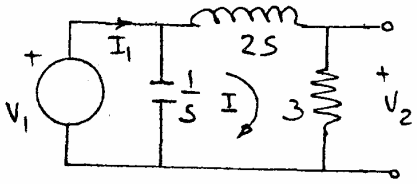
SOLUCION:

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$



$$g_{11} = \frac{\left(\frac{1}{s}\right)(2s+3)}{\frac{1}{s}+2s+3} = \frac{2s^2+3s+1}{2s+3}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$



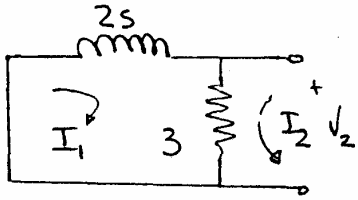
EN LA MALLA CENTRAL SE TIENE:

$$-V_1 + 2sI + V_2 = 0$$

$$-V_1 + \frac{2sV_2}{3} + V_2 = 0$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{3}{2s+3}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$



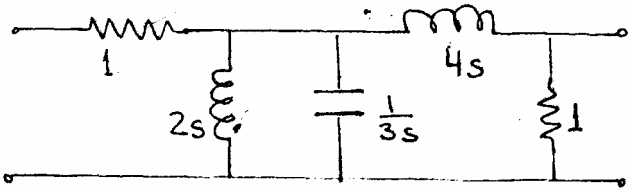
$$(2s+3)I_1 + 3I_2 = 0$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \frac{-3}{2s+3}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{(2s)3}{2s+3}$$

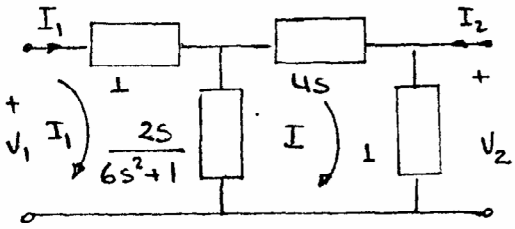
$$|g| = \begin{bmatrix} \frac{2s^2+3s+1}{2s+3} & \frac{-3}{2s+3} \\ \frac{3}{2s+3} & \frac{6s}{2s+3} \end{bmatrix}$$

4.- Hallar la matriz |a| del siguiente circuito.



SOLUCION:

EN EL DOMINIO DE LA FREC:



① $V_1 = \left(1 + \frac{2s}{6s^2+1}\right) I_1 - \frac{2s}{6s^2+1} I$

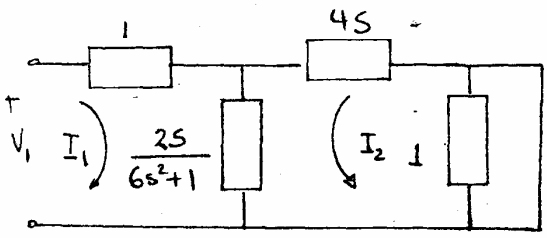
② $\left(1 + 4s + \frac{2s}{6s^2+1}\right) I - \frac{2s}{6s^2+1} I_1 = 0$

③ $V_2 = I$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{24s^3+14s^2+8s+1}{2s} \quad ; \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{24s^3+6s^2+6s+1}{2s}$$

$$V_1 = \left(1 + \frac{2s}{6s^2+1}\right) I_1 + \frac{2s}{6s^2+1} I_2$$

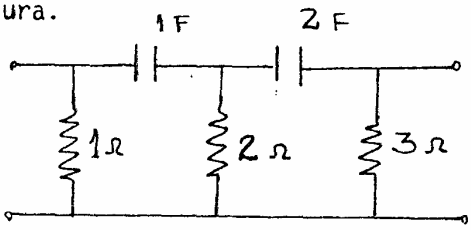
$$\left(4s + \frac{2s}{6s^2+1}\right) I_2 + \frac{2s}{6s^2+1} I_1 = 0$$



$$-D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = -\frac{24s^3+6s}{2s} \quad ; \quad -B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = -\frac{24s^3+8s^2+6s}{2s}$$

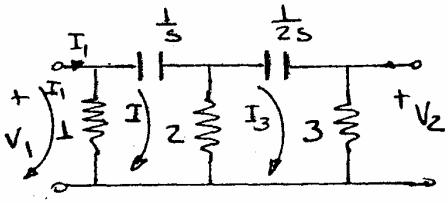
$$[a] = \begin{bmatrix} \frac{24s^3+14s^2+8s+1}{2s} & \frac{24s^3+8s^2+6s}{2s} \\ \frac{24s^3+6s^2+6s+1}{2s} & \frac{24s^3+6s}{2s} \end{bmatrix}$$

5.- Partiendo de la definición de sus parámetros hallar la matriz |Z| de la red de la figura.



SOLUCION:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$



$$Z_{11} = \frac{12s^2+12s+1}{22s^2+13s+1}$$

$$Z_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

LAS EC. SON:

$$\left(3+\frac{1}{s}\right)I - I_1 + 2I_3 = 0$$

$$\left(5+\frac{1}{2s}\right)I_3 - 2I = 0$$

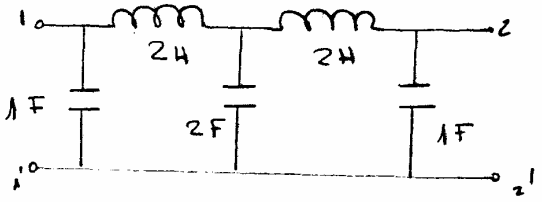
$$3I_3 = V_2$$

$$Z_{21} = Z_{12} = \frac{12s^2}{22s^2+13s+1}$$

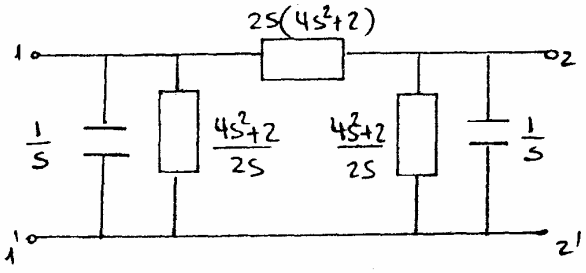
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{12s^2+21s+3}{22s^2+13s+1}$$

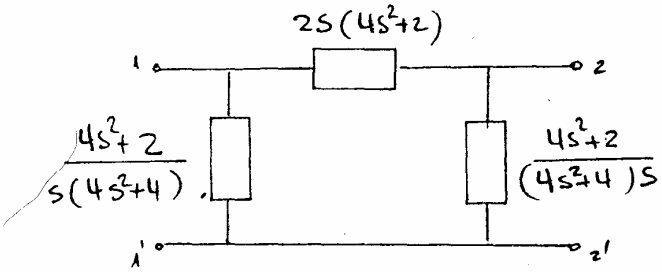
$$[Z] = \begin{bmatrix} \dfrac{12s^2+12s+1}{22s^2+13s+1} & \dfrac{12s^2}{22s^2+13s+1} \\ \dfrac{12s^2}{22s^2+13s+1} & \dfrac{12s^2+21s+3}{22s^2+13s+1} \end{bmatrix}$$

6.- Calcular la matriz |Y| del cuadripolo de la figura partiendo de la definición de sus parámetros.



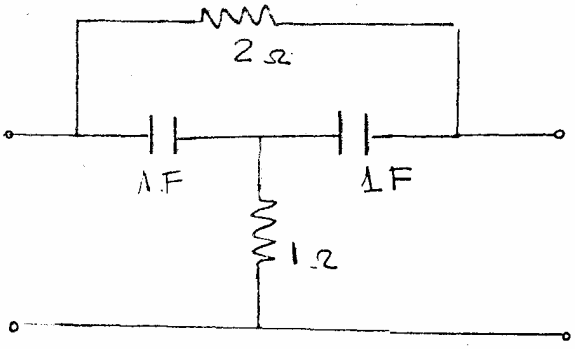
SOLUCION: TRANSFORMANDO LA ESTRELLA CENTRAL A DELTA





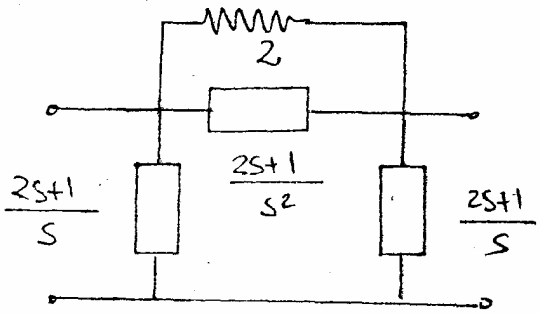
$$[Y] = \left[\begin{array}{c|c} \dfrac{8s^4+8s^2+1}{2s(4s^2+2)} & \dfrac{-1}{2s(4s^2+2)} \\ \hline \dfrac{-1}{2s(4s^2+2)} & \dfrac{8s^4+8s^2+1}{2s(4s^2+2)} \end{array}\right]$$

7. Hallar la matriz |Y| de la red de la figura.

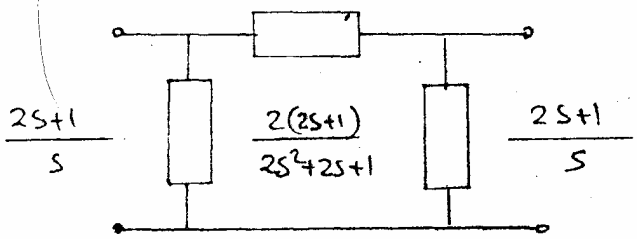


SOLUCION:

TRANSFORMANDO LA ESTRELLA CENTRAL A DELTA

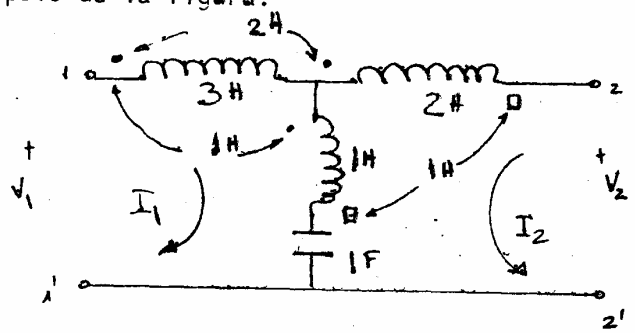


REDUCIENDO LAS RAMAS EN PARALELO



$$[Y] = \begin{bmatrix} \dfrac{2s^2+4s+1}{2(1+2s)} & \dfrac{-(2s^2+2s+1)}{2(1+2s)} \\ \dfrac{-(2s^2+2s+1)}{2(1+2s)} & \dfrac{2s^2+4s+1}{2(1+2s)} \end{bmatrix}$$

8. Partiendo de su definición determinar los parámetros de la matriz |a| del cuadripolo de la figura.



SOLUCION :

LAS ECUACIONES DE MALLA SON:

$$\left(4s+\frac{1}{s}\right)I_1+\left(s+\frac{1}{s}\right)I_2+2sI_1-2sI_2+sI_2-sI_2=V_1$$

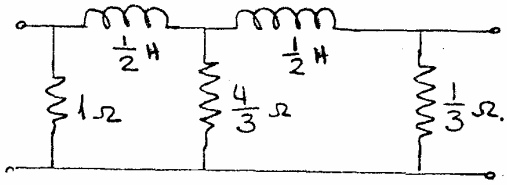
$$\left(3s+\frac{1}{s}\right)I_2-2sI_2-2sI_1+sI_1-sI_1+\left(s+\frac{1}{s}\right)I_1=V_2$$

$$A=\left.\frac{V_1}{V_2}\right|_{I_2=0}=\frac{6s^2+1}{1-s^2} \qquad C=\left.\frac{I_1}{V_2}\right|_{I_2=0}=\frac{s}{1-s^2}$$

$$B=\left.\frac{V_1}{I_2}\right|_{V_2=0}=\frac{5s^3+9s}{1-s^2} \qquad D=\left.\frac{I_1}{I_2}\right|_{V_2=0}=\frac{s^2+1}{1-s^2}$$

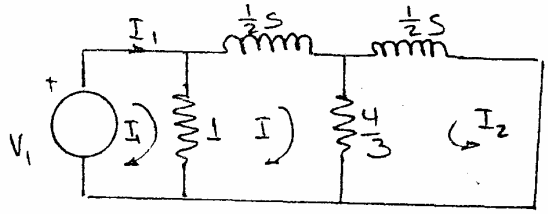
$$[a]=\begin{bmatrix}\dfrac{6s^2+1}{1-s^2} & \dfrac{s}{1-s^2}\\[2ex] \dfrac{5s^3+9s}{1-s^2} & \dfrac{s^2+1}{1-s^2}\end{bmatrix}$$

9. Partiendo de su definición calcular los parámetros de la matriz |h| del cuadripolo de la figura.



SOLUCION:

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$



$$h_{11} = \frac{3s^2+16s}{3s^2+22s+16}$$

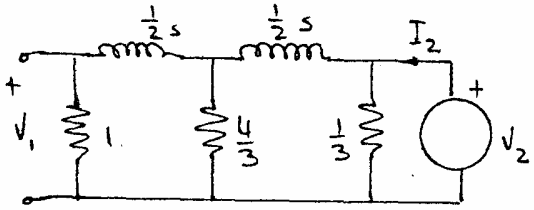
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$\left(\frac{s}{2}+\frac{4}{3}\right)I_2 + \frac{4}{3}I = 0$$

$$\left(1+\frac{4}{3}+\frac{s}{2}\right)I + \frac{4}{3}I_2 - I_1 = 0$$

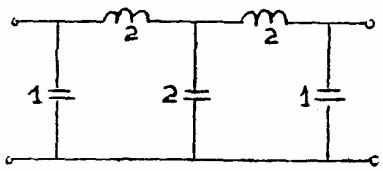
$$h_{21} = -h_{12} = \frac{-16}{3s^2+22s+16}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$h_{22} = \frac{9s^2+72s+76}{3s^2+22s+16}$$

10._ PROBLEMA.- Haciendo uso de la tabla, hallar la matriz $|Z|$ del cuadripolo de la figura.



SOLUCION:

$$Y_{11} = Y_{22} = \frac{8s^4+8s^2+1}{4s(2s^2+1)}$$

$$Y_{12} = Y_{21} = \frac{-1}{4s(2s^2+1)}$$

$$Z_{11} = Z_{22} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = Z_{21} = \frac{-Y_{12}}{\Delta Y}$$

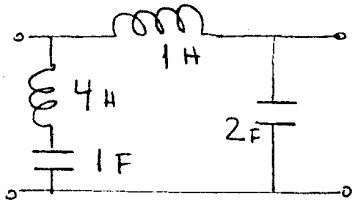
$$\Delta Y = \frac{64s^8+64s^6+16s^4+1}{16s^2(2s^2+1)^2}$$

$$Z_{11} = Z_{22} = \frac{16s^6+24s^4+10s^2+1}{4s(4s^6+8s^4+5s^2+1)}$$

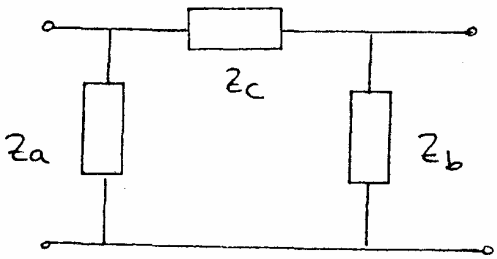
$$Z_{12} = Z_{21} = \frac{2s^2+1}{4s(4s^6+8s^4+5s^2+1)}$$

11.- De los siguientes cuadripolos calcular la matriz |Z| y |Y|.

c. a.:



SOLUCION:



$$Z_a = \frac{4s^2+1}{s}$$

$$Z_c = s$$

$$Z_b = \frac{1}{2s}$$

LA MATRIZ [Z]

$$Z_{11} = \frac{Z_a(Z_b+Z_c)}{Z_a+Z_b+Z_c} = \frac{8s^4+6s^2+1}{s(10s^2+3)}$$

$$Z_{12} = Z_{21} = \frac{Z_a Z_b}{Z_a+Z_b+Z_c} = \frac{4s^2+1}{s(10s^2+3)}$$

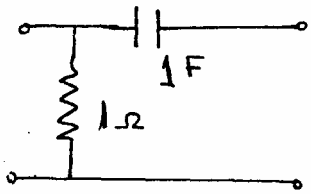
$$Z_{22} = \frac{Z_b(Z_a+Z_c)}{Z_a+Z_b+Z_c} = \frac{5s^2+1}{s(10s^2+3)}$$

$$y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{(5s^2+1)(10s^2+3)}{s(40s^4+22s^2+3)}$$

$$y_{12} = y_{21} = \frac{-Z_{12}}{\Delta Z} = \frac{-(4s^2+1)}{s(40s^4+22s^2+3)}$$

$$y_{22} = \frac{(4s^2+1)(2s^2+1)(10s^2+3)}{s(40s^4+22s^2+3)}$$

11.C.b :



SOLUCION :

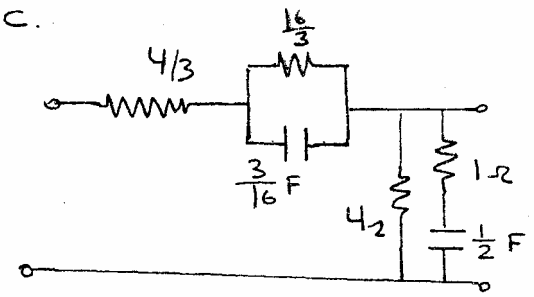
POR LA DEFINICION:

$$[Z] = \begin{bmatrix} 1 & 1 \\ 1 & \frac{S+1}{S} \end{bmatrix}$$

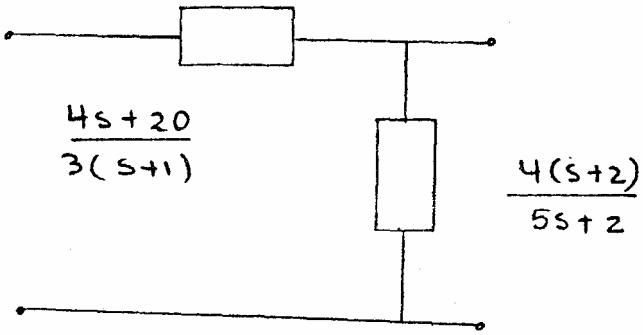
POR TRANSFORMACION:

$$[Y] = \begin{bmatrix} S+1 & -S \\ -S & S \end{bmatrix}$$

11. C.C.



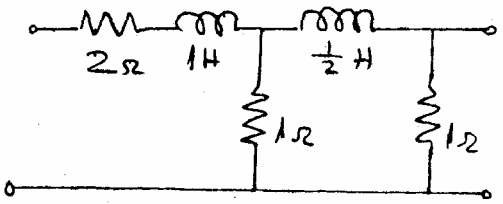
SOLUCION: REDUCIENDO RAMAS QUEDA:



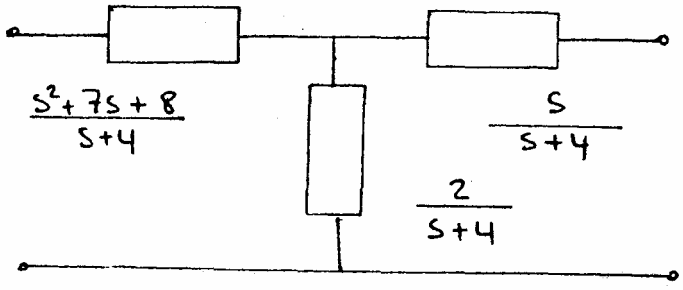
$$[Z] = \begin{bmatrix} \dfrac{32s^2+144s+64}{3(s+1)(5s+2)} & \dfrac{4s+8}{5s+2} \\[2ex] \dfrac{4s+8}{5s+2} & \dfrac{4s+8}{5s+2} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \dfrac{3(s+1)}{4(s+5)} & \dfrac{-3(s+1)}{4(s+5)} \\[2ex] \dfrac{-3(s+1)}{4(s+5)} & \dfrac{(2s+1)(s+4)}{(s+5)(s+2)} \end{bmatrix}$$

C.d..



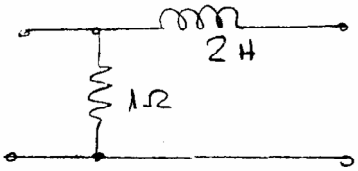
SOLUCION: TRANSFORMANDO LA Δ EN Y TENDREMOS:



$$[Z] = \begin{bmatrix} \frac{s^2+7s+1}{s+4} & \frac{2}{s+4} \\ \frac{2}{s+4} & \frac{s+2}{s+4} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{s+2}{(s+4)(s+1)} & \frac{-2}{(s+4)(s+1)} \\ \frac{-2}{(s+4)(s+1} & \frac{(s+5)(s+2)}{(s+4)(s+1} \end{bmatrix}$$

11.e.

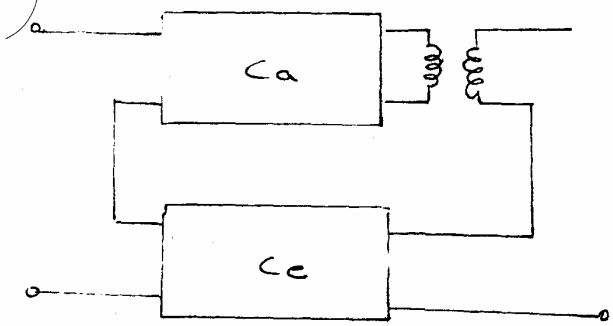


SOLUCION :

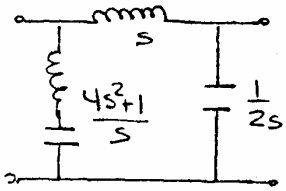
$$[Z] = \begin{bmatrix} 1 & 1 \\ 1 & 1+2S \end{bmatrix} \qquad [a] = \begin{bmatrix} 1 & 2S \\ 1 & 2S+1 \end{bmatrix}$$

$$[g] = \begin{bmatrix} 1 & 1 \\ 1 & 2S \end{bmatrix} \qquad [h] = \begin{bmatrix} \frac{2S}{1+2S} & \frac{1}{1+2S} \\ \frac{-1}{1+2S} & \frac{1}{1+2S} \end{bmatrix}$$

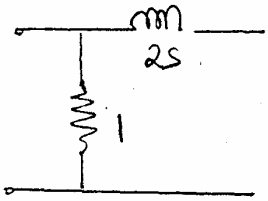
12.- Obtener la matriz |Z|de la conexión del cuadripolo de la figura y probar si es necesario el uso del transformador.



SOLUCION:



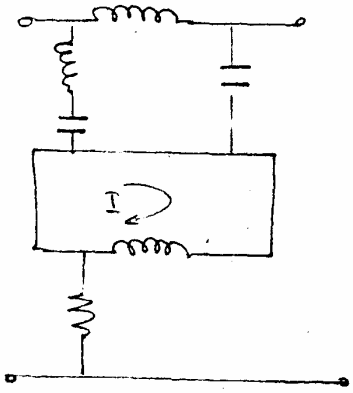
$$[Z_a] = \begin{bmatrix} \dfrac{8s^4+6s^2+1}{10s^3+3s} & \dfrac{4s^2+1}{10s^3+3s} \\ \dfrac{4s^2+1}{10s^3+3s} & \dfrac{5s^2+1}{10s^3+3s} \end{bmatrix}$$



$$[Z_e] = \begin{bmatrix} 1 & 1 \\ 1 & 1+2s \end{bmatrix}$$

SUMANDO LAS MATRICES [Z]

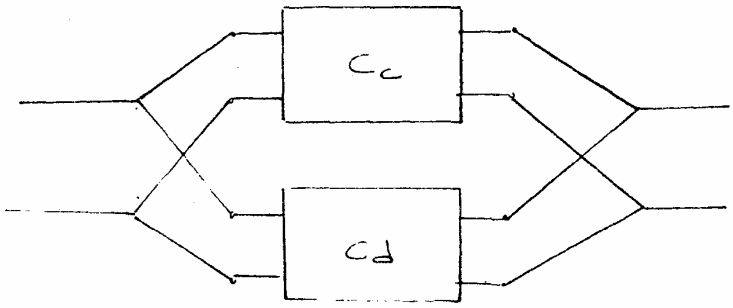
PRUEBA DE BRUNE



$$[Z] = \begin{bmatrix} \dfrac{8s^4+10s^3+6s^2+3s+1}{10s^3+3s} & \dfrac{10s^3+4s^2+3s+1}{10s^3+3s} \\ \dfrac{10s^3+4s^2+3s+1}{10s^3+3s} & \dfrac{20s^4+10s^3+11s^2+3s+1}{10s^3+3s} \end{bmatrix}$$

SI ES NECESARIO EL USO DEL TRANSFORMADOR IDEAL

13.- Obtener la matriz |Y| de la conexión mostrada en la figura.



SOLUCION:

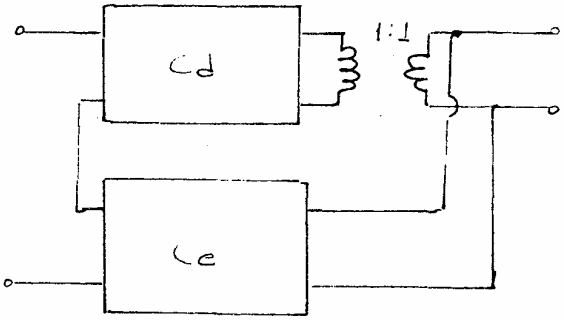
SUMANDO LAS MATRICES [Y] :

$$y_{11} = \frac{3(s+1)^2(s+4) + 4(s+2)(s+5)}{4(s+5)(s+4)(s+1)} = \frac{3s^3+22s^2+55s+52}{4(s+5)(s+4)(s+1)}$$

$$y_{12} = y_{21} = \frac{-3(s+1)^2(s+4) - 8(s+5)}{4(s+5)(s+4)(s+1)} = \frac{-(3s^3+18s^2+35s+52)}{4(s+5)(s+4)(s+1)}$$

$$y_{22} = \frac{(2s+1)(s+4)^2(s+1) + (s+2)^2(s+5)^2}{(s+2)(s+5)(s+4)(s+1)} = \frac{3s^4+33s^3+126s^2+106s+116}{(s+2)(s+5)(s+4)(s+1)}$$

14.- Calcular la matriz |h| de la conexión y probar si es necesario el transformador.



SOLUCION:

LA MATRIZ [h] DEL CUADRIPOLO Cd :

$$[h_d] = \begin{bmatrix} \frac{s^2+5s+4}{s+2} & \frac{2}{s+2} \\ \frac{-2}{s+2} & \frac{s^3+7s^2+11s+11}{(s+1)(s+4)(s+2)} \end{bmatrix}$$

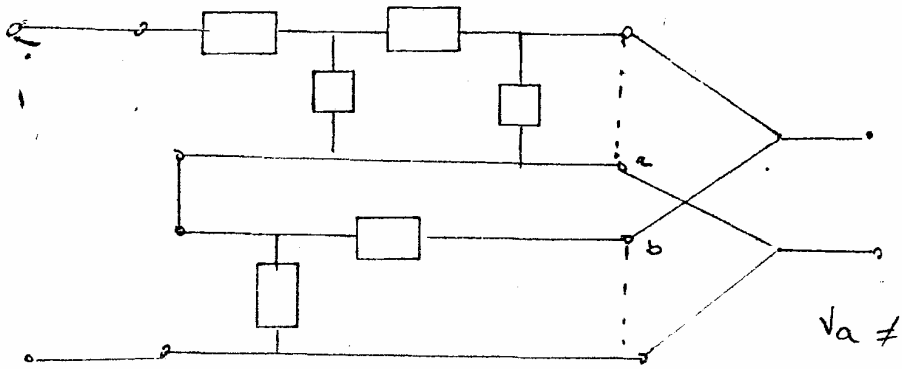
LA MATRIZ [h] DEL CUADRIPOLO Ce:

$$[h_e] = \begin{bmatrix} \frac{2s}{1+2s} & \frac{1}{1+2s} \\ \frac{-1}{1+2s} & \frac{1}{1+2s} \end{bmatrix}$$

LA MATRIZ [h] DE LA CONEXIÓN :

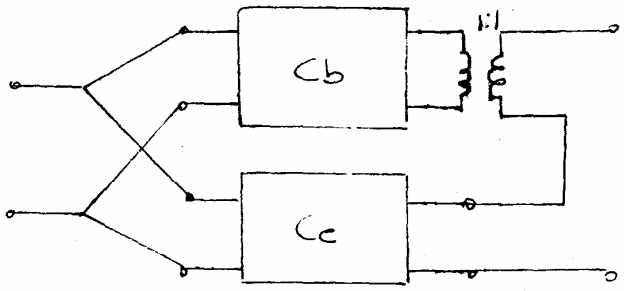
$$[h] = \begin{bmatrix} \frac{2s^3+13s^2+17s+4}{2s^2+5s+2} & \frac{3s+3}{2s^2+5s+2} \\ \frac{-(3s+3)}{2s^2+5s+2} & \frac{2s^4+19s^3+64s^2+76s+27}{2s^4+15s^3+32s^2+30s+8} \end{bmatrix}$$

PRUEBA DE BRUNE FALLA EN AMBOS TERMINALES POR LO TANTO ES NECESARIO EL TRANSFORMADOR IDEAL



$V_a \neq V_b$

15.- Calcular la matriz |g| debida a la conexión indicada, probar si es necesario el transformador.



SOLUCION :

LA MATRIZ [G] DEL CUADRIPOLO [Cb]

$$[g_b] = \begin{bmatrix} 1 & -1 \\ 1 & \frac{1}{s} \end{bmatrix}$$

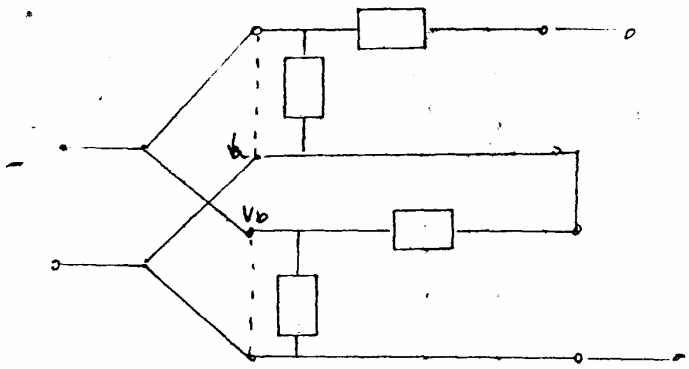
LA MATRIZ [G] DEL CUADRIPOLO [Ce]

$$[g_e] = \begin{bmatrix} 1 & -1 \\ 1 & 2s \end{bmatrix}$$

DE LA CONEXION :

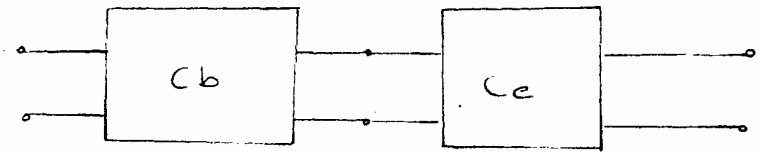
$$[g] = \begin{bmatrix} 2 & -2 \\ 2 & \frac{1+2s^2}{s} \end{bmatrix}$$

LA PRUEBA DE BRUNE FALLA POR LO TANTO ES NECESARIO EL TRANSFORMADOR



$V_a \neq V_b$

16.- Calcular la matriz |a|.



SOLUCION:

LA MATRIZ [a] DEL CUADRIPOLO [Cb]

$$[a_b] = \begin{bmatrix} 1 & \frac{1}{s} \\ 1 & \frac{s+1}{s} \end{bmatrix}$$

LA MATRIZ [a] DEL CUADRIPOLO [Ce]

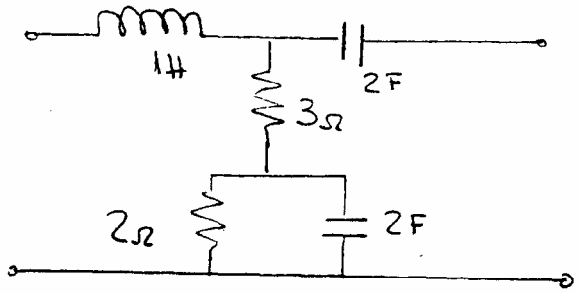
$$[a_e] = \begin{bmatrix} 1 & 2s \\ 1 & 1+2s \end{bmatrix}$$

LA MATRIZ [a] DE LA CONEXION EN CASCADA ES EL PRODUCTO

$$[a] = \begin{bmatrix} \frac{s+1}{s} & \frac{2s^2+2s+1}{s} \\ \frac{2s+1}{s} & \frac{4s^2+3s+1}{s} \end{bmatrix}$$

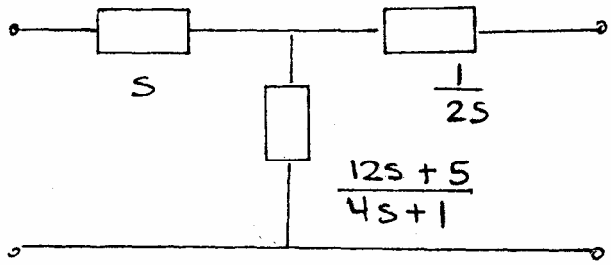
17. Hallar la matriz |Z| de cada una de las siguientes redes.

a)



SOLUCION:

REDUCIENDO LA RED A UNA T SE TIENE:



LUEGO

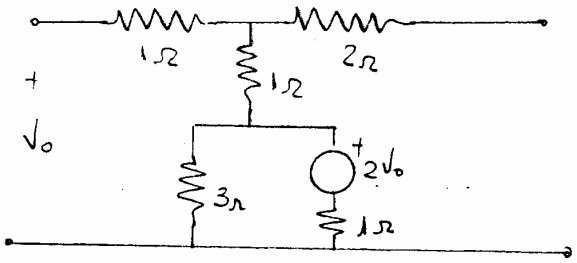
$$Z_{11} = S + \frac{12S+5}{4S+1} = \frac{4S^2+13S+5}{4S+1}$$

$$Z_{12} = Z_{21} = \frac{12S+5}{4S+1}$$

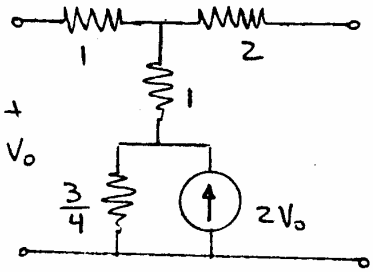
$$Z_{22} = \frac{1}{2S} + \frac{12S+5}{4S+1} = \frac{24S^2+14S+1}{2S(4S+1)}$$

$$[Z] = \begin{bmatrix} \frac{4S^2+13S+5}{4S+1} & \frac{12S+5}{4S+1} \\ \frac{12S+5}{4S+1} & \frac{24S^2+14S+1}{2S(4S+1)} \end{bmatrix}$$

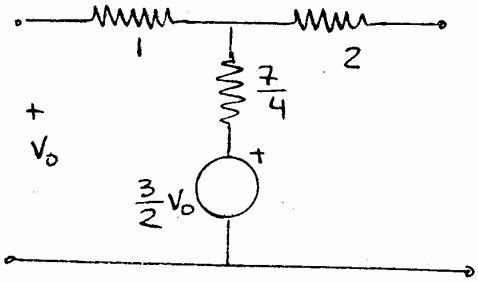
17.b.



SOLUCION: REDUCIENDO LA RED POR TRANSFORMACION DE FUENTES:



$\Rightarrow$



POR LA DEFINICION LA MATRIZ [Z] ES:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad V_1 = \left(1+\frac{7}{4}\right)I_1 + \frac{3}{2}V_1 \qquad -\frac{1}{2}V_1 = \frac{11}{4}I_1 \qquad \boxed{Z_{11} = -\frac{11}{2}\Omega}$$

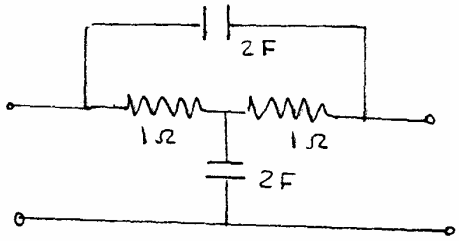
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad V_1 = \frac{7}{4}I_2 + \frac{3}{2}V_1 \qquad -\frac{1}{2}V_1 = \frac{7}{4}I_2 \qquad \boxed{Z_{12} = -\frac{7}{2}\Omega}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad V_2 = \frac{7}{4}I_1 + \frac{3}{2}V_1 \qquad V_2 = \frac{7}{4}I_1 + \frac{3}{2}\left(-\frac{11}{2}\right)I_1 \qquad \boxed{Z_{21} = -\frac{13}{2}\Omega}$$

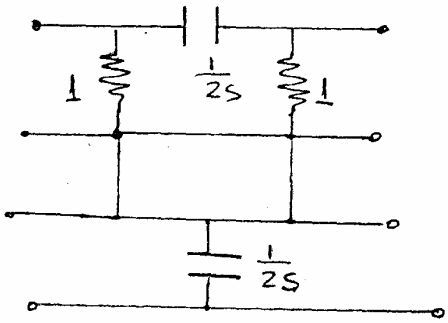
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad V_2 = 2I_2 + \frac{7}{4}I_2 + \frac{3}{2}V_o \qquad V_2 = \frac{15}{4}I_2 + \frac{3}{2}\left(-\frac{7}{2}I_2\right) \qquad V_2 = -\frac{6}{4}I_2 \qquad \boxed{Z_{22} = -\frac{3}{2}\Omega}$$

18.- Hallar la matriz |Y|de cada una de las siguientes redes.

a.



SOLUCION: CONSIDERANDO LA RED COMO CONEXION SERIE-SERIE



LAS MATRICES [Z] DE CADA UNA DE ELLAS SON:

$$[Z_\pi] = \begin{bmatrix} \frac{1+2s}{1+4s} & \frac{2s}{1+4s} \\ \frac{2s}{1+4s} & \frac{1+2s}{1+4s} \end{bmatrix} \qquad [Z_p] = \begin{bmatrix} \frac{1}{2s} & \frac{1}{2s} \\ \frac{1}{2s} & \frac{1}{2s} \end{bmatrix}$$

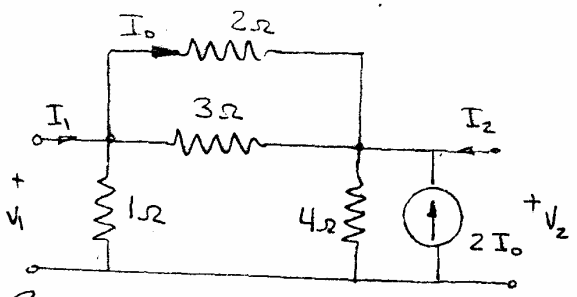
LA MATRIZ [Z] DE LA CONEXION SERIE - SERIE

$$[Z] = \begin{bmatrix} \frac{4s^2+6s+1}{2s(4s+1)} & \frac{4s^2+4s+1}{2s(4s+1)} \\ \frac{4s^2+4s+1}{2s(4s+1)} & \frac{4s^2+6s+1}{2s(4s+1)} \end{bmatrix}$$

LA MATRIZ [Y] EQUIVALENTE

$$[Y] = \begin{bmatrix} \frac{4s^2+6s+1}{2s+2} & \frac{-(4s^2+4s+1)}{2s+2} \\ \frac{-(4s^2+4s+1)}{2s+2} & \frac{4s^2+6s+1}{2s+2} \end{bmatrix}$$

18. b.



SOLUCION: POR LA DEFINICION:

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$-\frac{V_2}{3} = I_1 - I_o$$

$$-\frac{V_2}{3} = I_1 + \frac{V_2}{2}$$

$$-\frac{5}{6}V_2 = I_1$$

$$\boxed{y_{12} = -\frac{5}{6}}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$-\frac{1}{3}V_1 = I_2 + 3I_o$$

$$I_o = \frac{V_1}{2}$$

$$-\frac{1}{3}V_1 = I_2 + 3\frac{V_1}{2}$$

$$-\frac{11}{6}V_1 = I_2$$

$$\boxed{y_{21} = -\frac{11}{6}}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$V_1\left(1+\frac{1}{3}\right) - V_2\left(\frac{1}{3}\right) = I_1 - I_o$$

$$-V_1\frac{1}{3} + V_2\left(\frac{1}{3}+\frac{1}{4}\right) = I_2 + 3I_o$$

$$-\frac{4}{3}V_1 - \frac{V_2}{3} = I_1 - I_o$$

$$-\frac{1}{3}V_1 + \frac{7}{12}\cancelto{0}{V_2} = I_2 + 3I_o$$

$$\frac{4}{3}V_1 = I_1 - I_o \quad ; I_o = \frac{V_1 - V_2}{2}$$

$$\frac{4}{3}V_1 = I_1 - \frac{V_1}{2}$$

$$\frac{11}{6}V_1 = I_1$$

$$\boxed{y_{11} = \frac{11}{6}}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

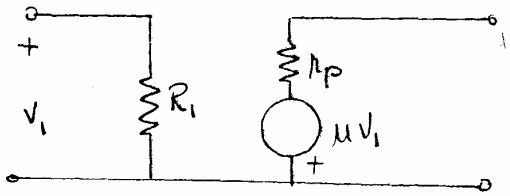
$$\frac{7}{12}V_2 = I_2 + 3I_o$$

$$\frac{7}{12}V_2 = I_2 - \frac{3}{2}V_2$$

$$\frac{25}{12}V_2 = I_2$$

$$\boxed{y_{22} = \frac{25}{12}}$$

19.- Determinar la matriz |Z| y la matriz |Y| del cuadripolo de la figura.



SOLUCION: POR LA DEFINICION LA MATRIZ [Z]

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad V_1 = I_1 R_1 \qquad \therefore \quad Z_{11} = R_1$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad V_2 = -\mu I_1 R_1 \qquad \therefore \quad Z_{21} = -\mu R_1$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad V_2 = I_2 r_p \qquad \therefore \quad Z_{22} = r_p$$

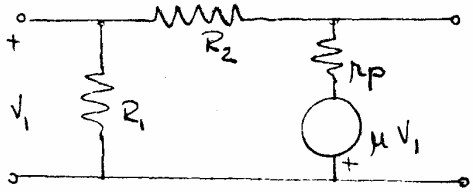
$$\Delta Z = r_p R_1$$

$$y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{1}{R_1} \qquad y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-\mu R_1}{R_1 r_p} = \frac{\mu}{r_p} \qquad y_{12} = 0$$

$$y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{R_1}{R_1 r_p} = \frac{1}{r_p}$$

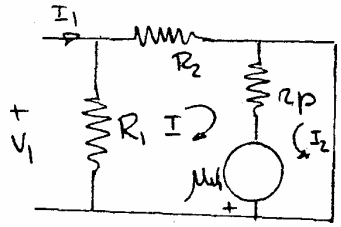
$$[Z] = \begin{bmatrix} R_1 & 0 \\ -\mu R_1 & r_p \end{bmatrix} \qquad [Y] = \begin{bmatrix} \frac{1}{R_1} & 0 \\ \frac{\mu}{r_p} & \frac{1}{r_p} \end{bmatrix}$$

20.- Determinar los parámetros |Z| y |Y| del cuadripolo activo.



SOLUCION: POR LA DEFINICION LA MATRIZ [Y]

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{11} = \frac{R_1+R_2}{R_1R_2}$$

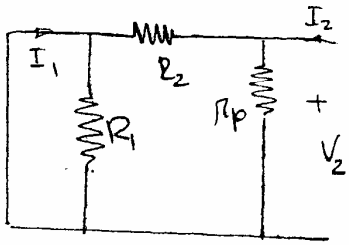


$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{21} = \frac{\mu R_2 - r_p}{R_2 r_p}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$I_1 R_2 + V_2 = 0$$

$$y_{12} = -\frac{1}{R_2}$$



$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{r_p+R_2}{r_p R_2}$$

MEDIANTE TABLAS SE OBTIENE LA MATRIZ [Z]

$$Z_{11} = \frac{R_1(R_2+r_p)}{R_2+r_p+R_1(1+\mu)}$$

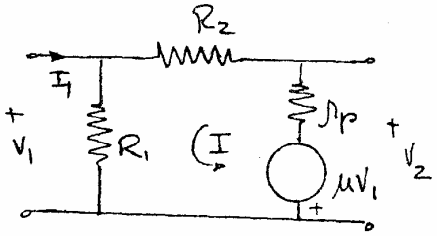
$$Z_{12} = \frac{R_1 r_p}{R_2+r_p+R_1(1+\mu)}$$

$$Z_{21} = \frac{R_1(r_p-\mu R_2)}{R_2+r_p+R_1(1+\mu)}$$

$$Z_{22} = \frac{r_p(R_1+R_2)}{R_2+r_p+R_1(1+\mu)}$$

21.- Haciendo uso de la definición de matrices determinar la matriz |a| del cuadripolo anterior.

SOLUCION:



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$(R_2 + r_p)I + V_1 + \mu V_1 = 0$$

$$\frac{V_2 - V_1}{R_2} = I$$

$$\frac{R_2 + r_p}{R_2} V_2 = \frac{R_2 + r_p - R_2 - \mu R_2}{R_2} \cdot V_1$$

$$A = \frac{r_p + R_2}{r_p - \mu R_2}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

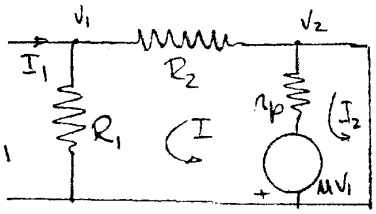
$$V_2 = I r_p - \mu V_1$$

$$V_1 = R_1(I_1 + I)$$

$$R_2 I = V_2 - V_1$$

$$\therefore \quad C = \frac{R_1(1+\mu) + R_2 + r_p}{R_1(r_p - \mu R_2)}$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$



$$V_1 = R_1 I_1 + R_1 I$$

$$r_p I_2 = R_1 I_1 + (R_1 + R_2 + r_p) I - \mu V_1$$

$$I = -\frac{V_1}{R_2}$$

$$B = \frac{R_2 r_p}{r_p - \mu R_2}$$

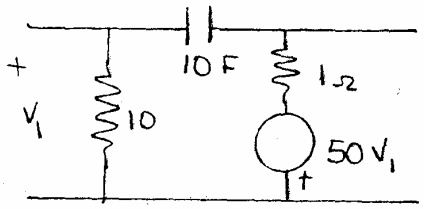
$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$V_2\left(\cancelto{0}{\frac{1}{R_2}} + \frac{1}{r_p}\right) - \frac{V_1}{R_2} = I_2 - \frac{\mu V_1}{r_p}$$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \cancelto{0}{\frac{V_2}{R_2}} = I_1$$

$$D = \frac{r_p(R_1 + R_2)}{R_1(r_p - \mu R_2)}$$

22.- Determinar la matriz |h| para el cuadripolo de la figura.

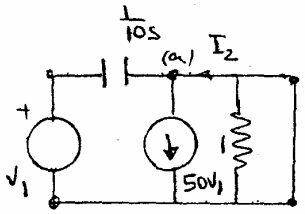


SOLUCION:

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{10 \times \frac{1}{10S}}{10 + \frac{1}{10S}} = \frac{10}{100S+1}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad ; \quad V_1\left(\frac{1+100S}{10}\right) = I_1$$

En el nod. a: $(50 - 10S)V_1 = I_2$

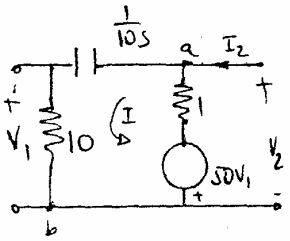


$$h_{21} = \frac{500 - 100S}{100S + 1}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$V_1 = 10\,I$

$V_2 = \frac{1}{10S} I + V_1$



$$h_{12} = \frac{100S}{100S+1}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

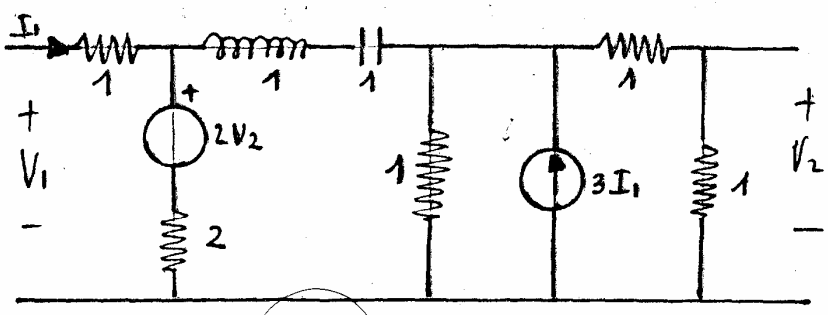
$V_1 = \frac{100S}{100S+1} V_2$

$(50-10S)V_1 + V_2(10S+1) = I_2$

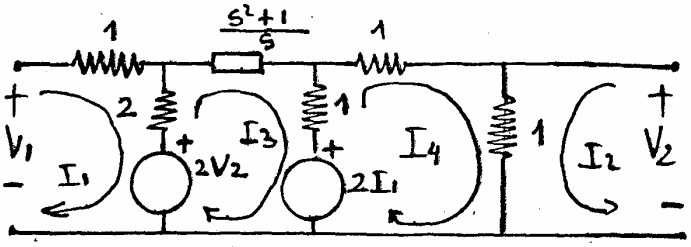
$$h_{22} = \frac{5110S+1}{100S+1}$$

$$[h] = \begin{bmatrix} \frac{10}{100S+1} & \frac{100S}{100S+1} \\ \frac{500-100S}{100S+1} & \frac{5110S+1}{100S+1} \end{bmatrix}$$

23.- Calcular $|Z|$ para el cuadripolo de la figura



SOLUCION.-



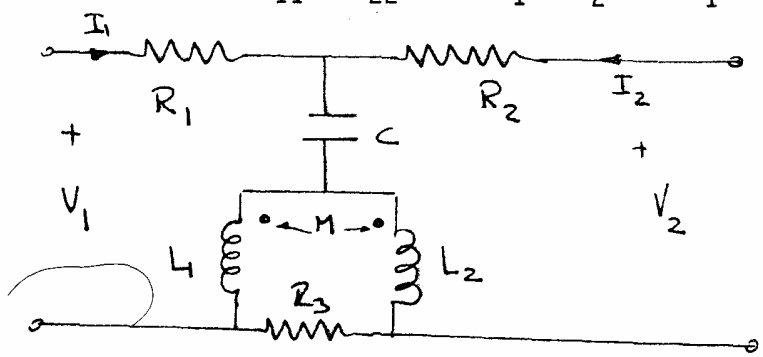
$$\begin{bmatrix} 3 & -2 & 0 & 0 \\ -2 & 3+\frac{S^2+1}{S} & -1 & 0 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \\ I_4 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 - 2V_2 \\ 2V_2 - 2I_1 \\ 2I_1 \\ V_2 \end{bmatrix}$$

$$[Z] = \frac{1}{3(S+1)^2} \begin{bmatrix} 15S^2 + 22S + 15 & 4S^2 + 4S + 4 \\ 3S^2 + 8S + 3 & 2S^2 + 5S + 2 \end{bmatrix}$$

24.- Para el circuito de la figura

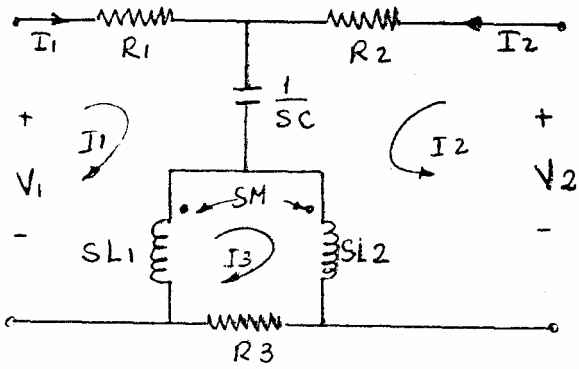
a) Hallar $|Z|$ y $|Y|$

b) Demostrar que $Z_{11} = Z_{22}$ si $R_1 = R_2$ y $L_1 = L_2$



SOLUCION:

En el dominio de la frecuencia:



Mediante el análisis matricial de mallas:

$$\begin{bmatrix} V_I \\ V_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \left(R_I + \frac{I}{SC} + SL_I\right) & \left(\frac{I}{SC} + SM\right) & \left(-SL_I + SM\right) \\ \left(\frac{I}{SC} + SM\right) & \left(R_2 + \frac{I}{SC} + SL_2\right) & \left(SL_2 - SM\right) \\ \left(-SL_I + SM\right) & \left(SL_2 - SM\right) & \left(SL_I + SL_2 + R_3 - 2SM\right) \end{bmatrix} \begin{bmatrix} I_I \\ I_2 \\ I_3 \end{bmatrix}$$

$$Z_{II} = \left.\frac{V_I}{I_I}\right|_{I_2 = 0} \quad ; \quad V_I = (R_I + I/SC + SL_I)\, I_I + (-SL_I + SM) I_3$$

$$0 = (-SL_I + SM) I_I + (SL_2 - SM) I_2 + (SL_I + SL_2 + R_3 - 2SM) I_3$$

Para $I_2 = 0$

entonces : $$I_3 = \frac{SL_I - SM}{SL_I + SL_2 + R_3 - 2SM} \cdot I_I$$

$$V_I = \left[(R_I + I/SC + SL_I) + \frac{-(SL_I - SM)^2}{SL_I + SL_2 + R_3 - 2SM}\right] I_I$$

$$Z_{II} = \frac{V_I}{I_I} = (R_I + I/SC + SL_I) + \frac{-(SL_I - SM)^2}{SL_I + SL_2 + R_3 - 2SM}$$

$$Z_{22} = \frac{V_2}{I_2}\Bigg|_{I_I = 0}$$ entonces con esta condición $I_I = 0$

$$V_2 = (R_2 + I/SC + SL_2)I_2 + (SL_2 - SM)I_3$$

$$I_3 = \frac{-(SL_2 - SM)}{SL_I + SL_2 + R_3 - 2SM} \cdot I_2$$

$$V_2 = \left[(R_2 + I/SC + SL_2) + \frac{-(SL_2 - SM)^2}{SL_I + SL_2 + R_3 - 2SM}\right] \cdot I_2$$

$$Z_{22} = \frac{V_2}{I_2} = (R_2 + I/SC + SL_2) + \frac{-(SL_2 - SM)^2}{SL_I + SL_2 + R_3 - 2SM}$$

$$Z_{I2} = \frac{V_I}{I_2}\Bigg|_{I_I = 0}$$

$$V_I = (I/SC + SM)I_2 + (-SL_I + SM)I_3$$

reemplazando el valor de I_3 deducido anteriormente para $I_I = 0$

$$Z_{I2} = (I/SC + SM) + \frac{(-SL_I + SM)(-SL_2 + SM)}{SL_I + SL_2 + R_3 - 2SM}$$

$$Z_{2I} = \frac{V_2}{I_I}\Bigg|_{I_2 = 0}$$

$$V_2 = (I/SC + SM)I_I + (SL_2 - SM)I_3$$

reemplazando el valor de I_3 deducido anteriormente para $I_2 = 0$

$$Z_{2I} = (I/SC + SM) + \frac{(SL_2 - SM)(SL_I - SM)}{SL_I + SL_2 + R_3 - 2SM}$$

Luego la matriz $[Z]$:

$$[Z] = \begin{bmatrix} \left(R_1 + \frac{1}{SC} + SL_1\right) + \frac{-(SL_1 - SM)^2}{SL_1 + SL_2 + R_3 - 2SM} & \left(\frac{1}{SC} + SM\right) + \frac{(-SL_1 + SM)(-SL_2 + SM)}{SL_1 + SL_2 + R_3 - 2SM} \\ \left(\frac{1}{SC} + SM\right) + \frac{(SL_2 - SM)(SL_1 - SM)}{SL_1 + SL_2 + R_3 - 2SM} & \left(R_2 + \frac{1}{SC} + SL_2\right) + \frac{-(SL_2 - SM)^2}{SL_1 + SL_2 + R_3 - 2SM} \end{bmatrix}$$

Para obtener $[Y]$ lo hacemos a partir de la matriz $[Z]$, haciendo uso de las tablas correspondientes:

$$[Y] = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{I2}}{\Delta Z} \\ \frac{-Z_{2I}}{\Delta Z} & \frac{Z_{II}}{\Delta Z} \end{bmatrix}$$

b) Si $R_I = R_2$ y $L_I = L_2$ por demostrar que $Z_{II} = Z_{22}$

$R_I = R_2 = R$; $L_I = L_2 = L$

reemplazando estos valores en Z_{II} y Z_{22}

$$Z_{II} = (R + I/SC + SL) + \frac{-(SL - SM)^2}{2SL + R_3 - 2SM}$$

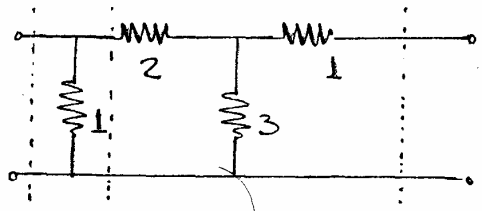
$$Z_{22} = (R + I/SC + SL) + \frac{-(SL - SM)^2}{2SL + R_3 - 2SM}$$

De lo que podemos observar que :

$Z_{II} = Z_{22}$ para estas condiciones. l.q.q.d.

25.- Hallar |Z|, |Y|, |h| y |a| para las dos redes de la figura.

a)



SOLUCION:

CONSIDERANDO EL CUADRIPOLO COMO LA CONEXION DE LA CASCADA DE UN CUADRIPOLO PARALELO Y UN CUADRIPOLO ENT:

$$[a'] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad [a''] = \begin{bmatrix} 1+\frac{2}{3} & \frac{2}{3}+3 \\ \frac{1}{3} & 1+\frac{1}{3} \end{bmatrix}$$

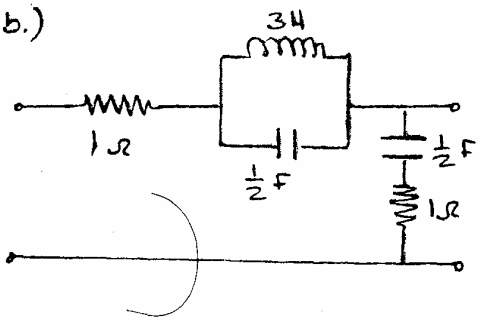
$$[a_T] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{5}{3} & \frac{11}{3} \\ \frac{1}{3} & \frac{4}{3} \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & \frac{11}{3} \\ 2 & 5 \end{bmatrix}$$

MEDIANTE LA UTILIZACION DE TABLAS:

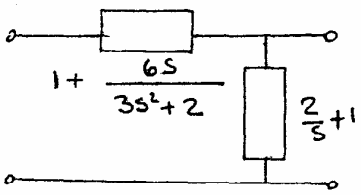
$$[Z] = \begin{bmatrix} \frac{5}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{15}{11} & -\frac{3}{11} \\ -\frac{3}{11} & \frac{5}{11} \end{bmatrix} \qquad [h] = \begin{bmatrix} \frac{11}{15} & \frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

25 b.)



$\Rightarrow$



SOLUCION: LA MATRIZ [Z] ES:

$$Z_{11} = \frac{6s^3+12s^2+4s+4}{s(3s^2+2)}$$

$$Z_{12} = Z_{21} = \frac{s+2}{s}$$

$$Z_{22} = \frac{s+2}{s}$$

$$\Delta Z = \frac{3s^3+12s^2+14s+4}{s(3s^2+2)}$$

LA MATRIZ [Y]:

$$Y_{11} = \frac{3s^2+2}{3s^2+6s+2}$$

$$Y_{12} = Y_{21} = -\frac{3s^2+2}{3s^2+6s+2}$$

$$Y_{22} = \frac{6s^3+12s^2+4s+4}{3s^3+12s^2+14s+4}$$

LA MATRIZ [h]:

$$h_{11} = \frac{3s^2+6s+2}{3s^2+2}$$

$$h_{12} = 1$$

$$h_{21} = -1$$

$$h_{22} = \frac{s}{s+2}$$

LA MATRIZ [a] =

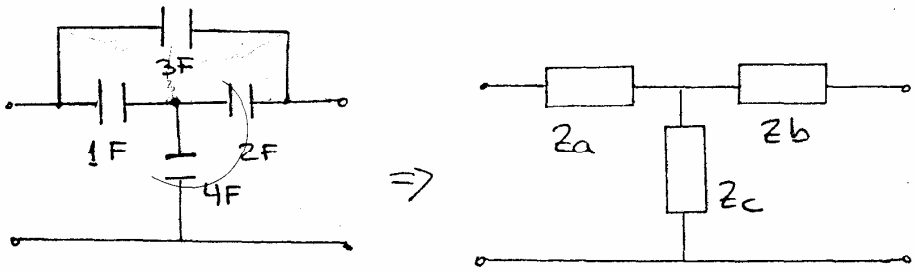
$$A = \frac{6s^3+12s^2+4s+4}{(s+2)(3s^2+2)}$$

$$B = \frac{3s^2+6s+2}{3s^2+2}$$

$$C = \frac{s}{s+2}$$

$$D = 1$$

26.- El circuito de la figura tiene una matriz |Z| que es equivalente a la red en T de la derecha. Hallar Za, Zb y Zc.



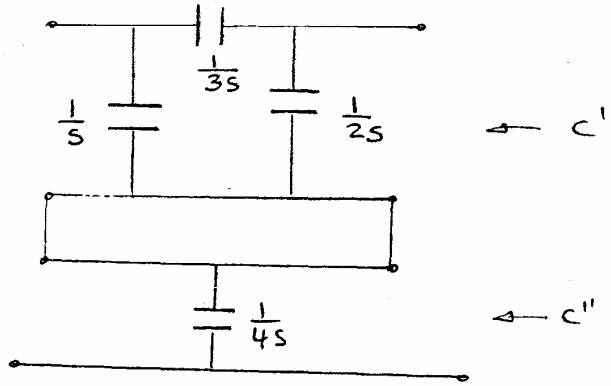
SOLUCION: CONSIDERANDO AL CUADRIPOLO DADO COMO LA CONEXION SERIE-SERIE DE UNA π CON UNA RAMA PARALELO:

$Z_{11}' = \frac{5}{11s}$

$Z_{12}' = Z_{21}' = \frac{3}{11s}$

$Z_{22}' = \frac{4}{11s}$

$Z_{11}'' = Z_{22}'' = Z_{12}'' = Z_{21}'' = \frac{1}{4s}$



LA SUMA DE LAS MATRICES [Z]

$$[Z] = \begin{bmatrix} \frac{31}{44s} & \frac{23}{44s} \\ \frac{23}{44s} & \frac{27}{44s} \end{bmatrix}$$

DE DONDE SE DEBERA CUMPLIR QUE:

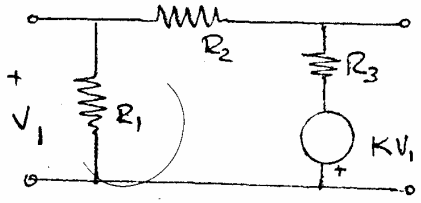
$$Z_c = \frac{23}{44s}$$

$$Z_a + Z_c = \frac{31}{44s} \quad \therefore \quad Z_a = \frac{2}{11s}$$

$$Z_b + Z_c = \frac{27}{44s} \quad \therefore \quad Z_b = \frac{1}{11s}$$

27.- Hallar los parámetros |h| de los circuitos de la figura.

a)



SOLUCION :

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{11} = \frac{R_1 R_2}{R_1 + R_2}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad V_1\left(\frac{K}{R_3} - \frac{1}{R_2}\right) = I_2$$

$$I_2\left(\frac{R_2 R_3}{KR_2 - R_3} \cdot \frac{R_1 + R_2}{R_1 R_2}\right) = I_1$$

$$h_{21} = \frac{R_1 (KR_2 - R_3)}{R_3 (R_1 + R_2)}$$

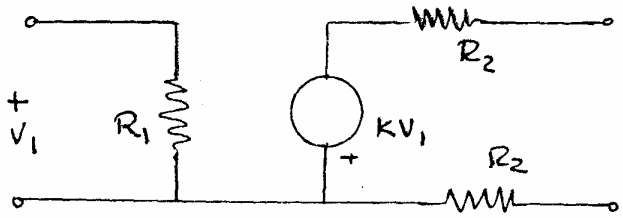
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad V_1\left(\frac{R_1 + R_2}{R_1 R_2}\right) = \frac{V_2}{R_2} \qquad h_{12} = \frac{R_1}{R_1 + R_2}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \qquad V_1 = V_2 \frac{R_1}{R_1 + R_2}$$

$$\frac{V_2 R_1}{R_1 + R_2}\left(K - \frac{1}{R_2}\right) + V_2\left(\frac{R_2 + R_3}{R_2 R_3}\right) = I_2$$

$$h_{22} = \frac{KR_1 + R_1 + R_2 + R_3}{R_3 (R_1 + R_2)}$$

27 b)



SOLUCION:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad V_1 = I_1 R_1 \qquad Z_{11} = R_1$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad Z_{12} = 0$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad V_2 = -KV_1 \qquad V_2 = -KI_1R_1 \qquad Z_{21} = -KR_1$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad Z_{22} = 2R_2$$

$$\Delta Z = 2R_1R_2$$

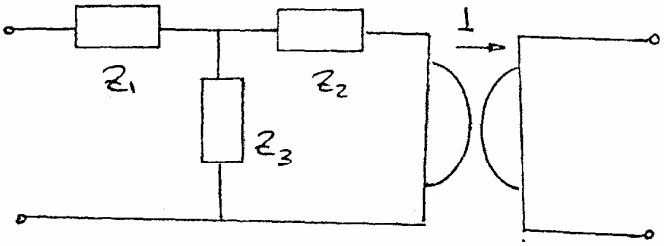
$$h_{11} = R_1$$

$$h_{12} = 0$$

$$h_{21} = \frac{KR_1}{2R_2}$$

$$h_{22} = \frac{1}{2R_2}$$

28.- Hallar la matriz |a| del circuito de la figura.



SOLUCIÓN:

LA MATRIZ [Z] DEL GIRADOR:

$$[Z] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \Delta z = 1$$

$A = 0$
$B = 1$
$C = 1$
$D = 0$

LA MATRIZ [Z] DELA T:

$$Z_{11} = Z_1 + Z_3$$

$$Z_{12} = Z_{21} = Z_3$$

$$Z_{22} = Z_2 + Z_3$$

$$\Delta z = Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3$$

$$A = \frac{Z_1 + Z_3}{Z_3}$$

$$B = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_3}$$

$$C = \frac{1}{Z_3}$$

$$D = \frac{Z_2 + Z_3}{Z_3}$$

LA MATRIZ [a] DE LA CASCADA:

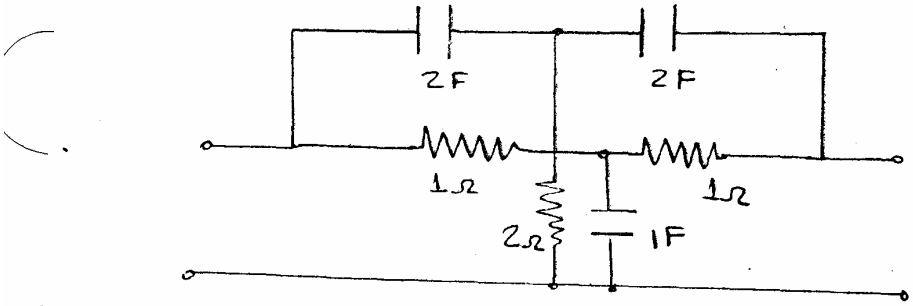
$$A = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_3}$$

$$B = \frac{Z_1 Z_3}{Z_3}$$

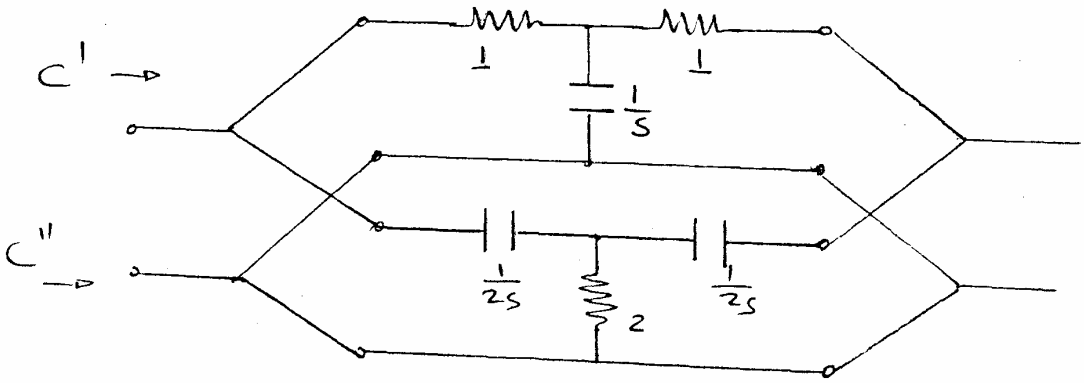
$$C = \frac{Z_2 + Z_3}{Z_3}$$

$$D = \frac{1}{Z_3}$$

29.- Hallar la matriz |Y| de la doble T de la figura.



SOLUCION: CONSIDERANDO COMO LA CONEXION PARALELO-PARALELO DE DOS CUADRIPOLOS:



$$[Z'] = \begin{bmatrix} 1+\frac{1}{S} & \frac{1}{S} \\ \frac{1}{S} & 1+\frac{1}{S} \end{bmatrix} \qquad [Y'] = \begin{bmatrix} \frac{S+1}{S+2} & \frac{-1}{S+2} \\ \frac{-1}{S+2} & \frac{S+1}{S+2} \end{bmatrix}$$

$$[Z''] = \begin{bmatrix} 2+\frac{1}{2S} & 2 \\ 2 & 2+\frac{1}{2S} \end{bmatrix} \qquad [Y''] = \begin{bmatrix} \frac{2S(4S+1)}{8S+1} & \frac{-8S^2}{8S+1} \\ \frac{-8S^2}{8S+1} & \frac{2S(4S+1)}{8S+1} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{8S^3+26S^2+13S+1}{8S^2+17S+2} & \frac{-(8S^3+16S^2+8S+1)}{8S^2+17S+2} \\ \frac{-(8S^3+16S^2+8S+1)}{8S^2+17S+2} & \frac{8S^3+26S^2+13S+1}{8S^2+17S+2} \end{bmatrix}$$

32.- Sabiendo que un cuadripolo tiene matriz

$$|Y| = \begin{vmatrix} \frac{2(s+1)}{s} & -2 \\ -2 & \frac{2(s+1)}{s} \end{vmatrix}$$

y que la carga en los terminales 2-2' es $Z_L = 2\Omega$ mientras que la diferencia de potencial en los terminales de entrada vale $V_1 = 5$ Volt. , calcular:

a) La impedancia de entrada

b) las corrientes I_1, I_2 y la tensión V_2

SOLUCION:

a) impedancia de entrada $= Z_I = \dfrac{I + Y_{22}Z_L}{Y_{11} + \Delta Y Z_L}$

$$\Delta Y = \left[\frac{2(S+I)}{S}\right]^2 - 4$$

$$\Delta Y = \frac{4\,(2S+I)}{S^2}$$

$$Z_1 = \frac{I + 2\,\frac{(S+I)}{S}.\,2}{2\,\frac{(S+I)}{S} + 4\,\frac{(2S+I)}{S^2}\,2}$$

$$Z_I = \frac{S\,(5S+4)}{2S^2 + I8S + 8}$$

b) $Z_I = V_I/I_I \qquad I_I = \dfrac{V_I}{Z_I} = \dfrac{5}{\dfrac{S(5S+4)}{2S^2 + I8S + 8}}$

$$Z_I = \frac{I0\,(S^2 + 9S + 4)}{S(\,5S + 4)}$$

$- I_2 = \dfrac{V_I}{AZ_L + B}$; Calculamos matriz $[a]$ a partir de matriz $[Y]$

$$[a] = \begin{bmatrix} \frac{S+I}{S} & \frac{I}{2} \\ \frac{2(2S+I)}{S^2} & \frac{S+I}{S} \end{bmatrix}$$

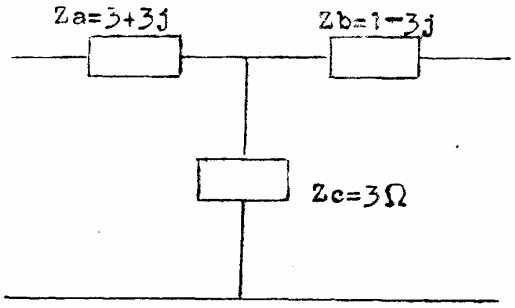
$$- I_2 = \frac{5}{\frac{(S + I)}{S} 2 + \frac{I}{2}}$$

$$- I_2 = \frac{IOS}{5S + 4}$$

$$V_2 = - I_2 Z_L = \frac{IOS}{5S + 4} \times 2$$

$$V_2 = \frac{20S}{5S + 4}$$

33.- Dado



Hallar las impedancias iterativas.

SOLUCION

La matríz [Z] de este cuadripolo será:

$$[Z] = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix} = \begin{bmatrix} 6 + 3j & 3 \\ 3 & 4 - 3j \end{bmatrix}$$

De esta matríz se obtienen las siguientes tablas:

$Z_{11} = 6 + 3j$; $Z_{12} = Z_{21} = 3$; $Z_{22} = 4 - 3j$; $\Delta Z = 24 - 6j$

Las impedancias iterativas las hallamos utilizando las fórmulas:

$$Z_{I_1} = \frac{Z_{11} - Z_{22} \pm \sqrt{(Z_{11} - Z_{22})^2 + 4\Delta Z}}{2}$$

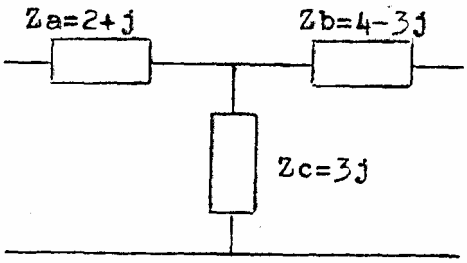
$$Z_{I_2} = \frac{Z_{22} - Z_{11} \pm \sqrt{(Z_{11} - Z_{22})^2 + 4\Delta Z}}{2}$$

$$Z_{I_1} = \frac{2 + 6j \pm \sqrt{64}}{2} \quad ; \quad Z_{I_2} = \frac{-2 - 6j \pm \sqrt{64}}{2}$$

Para que ambas impedancias tengan parte real positiva tomamos al radical signo positivo, con lo cual:

$$Z_{I_1} = 5 + 3j \quad ; \quad Z_{I_2} = 3 - 3j$$

34.- Determinar las impedancias, imágen y la constancia de transmisión de la figura



SOLUCION

La matríz Z del cuadripolo será:

$$[Z] = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix} = \begin{bmatrix} 2 + 4j & 3j \\ 3j & 4 \end{bmatrix}$$

$\Delta Z = 17 + 16j$; $Z_{11} = 2 + 4j$; $Z_{21} = Z_{12} = 3j$; $Z_{22} = 4$

$Z_{i_1} = \sqrt{\dfrac{Z_{11}}{Y_{11}}}$; donde $Z_{11} = 2 + 4j$ y $Y_{11} = \dfrac{Z_b + Z_c}{\Delta Z} = 0,125 - 0,117j$

$\therefore \quad Z_{i_1} = 3,0502 + 4,0984j$

$Z_{i_2} = \sqrt{\dfrac{Z_{22}}{Y_{22}}}$; donde $Z_{22} = 4$ y $Y_{22} = \dfrac{Z_a + Z_c}{\Delta Z} = 0,179\ 0,066j$

$\therefore \quad Z_{i_2} = 4,4988 - 0,8002j$

$\text{th}\,\gamma_i = \sqrt{\dfrac{\Delta Z}{Z_{11} Z_{22}}}$ $\qquad \text{th}\,\gamma_i = 1,1247 - 0,2j = p + qj$

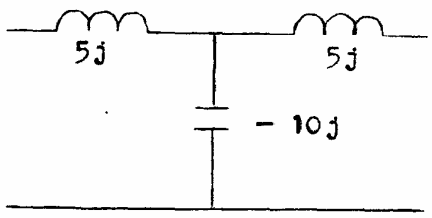
$\alpha_i = \dfrac{1}{4} \ln \dfrac{(1 + p)^2 + q^2}{(1 - p)^2 + q^2}$ $\qquad \alpha_i = 1,1015$ nepers

$\beta_i = \dfrac{1}{2} \text{arctg} \dfrac{2q}{1 - p^2 - q^2}$ $\qquad \beta_i = 26,3388°$

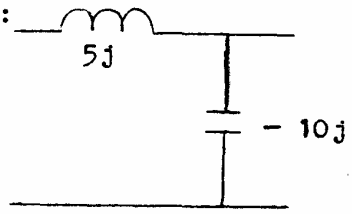
$\gamma_i = 1,1015 + 26,3388j$

35.- Calcular las impedancias imágen (o impedancia característica si es simétrica) y la constante de Propagación de los cuadripolos siguientes:

(a) :



(b):



SOLUCION

(a).- la matríz [Z] será:

$$[Z] = \begin{bmatrix} -5j & -10j \\ -10j & -5j \end{bmatrix}$$

$$\Delta Z = 75 \quad ; \quad Z_o = \sqrt{75} = 8,6603\ \Omega$$

$$\text{th}\,\gamma_i = \sqrt{\frac{\Delta Z}{Z_{11}Z_{22}}} = \sqrt{\frac{75}{-25}} = \sqrt{3}\ j \quad \therefore \quad p = 0 \quad y \quad q = 1,7321$$

$$\alpha_i = \frac{1}{4}\ln\frac{(1+p)^2 + q^2}{(1-p)^2 + q^2} = 0 \qquad \beta_i = \frac{1}{2}\,\text{arctg}\,\frac{2q}{1 - p^2 - q^2} = -30°$$

$$\therefore \qquad \gamma_i = -30\ j$$

(b).-

$$[Z] = \begin{bmatrix} -5j & -10j \\ -10j & -10j \end{bmatrix} \qquad \therefore \qquad \Delta Z = 50$$

$$Z_{i_1} = \sqrt{\frac{Z_{11}}{Y_{11}}} \qquad \text{pero} \qquad Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{-10j}{50} = -0,2\ j \quad \therefore \quad Z_{i_1} = 5\Omega$$

$$Z_{i_2} = \sqrt{\frac{Z_{22}}{Y_{22}}} \qquad \text{pero} \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{-5j}{50} = -0,1\ j \qquad Z_{i_2} = 10\Omega$$

$$\text{th}\,\gamma_i = \sqrt{\frac{\Delta Z}{Z_{11}Z_{22}}} = j \quad \therefore \quad p=0 \ ; \ q=1$$

$$\alpha_i = \frac{1}{4}\ln\frac{(1+p)^2 + q^2}{(1-p)^2 + q^2} = 0 \quad ; \quad \beta_i = \frac{1}{2}\,\text{arctg}\,\frac{2q}{1 - p^2 - q^2} = 45°$$

$$\gamma_i = 45\ j$$

38.- *Una T simétrica presenta a cierta frecuencia impedancia característica* $Z_0 = 5\angle 36°52'$ y constante de propagación $\gamma_0 = 1{,}7628 + j\frac{\pi}{2}$. Calcular los valores que corresponde a Z_1 y Z_2.

SOLUCION

$$Z_2 = \frac{Z_0}{Sh\ \gamma_0} \quad \therefore$$

$$\frac{Z_1}{2} = Z_0 Cth\ \gamma_0 - Z_2 \quad \therefore$$

$$Z_0 = 4 + 3j$$

$$e^{\gamma_0} = e^{1.7628}\, e^{j90°} = 5.8287j$$

$$e^{2\gamma_0} = e^{3.5286}\, e^{j180} = -33.9774$$

$$Sh\gamma_0 = \frac{e^{2\gamma_0} - 1}{2e^{\gamma_0}} = 3\angle 90°$$

$$cth\gamma_0 = \frac{e^{2\gamma_0} + 1}{e^{2\gamma_0} - 1} = 0.9428$$

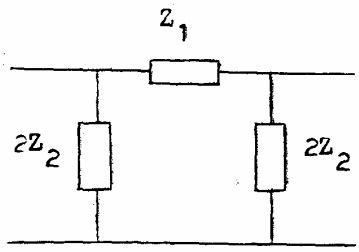
$$Z_2 = \frac{5\angle 37°}{3\angle 90°} = 1.666\angle -53°$$

$$\frac{Z_1}{2} = 5\angle 37° \cdot 0.9428 - 1.666\angle -53°$$

$$Z_2 = 1 - \frac{4}{3}j$$

$$Z_1 = 5{,}5425 + 8{,}3236\ j$$

39.- Una π simétrica presenta a cierta frecuencia impedancia característica $Z_0 = 5\angle 53°8'$ y constante de propagación $\gamma_0 = 1{,}7628 + j\frac{\pi}{2}$. Calcular los valores que corresponden a Z_1 y Z_2.



SOLUCION :

$$Z_0 = 5\angle 53° = Z_{i_1} = Z_{i_2}$$

$$Z_1 = \sqrt{Z_{i_1} Z_{i_2}} \cdot \operatorname{sh}\gamma_0$$

$$\operatorname{sh}\gamma_0 = 3\angle 90°$$

$$Z_1 = 5\angle 53° \cdot 3\angle 90° = 15\angle 143°$$

$$2Z_2 = \frac{Z_0 \operatorname{sh}\gamma_0}{\sqrt{\frac{Z_{i_2}}{Z_{i_1}}}\operatorname{ch}\gamma_i - 1}$$

$$Z_2 = 0.7476 + 2.3855j$$