Adaptive Control using Multiple Models Switching and Tuning

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Abstract— **The purpose of this paper is to marry the two concepts of multiple model adaptive control and safe adaptive control. In its simplest form, multiple model adaptive control involves a supervisory switching among one of a finite number of controllers as more is learnt about the plant, until one of the controllers is finally selected and remains unchanged. Safe adapative control is concerned with ensuring that when the controller is changed the closed-loop is never unstable. This paper introduces a receding horizon multiple model, switching and tuning control scheme based on an on-line redesign of the controller.. This control scheme has a natural two-stage adaptive control algorithm: identification of the closest model and design of the control law. The computational complexity aspects of this approach to adaptive control are discussed briefly. A nonlinear system is used to illustrate the ideas.**

I. Introduction

Adaptive control systems has been investigated for over four decades. Since the begining, for the sake of mathematical tractability, adaptive control theorists confined their attention to time invariant systems with unknown parameters or slow drifts in the parameters [1], [12]. The accepted philosophy was that if an adaptive system was fast and accurate when the plant parameters were constant but unknown, they would also prove satisfactory when parameters varied with time, provided the latter occurred on a rather slower time-scale. Based on these general principles, adaptive control was extensively studied and numerous robust adaptive control algorithms were derived [7].

In this framework, the problem of selecting the best controller according to the performance index J can be addressed, along a dual control approach, by introducing a state variable representing the unknown parameter vector, and solving the resulting optimal control problem on the augmented state–space representation of the process. The optimal controller incorporates a self–adjusting mechanism, in that it selects a control input that compromises the control objective versus estimation needs (dual action, see e.g. [6]). However, such an optimal dual control approach is generally difficult to implement because it is computationally excessive.. Besides, extensive computer simulations have revealed that when there are large errors in the initial parameters estimates, the system exhibits a poor performance during the transient phase, exhibiting oscillatory behaviour with large amplitude.

A computationally feasible, though sub-optimal, approach to the design of self-adjusting controllers is the so called switching control design method originally introduced in [9] and further developed in e.g. [5], [11], [13]. The switching control scheme consists of an inner loop, where a candidate controller is connected in closed-loop with the process. There is also an outer loop where a supervisor decides which controller to select and when to switch to a different one, based on the input-output data.

The switching times are chosen so as to avoid switching that is too fast with respect to the system settling time, thus causing instability. As for the controller selection, it is based on an '*estimator-based*' procedure [11] typically. Specifically, at any switching time, a performance signal is computed for each admissible model parameter. The supervisor then selects the candidate controller associated with the model that minimizes the performance signal. Implementation and analysis of the switching control scheme is simplified by considering a finite number of candidate controllers. This set is called a "finite controller cover" [2].

In standard switching control schemes, the compromise between robustness and performance is made off-line when the controller cover is designed. If the controller cover consists of a large number of controllers, each one stabilizing a wide set of models, then stability is generally rapidly achieved, even before a large amount of information has been accrued, but in the long run the resulting performance is low typically. In contrast, if the controller cover consists of a large number of controllers, each one tailored to a narrow set of models, a high performance control system is potentially achieved, but poor performance will possibly occur until there is sufficient data to obtain an accurate estimate of the process model.

In this paper, a new multiple models, switching and tuning control strategy, based on a receding horizon technique is proposed. The proposed control algorithm exploits the advantage of superstable systems to derive a linear optimization problem that designs the controller every sample. This problem is convex in the controller's parameter and allows to include constraints on the system states.

The paper is structured as follow: the class of superstable system is introduced and some properties of this class of system are analysed in Section II. The main property of this class of systems is that they admit non-asymptotic estimates for the outputs for arbitrary initial conditions. An optimization design procedure based on this bound is proposed in Section III. One key advantage is that the proposed performance index is quasi convex with respect to the controller coefficients. Robustness issues in the optimization problem are also considered in this section. Finally, the objective function is analysed from the multiobjective point of view. In Section IV the multiple models, switching and tuning control approach is suggested by modifying the objective function and the constraints employed by the predictive feedback controller. Section V shows the results obtained from the application of the proposed algorithm to a nonlinear continuous stirred tank reactor. Finally, the conclusions are presented in Section VI.

II. Superstable Systems

Given that the local approximation to the process model is given by its state-space discrete model

$$x(k+1) = Ax(k) + Bw(k) \quad x(0) = x^0, \tag{1}$$

where $x(k) \in R^n$, $w(k) \in R^m$, $A \in R^{n\times n}$ and $B \in R^{n\times m}$. The ∞ and 1 norms for the vectors $x \in R^n$ and matrices $A \in R^{n\times m}$ are given by

$$\|x(k)\|_\infty = \max_{i\in[1,n]} |x(k)|, \|A\|_1 = \max_{i\in[1,m]} \sum_{j=1}^{n} |a_{ij}|.$$

Definition 1: The system (1) is superstable if $\|A\|_1 < 1$.

The superstability of the system implies its stability

$$\rho(A) \leq \|A\|_1 < 1,$$

where $\rho(A) = \max_{i\in[1,n]} |\lambda_i(A)|$ is the spectral radius of A.

Discrete–time superstable systems enjoy numerous important properties [14]. The main one is that they admit simple non–asymptotic estimates for arbitrary initials conditions. For instance, there exists a constant η such that if initial conditions are less than or equal to μ, and inputs are bounded in l_∞ norm, then the outputs do not exceed η for all time steps.

Lemma 1: Assume a closed-loop system, described by (1) with the initial conditions $\|x(0)\|_\infty \leq \mu$ and bounded disrtubances $\|w(k)\|_\infty \leq 1$. Suppose that the system is superstable and the equalized performance of the system [3] is given by

$$\eta \;\; = \;\; \|B\|_1 / (1 - \|A\|_1) \tag{2}$$

Then, the closed-loop system responses are bounded by

$$\|x(k)\|_\infty \;\; \leq \;\; \eta + \|A\|_1^k max\{0, \mu - \eta\} \qquad \forall k \geq 0 \tag{3}$$

In particular, if $\|B\|_1 = 0$ then

$$\|x(k)\|_\infty \;\; \leq \;\; \|A\|_1^k \mu \tag{4}$$

Proof: We have $\|x(k+1)\|_\infty \leq \|A\|_1 \|x(k)\|_\infty + \|B\|_1 \|w(k)\|_\infty$, by induction this implies $\|x(k)\|_\infty \leq \eta + \|A\|_1^k (\|x(0)\|_\infty - \eta)$ and hence (3). ■

For superstable systems, the output can be estimated for all time steps, not only its asymptotic values. Besides, for any $c > 1$ a k_0 can be found such that $|x(k)| \leq c\eta$ $\forall k > k_0$. In contrast, for stable systems only asymptotic estimates of the output can be guaranteed, while the effect of non–zero initial conditions may be very large.

An additional additional advantage of superstable systems is their robustness with respect to outliers in inputs. Suppose that the disturbance $w(k)$ is bounded for all samples except one, $|w(k)| \leq 1 \quad \forall k \neq N$ and $|w(N)| = \sigma > 1$. Then, if the system is superstable and $\|x(0)\|_\infty \leq \mu$, the effect of the outlier is attenuated after enough steps (see Theorem 2 [3])

$$|x(k+N)| \leq 2\mu \qquad \forall k \geq 1 + \frac{\ln\mu - \ln\sigma\,\|B\|_1}{\ln\|A\|_1}. \tag{5}$$

The inputs and outputs can be written as $w(k) = w_1(k) + w_2(k)$, $x(k) = x_1(k) + x_2(k)$ where $w_2(k) = 0$ $\forall k \neq N$, $w_2(N) = \sigma$, $x_2(k) = 0$ $\forall k \neq N$. Then $\|x_1(k)\|_\infty \leq \eta$ and $\|x_2(k+N)\|_\infty \leq \|A\|_1^k \mu \leq \eta$ $\forall k \geq 0$.

These results can be easily extended to time–varying and nonlinear systems by analyzing the behavior of the so called frozen systems. The superstability of frozen *LTI* systems implies superstability of *LTV* system

$$x(k+1) = A_k x(k) + B_k w(k)$$

and guarantees equalized performance η of the *LTV* system. It is well known that this does not hold for the more general class of stable systems. Besides, the convergence $x(k) \to 0$ for any *LTV* system, with no external disturbances, has been shown [15].

A. Properties of equalized performance η

The equalized performance η has been employed as a closed–loop system performance index for controller design [3], [14]. Assuming that the closed–loop system $\phi(z) = (zI - A)^{-1}B$ is superstable, it is easy to show that:

1) $\eta(\phi)$ is an upper bound for the H_∞ norm $\|\phi\|_\infty = \max_\omega \frac{|B(e^{j\omega})|}{|1+A(e^{j\omega})|} \leq \frac{\max_\omega |B(e^{j\omega})|}{\min_\omega |1+A(e^{j\omega})|} \leq \eta(\phi)$. The sharpness of this estimate depends on the sign of coefficients, for example when all coefficients are positive $\eta(\phi)$ is a very conservative.
2) $\eta(\phi)$ is an upper bound for the l_1 norm: Indeed, the function $\phi(z)$ is analytic in the unit disk, therefore $\|\phi\|_1 = \sum_{i=0}^{\infty} |\phi_i|$ and
$$\left|\phi(q^{-1})w(k)\right| \leq \|\phi\|_1 |w(k)| = \eta(\phi) |w(k)|,$$
thus, $\left\|\phi(q^{-1})\right\|_1 \leq \eta(\phi)$.

In the next section a quasi-convex optimization program based on the equalized performance η, for design of controllers, will be proposed.

III. The Controller Design

Consider a discrete-time system described by

$$\begin{aligned} x(k+1) = &\; Ax(k) + Bu(k) + D_1 w(k), \\ y(k) = &\; Cx(k) + D_2 w(k), \end{aligned} \tag{6}$$

there is a variety of problem formulations for this system. In this work the static output feedback stabilization problem will be consider ($u(k) = \mathscr{K} y(k)$). Depending on the system output, the transfer can be of the form

$$\begin{aligned} x(k) &= (zI - A - B\mathscr{K}C)^{-1}(D_1 + B\mathscr{K}D)\, w(k) \\ u(k) &= KC(zI - A - B\mathscr{K}C)^{-1}(D_1 + B\mathscr{K}D)\, w(k) \end{aligned}$$

then, it is required that the closed-loop system matrix $A + B\mathscr{K}C$ to be superstable and minimize the desired performance index, or a mixture of them,

$$\eta_x(\mathscr{K}) = \frac{\|D_1 + B\mathscr{K}D_2\|_1}{1 - \|A + B\mathscr{K}C\|_1}, \tag{7}$$

$$\eta_u(\mathscr{K}) = \frac{\|\mathscr{K}CD_1 + \mathscr{K}CB\mathscr{K}D_2\|_1}{1 - \|A + B\mathscr{K}C\|_1}. \tag{8}$$

The main feature of the above optimization problem is its reductibility to a one parameter family of linear or quadratic programming problems. In the following the $\eta_x(\mathscr{K})$ will be employed.

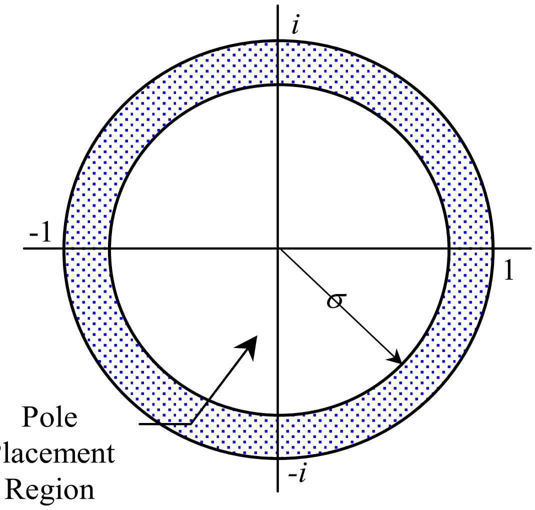

Fig. 1. Effect of σ on the pole placement region

The minimization of (7) is equivalent to a parametric linear programming problem

$$\begin{aligned} \min_{0 \leq \sigma < 1} \min_{\mathscr{K}} \tfrac{1}{1-\sigma} \|D_1 + B\mathscr{K}D_2\|_1 \\ \|A + B\mathscr{K}C\|_1 \leq \sigma \end{aligned} \tag{9}$$

Proof: First, minimization of a fraction of the form $\alpha(x)/(1-\beta(x))$ subject to $0 \leq \beta(x) < 1$, over x, is obviously equivalent to minimization of $\hat{\alpha}(x)/(1-\mu)$ subject to $0 \leq \mu < 1$, where $\hat{\alpha}(x)$ is a solution of $\min \alpha(x)/(1-\beta(x))$ s.t. $0 \leq \beta(x) < 1$. Second, note the problem (9) is linear programing for a fix σ. Indeed, the coefficients of $D_1 + B\mathscr{K}D_2$ and $A + B\mathscr{K}C$ are affine functions of the parameters of $\mathscr{K}$. In the deterministic case ($D_2 = 0$) the optimization problem (9) is equivalent to

$$\min_{\mathscr{K}} \|A + B\mathscr{K}C\|_1. \tag{10}$$

■

The parameter σ defines the boundaries of the region where the eigenvalues of the closed-loop system can be placed (see Figure 1). This means that the eigenvalues will be located in a the circle of radius σ. When $\sigma = 1$, the eigenvalues can be placed inside of the unitary circle. This parameter can be employed to improve the robustness of the optimization procedure against the uncertainties.

The performance index (7) and the *LP*–like design problem (9) have been introduced in the works [3],

[14]. It guarantees the optimal rejection of bounded disturbances for non–zero initial conditions and provides an opportunity for direct optimization in the space of controller coefficients, in contrast with all other techniques (H_∞, l_1,etc.) where the solution is sought in the Youla parameter space. These problems can be solved on–line, based on a receding horizon technique, such that the information available in the system output is included in the design of the control law. Due to this fact, the controller will act as regulator, driving the system from its current state, $x(k)$, to the next one, $x(k+1)$. In this way, the constraints can be rewritten as in (3inf(k)rh}) leading to linear constraints of the type

$$\|A + B\mathscr{K}C\|_1 \leq \left(\frac{\bar{x}}{\mu}\right)^{\frac{1}{k}}, \tag{11}$$

where $\bar{x}$ is the constraint value and .

A. Robust Design

Assuming that a set $\mathscr{W}$, of M plants

$$\begin{aligned} x_l(k+1) = &\quad A_l x(k) + B_l u(k) \qquad l = 1, \ldots, M, \\ y_l(k) = &\quad C_l x(k), \end{aligned}$$

is able to represent the behaviour of system in a given operating region. The problem is to find a controller K which stabilizes all the plants simultaneously. This simultaneous stabilization problem is known to be *NP*-hard for $m > 2$ [4], and there are no effective algorithms to solve it. The problem can be solved in this framework by considering a set of stability inequalities for each model

$$\|A_l + B_l\mathscr{K}C_l\|_1 < \sigma \quad l = 1, \ldots, M. \tag{12}$$

This is a system of linear inequalities with respect to coefficients of the gain $\mathscr{K}$. Hence, the problem of simultaneous super stabilization has a solution if and only if the system of linear inequalities (12) is non-empty. If the set of solutions is non–empty, a η–optimization problem can be solved. For instance,the optimal control design problem can be solved through the following linear programming problem

$$\begin{aligned} \min_K \textstyle\sum_{l=1}^{M} \lambda_l \|B_l\mathscr{K}\|_1 \\ \|A_l + B_l\mathscr{K}C_l\|_1 \leq \sigma_l, \qquad l = 1, \ldots, M \end{aligned} \tag{13}$$

such that $\sum_{i=1}^{M} \lambda_i = 1$ and $\lambda_i \geq 0 \quad \forall i$.

This type of problem can also arise for the optimal control design of a single plant when several objectives are considered simultaneously during the controller design. For example minimize the error while the control energy is bounded or minimize the sensitivity function of the system $S(z)$, while the complementary sensitivity function $T(z)$ is bounded or minimized.

IV. MULTIPLE MODELS, SWITCHING AND TUNING

In adaptive control, the system is assumed to be linear with unknown parameters that have small variations.. However, simulation studies and industrial applications [8] have also revealed that the transient error of these adaptive systems are significantly larger than in the linear case, due to large and abrupt variations of the parameters, so that the multiple models, switching and tuning approach is relevant.

This control strategy is based on the idea of describing the dynamics of the system, using different models for different operating regimes. It also requires a suitable strategy for finding the model that is closest (in some sense) to the current plant dynamics. This model is used to construct the control law for the current sample that achieve the desired control objective. The suggested approach is to consider the control assembled in two stages: first, the closest models to the current dynamic model is identified, followed by a control design based on this model. The identification is defined via a finite optimization problem, while the design is defined via an infinite horizon. The objective of this work is the control design stage, therefore in the following the switching variables $\mathscr{S}(k) = \left[\begin{array}{ccc} S_1(k) & \cdots & S_M(k) \end{array}\right]$ are assumed be given. The structure of the resulting *MMST* controller is showed in Figure 2. The switching variables $\mathscr{S}(k)$, which are external inputs of the optimiser, are calculated independently of control law every sample of the control design.

To introduce the switching into the control design problem (13), the objective function and design constraints are modified by replacing the weight λ_l with the switching variables $S_l(k)$

$$\textstyle\sum_{l=1}^{M} S_l(k)\, \|B_l\mathscr{K}\|_1,$$

and including them in the design constraints

$$g_z(S_l(k), x(k), \mathscr{K}) \leq 0 \quad z = x, u, y$$

In this way, the control law is designed, only employing the closest model to the current plant dynamic, which is used to measure the performance and evaluate the constraints, while the superstability of the set $\mathscr{W}$ is guaranteed. Thus, a better closed-loop performance than a robust approach is obtained because a less conservative model is used to design the control law. However, note that the stability of the nonlinear system is also guaranteed because the control law satisfies the super stability condition for all models of $\mathscr{W}$ simultaneously.

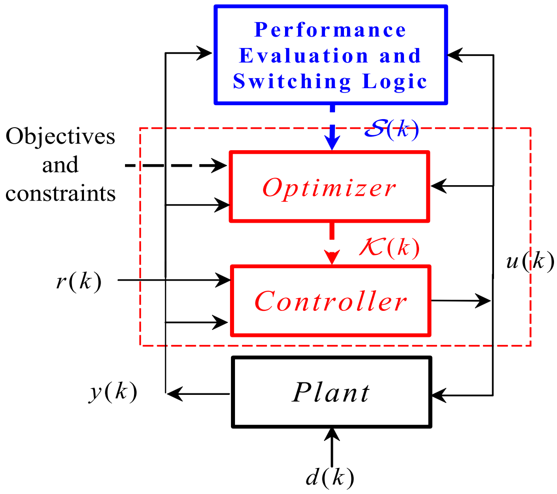

Fig. 2. Controller structure

Thus, the resulting control law will stabilise the system in the whole-operating region and will obtain the best performance for the current operating point.

V. Simulations and Results

Consider the problem of controlling a continuous stirred tank reactor (*CSTR*) in which an irreversible exothermic reaction $A \longrightarrow B$ occurs in a constant volume reactor. This nonlinear system was originally used by Morningred, Paden, Seborg and Mellichamp [10] for testing discrete control algorithms. The system is modelled by the following equations

$$
\begin{aligned}
\frac{dCa}{dt} &= \frac{q}{V}\left[Ca_0 - Ca\right] - k_0 Ca \exp\left(\frac{-E}{RT}\right), \\
\frac{dT}{dt} &= \frac{q}{V}\left[T_0 - T\right] - \frac{k_0 \Delta H}{\rho c_p} Ca \exp\left(\frac{-E}{RT}\right) \\
&\quad + \frac{\rho_C c_{p_C}}{\rho c_p V} q_C \left[1 - \exp\left(\frac{-hA}{q_C \rho_C c_{p_C}}\right)\right]\left[T_{CO} - T\right],
\end{aligned}
$$

the nominal values of the variables and parameters can be found in Morningred's paper [10]. The objective is controlling the output concentration, Ca, using the coolant flow rate, q_C, as the manipulated variable, and the inlet coolant temperature, T_{CO}, and the feed concentration, Ca_O, are the disturbances. The output concentration has a measured time delay of $T_d = 0.5$ min.

The nonlinear nature of the system is shown in figure 3, for the open-loop response to changes in the manipulated variable. It shows the dynamic responses to the following sequence of changes in the manipulated variable q_C: $+10\ lt\,min^{-1}, -10\ lt\,min^{-1}, -10\ lt\,min^{-1}$. Besides, the *CSTR* becomes uncontrollable when q_C is

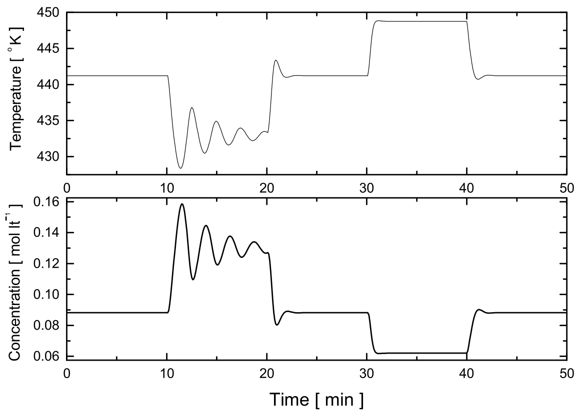

Fig. 3. Open-loop responses of the CSTR

TABLE I

Vertices of the polytopic model

Change	Model Obtained
Model 1 $q_C = 110, \Delta q_C = 10$	$G_{p1}(z) = \frac{0.1859\ 10^{-3} z^{-5}}{z^2 - 1.9835z + 0.9406}$
Model 2 $q_C = 110, \Delta q_C = -10$	$G_{p2}(z) = \frac{0.2156\ 10^{-3} z^{-5}}{z^2 - 1.7272z + 0.7793}$
Model 3 $q_C = 110, \Delta q_C = -10$	$G_{p3}(z) = \frac{0.1153\ 10^{-3} z^{-5}}{z^2 - 1.7104z + 0.7547}$
Model 4 $q_C = 110, \Delta q_C = 10$	$G_{p4}(z) = \frac{0.8305\ 10^{-4} z^{-5}}{z^2 - 1.7922z + 0.8241}$

bigger than 113 $lt\,min^{-1}$. Four linear models were determined from the composition of responses shown in figure 3, using a subspace identification algorithm [16]. Notice that these changes imply three different operating points corresponding to the following stationary manipulated flow-rates: 100 $lt\,min^{-1}$, 110 $lt\,min^{-1}$, and 90 $lt\,min^{-1}$. Table I shows the four process transfer functions that define the polytopic model associated to the nonlinear behavior in the operating region being considered. Like in Morningred's work, the sampling time period was fixed in 0.1 *min*, which gives about four sampled-data points in the dominant time constant when the reactor is operating in the high concentration region.

The controller must be able to follow the reference and keep the system's controllability over the whole operational region. Hence, assuming a hard constraint is used on the coolant flow rate at 110 $ltmin^{-1}$, an additional restriction for the more sensitive model (Model 1) must be considered for the deviation variable $u(k)$:

$$u_1(k+1) \quad \leq \quad 10 \qquad \forall i \geq 0 \qquad (14)$$

In addition, a settling time of 5 *min* are demanded (the error must be lower than 10^{-3} $mol\, lt^{-1}$), thus the following constraints are included

$$|e_l(k)| \leq 10^{-3} \quad \forall k \geq N_o + 50. \qquad (15)$$

where N_o is the time instant when the setpoint change happens. This assumes that the nominal absolute value for the manipulated variable is around 100 $lt\ min^{-1}$ and that the operation is kept inside the polytope whose vertices are defined by the linear models. The constraints (14) and (15) are then included in the optimization problem (13).

A traditional *MMST* controller was developed using the models showed in table I, therefore four linear controllers were obtained. Each controller was developed using robust tuning methods employing two models simultaneously: the model corresponding to the operating region and the previous in order to guarantee the stability of the system.

The switching criterion employed by both adaptive controller is

$$S_i(k) = \alpha e_i(k) + \beta \sum_{j=N_0}^{k} \rho^{k-j} e_i^2(j) \qquad (16)$$

with parameters given by

$$\alpha = 0.7,\ \beta = 0.4,\ \rho = 0.2. \qquad (17)$$

The indexes $S_i(k) i = 1,2,3,4$ are initialise each time that a setpoint happens.

A robust *MPC* based on the worst–case minimization was developed to compare the closed–loop responses. The predictor was built using the model 2 assuming that the parameters are corrupted by some error ε_i $i = 0,1,\ldots,p$ due to modelling error, i.e. $\alpha_i = \alpha_i^2 + \varepsilon_i$ $i = 0,1,\ldots,p$, such that $\alpha_i \subset \left[\alpha_i^2 - \varepsilon_i, \alpha_i^2 + \varepsilon_i\right]$. The uncertainty bound ε_i was calculated form the vertex of the polytopic models

$$\varepsilon_i = \max_{j=1,3,4} \left(\left| \alpha_i^j - \alpha_i^2 \right| \right).$$

The noise for the remaining parameters of the model have been computed in a similar way. Here it is assumed that the parameters' noise is an independently identical uniform distributed variable. The remaining tuning parameters (the optimization horizon N, the control horizon N_U, control weight R, and the error weight Q) were setting to

$$N = 200, N_U = 5, R = 5\,10^{-3} I, Q = I.$$

The optimization problem was solved, at each step, using a min–max algorithm.

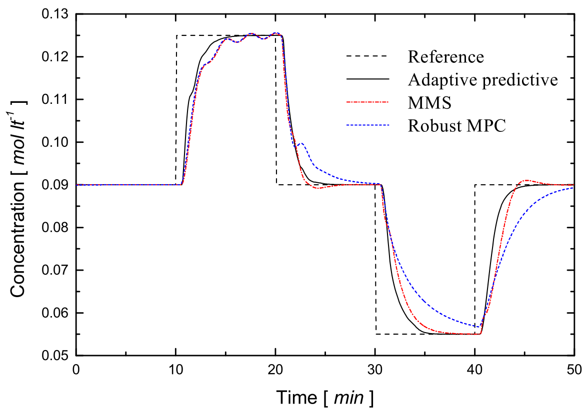

Fig. 4. Closed-loop responses to setpoint

The simulation tests are similar to Morningred's work [10] and consists of a sequence of step changes in the reference value. The set point was changed in intervals of 10 *min*. from 0.09 $mol\ lt^{-1}$ to 0.125, returns to 0.09, then steps to 0.055 and finally returns to 0.09 $mol\ lt^{-1}$. Figure 4 shows the closed–loop simulation results sustained by the controller described in the previous paragraph. As can be seen in this figure the adaptive controller provides a response without overshoot and faster settling time for all the operating region. The robust *MPC* always provides a slower response without overshoot, and in some cases it fails to achieve the setpoint value in the time of setpoint changes. Finally, the switching controller always achieves the setpoint but with an overshot and a bigger settling time than the predictive feedback controller. The reason for this result is that the adaptive controller: (a) employes more to compute the control actions it is able to adapt it to each operating region, by changing the model, and (b) include the feedback information available at each sample in the design the control law.

The good performance of the adaptive controller proposed in this work is due to the combination of a switching scheme with the on-line design of the controller. In this way, the adaptive controller is able to identify the local model and to optimize the closed-loop response

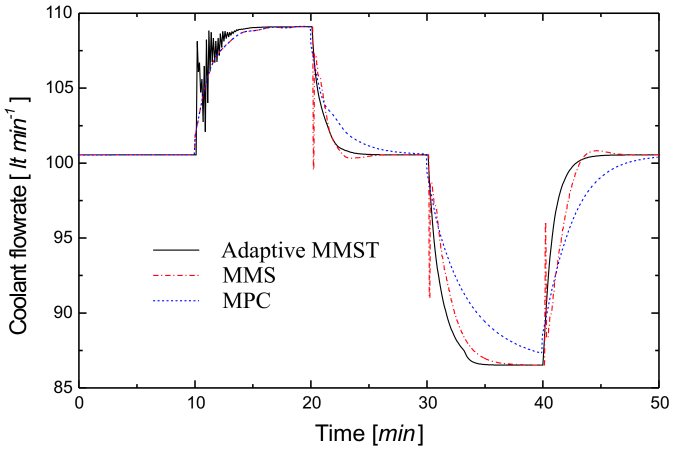

Fig. 5. Manipulated movements

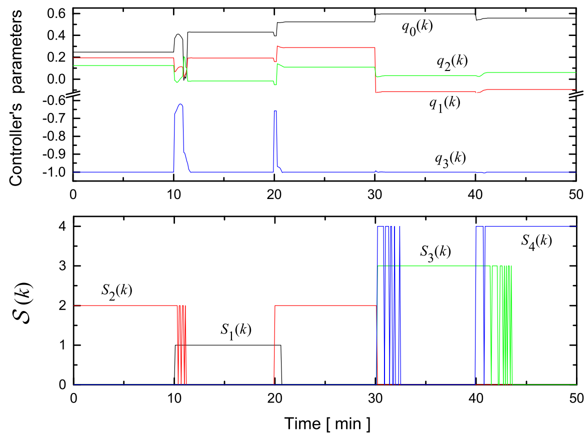

Fig. 6. Controller gains sequences (upper figure) and the switching-indices sequences(lower figure)

whilst at the same time satisfying the constraints by modifying the controllers gains (Figure 6-upper figure). The parameters of the adaptive controller are modified with changes in the reactors operating region. They revealed an initial transient behavior, after each change, before achieving their steady state values. The major changes happen during the transitions from and to model 1 because it is the different behavior (see Figure 3). This fact can be appreciated in the behavior of the switching variables S(k)(Figure 6 -lower figure), which show jitter during the first, third and fourth reference changes. These transitions correspond to switches between models 21; 23 and 34; which have similar dynamics and only differ in the gain.

The switching controller has a better performance than the *MPC* and a poorer one than the adaptive controller. The adaptive nature of the controller leads to a better global performance than a worst–case design whilst guaranteeing the robust stability of the system. However, the response shows an overshoot in one operating region (first model) and a bigger settling time in other (third model). The reason for this is that the controller has fixed its parameters for each operating region. The *MPC* controller has a poorer performance than the predictive feedback and the fixed–structure controller because the worst–case scenario is considered all the time. The conservative nature of the min–max, added to the difference in the models, results in a slower response with a consequential loss of performance.

VI. CONCLUSIONS

A simple framework for the design of robust adaptive controllers with multiple models was presented. The approach was to relate the control law performance to the prediction of performance. The resulting controller identifies, at each sample, the closest linear model to the actual operational point of the controlled system, and reconfigures the control law such that it ensures robust stability of the closed-loop system. The reconfiguration of the controller is carried out by switching the function used to measure the closed-loop performance and the constraints.

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