Mathematical Model of a Levitation System Based on Eddy Currents

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***Abstract*— This paper contains a synthesis to model an electromagnetic levitation system using a direct evaluation method of electromagnetic and electromotive forces, taking the Maxwell equations that relate electric and magnetic fields. The aim of this work is to contribute to researches focussed on developing and optimising mechanisms to provide solutions for applications where mechanical friction must be reduced or eliminated. A prototype is also presented; it will support experimental requirements for curses on automatic control and physical experimentation principles as well as models related with electrostatics and magnetodynamics.**

***Index terms*— Electromagnetic levitation, Eddy currents, Maxwell equations, state space.**

I. Introduction

WHEN a conductor material is reached by a time varying magnetic field $\vec{B}$, an electric field $\vec{E}$ is induced, according to Faraday's Law $\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$, which generates an electric current given by Ampere's Law $\vec{J} = \sigma \vec{E}$. In Fact, this is the basic principle on which all Eddy-current devices are based.

When the current $\vec{J}$ flows through a closed path, the following phenomena are presented [14]: First, they produce heat because of ohmic losses of the material (inductive heating); second, a magnetic field is generated, which opposes to the incident field; and third, repulsive forces are created by the interaction of the incident and induced magnetic fields, $f = \vec{J} \, x \, \vec{B}$.

Neglecting the inductive heating and considering repulsion forces and magnetic fields, a dynamic model for an aluminum disc levitation system is deduced [9], as shown in Fig. 1.

A *non-linear mathematical description* is obtained using a direct evaluation method of mechanic and electromagnetic forces [10] and rules that relate electric and magnetic fields (synthesis of Maxwell equations) [8][11].

Energy loss due to friction is one of the problems that appear when objects are moved over a surface. Friction produces, among other problems, noise, dust and wearing down of surfaces.

Hence, research and technological developments are important in order to reduce or eliminate friction.

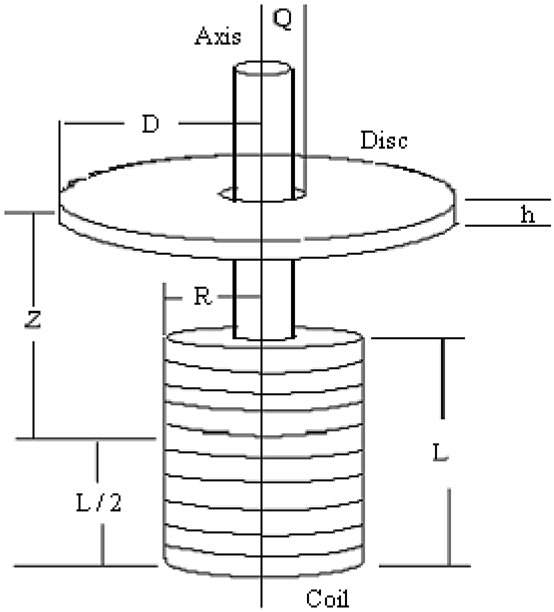

Fig. 1. Scheme for the levitation system

Fig. 1 shows an electromagnetic levitation system. It is designed to research in clean environments, and with a minimum of mechanical contact [1][2].

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From an academic point of view, levitation systems satisfy experimental requirements for teaching automatic control, modern control [3][4][5][12] and electromagnetic principles [14][20], among others.

This work also intends to motivate researches into technological solutions for mechanical contact free applications.

II. Mathematical Model

First, we introduce following assumptions in order to derive a nominal model [13]:

- Uniform magnetic induction through the air gap.
- Absence of magnetic dispersion flows.
- Losses caused by Eddy currents and hysteresis are ignored.
- Iron core concentrates magnetic flow and limits the degrees of freedom of the disc.
- Relative permeability of the core, μ_r, is considered constant and linear.

In order to lift up the disc to an equilibrium position Z_o, electromagnetic force F_{em} is applied to the disc and should be equal to gravitational force F_g acting on the disc but in the opposite direction.

$$F_{em} = F_g \quad (1)$$

Gravitational force is calculated as the product of the mass of the disk m multiplied by gravitational acceleration g:

$$F_g = m \cdot g \quad (2)$$

According to Lorentz's force, electromagnetic force acting on the disc is:

$$\vec{F}_{em} = \frac{2\left(D^3 - Q^3\right)}{3\left(D^2 - Q^2\right)}\left(\vec{i}_{disc} \times \vec{B}_{coil}\right) \quad (3)$$

Alternating current through the coil I_{coil} generates the external electromagnetic field $\vec{B}_{coil}$. Using Biot-Savart's law, the electromagnetic field at a point outside the core in direction Z caused by a coil with N turns, and radius R and a current I_{coil} is [8]:

$$\vec{B}_{coil} = \frac{\mu \cdot N \cdot R^2 \cdot I_{coil}}{2\left(R^2 + Z^2\right)^{3/2}} \quad (4)$$

According to Lenz's law, the current induced in the disc generates a magnetic field opposite to the field created by the induced current as shown in Fig. 2.

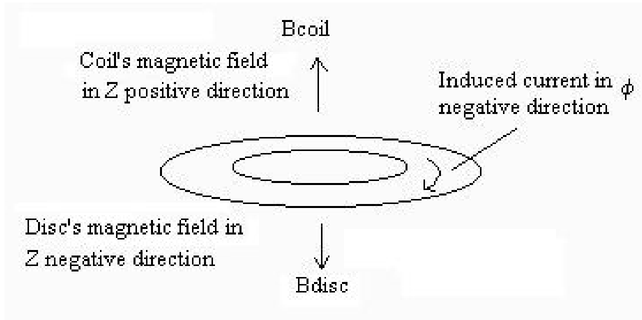

Fig. 2. Current and fields

With (3) and considering directions of the electromagnetic field caused by the coil and the induced current, it is deduced that the force that lifts the disk results from the interaction of the current through the disc and the radial component of the inducer's electromagnetic field so the electromagnetic force is:

$$\vec{F}_{em} = -\frac{2 \cdot \left(D^3 - Q^3\right) \cdot I_{disc}}{3\left(D^2 - Q^2\right)} \cdot \frac{\mu \cdot N \cdot R \cdot I_{coil} \cdot Z}{2\left(R^2 + Z^2\right)^{3/2}} \quad (5)$$

Using Newton's second law, the dynamic equation of movements is:

$$\frac{d^2}{dt^2} Z = -\frac{2\left(D^3 - Q^3\right)}{3\left(D^2 - Q^2\right)m} \cdot I_{disc} \cdot \frac{\mu \cdot NRI_{coil} Z}{2\left(R^2 + Z^2\right)^{3/2}} - g \quad (6)$$

The aluminium disc can be considered as a circuit with inductance L_{disc} and resistance R_{disc}, when an induced current I_{disc} flowing through it, due to a voltage originated by the incident field $\vec{B}_{coil}$. According to Faraday's law, the induced voltage is:

$$V_{ind} = -\frac{A_{core} \cdot \mu \cdot NR^2}{2}\left[\frac{V_0 \sin \varpi .t - R_{Coil} I_{Coil}}{L_{Coil}(R^2 + Z^2)^{3/2}} - \frac{3ZI_{Coil}}{(R^2 + Z^2)^{5/2}} \frac{d}{dt} Z\right] \quad (7)$$

The relation between the current through the coil and the voltage source $V_o \sin \omega .t$ is:

$$\frac{d}{dt} I_{Coil} = \frac{V_0 \sin \varpi . t - R_{Coil} I_{Coil}}{L_{Coil}} \quad (8)$$

$$R_{disc} = \frac{2\pi\rho}{h \cdot Ln\left(D/Q \right)} \quad (9)$$

$$L_{disc} = \frac{\mu \cdot \pi \cdot Q}{D + Q} \quad (10)$$

An expression for the current I_{Disc} is:

$$\frac{d}{dt} I_{disc} = -\frac{R_{disc}}{L_{disc}} I_{disc} + \left[-\frac{A_{core} \mu \cdot NR^2}{2L_{Disc}} \right] \left[\frac{V_0 \sin \omega t - R_{coil} I_{coil}}{L_{coil} \left(R^2 + Z^2 \right)^{3/2}} - \frac{3ZI_{coil}}{\left(R^2 + Z^2 \right)^{5/2}} \frac{d}{dt} Z \right] \quad (11)$$

Using the previous equations, a space-state representation is generated, taking as state variables the current through the coil, the current through the disc, height of the disc and its relative vertical velocity.

$$x = \left[I_{coil}, I_{disc}, Z, v \right]^T; \; v = \frac{d}{dt} Z \quad (12)$$

$$\dot{x}_1 = \frac{V_0 \sin \omega t - R_{coil} x_1}{L_{coil}} \quad (13)$$

$$\dot{x}_2 = -\frac{R_{disc}}{L_{disc}} x_2 + \left[-\frac{NR(D+Q)}{2} \right] \left[\frac{V_0 \sin \omega t - R_{coil} x_1}{L_{coil} \left(R^2 + x_3^2 \right)^{3/2}} - \frac{3x_3 x_1 x_4}{\left(R^2 + x_3^2 \right)^{5/2}} \right] \quad (14)$$

$$\dot{x}_3 = x_4 \quad (15)$$

$$\dot{x}_4 = -\frac{2(D^3 - Q^3)}{3(D^2 - Q^2)m} x_2 \frac{\mu . NRx_1 x_3}{2(R^2 + x_3^2)^{3/2}} - g \quad (16)$$

III. Simulation and Analysis

Table 1 presents the parameters and values for simulation. The non-linear space state model used for simulations is shown in (17), (18), (19) and (20).

TABLE I
MODEL PARAMETERS

Symbol	Quantity	Value (SI)
μ	permeability	$4\pi \times 10^{-7}$ Wb/(A·m)
D	External radius disc	0.062125 m
Q	Internal radius disc	0.0131 m
h	Thickness disc	0.0016 m
N	Number of Turns	450
R	Radius coil	0.06 m
m	Mass	0.0565 Kg
g	Acceleration due to Gravity	9.8 m/s^2
μ_r	Relative permeability	5500 Wb/(A·m)
L_{coil}	Coil inductance	0.0470 H
R_{coil}	Coil Resistance	10.7 Ω
ρ	Resistivity	2.75e-8 Ω·m
A_{core}	Core area	0.0121 m^2
R_{disc}	Disc Resistance	6.9380e-5 Ω
L_{disc}	Disc inductance	9.0062e-9 H
f	Source frequency	60 Hz
V_0	Voltage peak	V
I_{coil}	Coil Current	A
I_{disc}	Induced current	A
Z	Disc height	m
v	Disc velocity	m/s

$$\dot{x}_1 = 21.2766 V_0 \sin 377t - 227.6596 x_1 \quad (17)$$

$$\dot{x}_2 = -7706.6 x_2 - 0.0609 \left[\frac{V_0 \sin 377t - 10.7 x_1}{0.0470 \left(0.0036 + x_3^2 \right)^{3/2}} - \frac{3x_3 x_1 x_4}{\left(0.0036 + x_3^2 \right)^{5/2}} \right] \quad (18)$$

$$\dot{x}_3 = x_4 \quad (19)$$

$$\dot{x}_4 = -0.0709 \frac{x_2 x_1 x_3}{\left(0.0036 + x_3^2 \right)^{3/2}} - 9.8 \quad (20)$$

From simulations, is deduced the following:

In Figure 3, the induced current through the disc and the current through the coil are not in phase. The angle depends on the resistance and inductance of the disc, but this angle is always between 90 and 180 degrees. This is due to two factors: first, the induced voltage on the disc is 90 degrees out of phase with respect to the current in the coil (this may be explained by applying Faraday and Lenz's laws), and second, the disc behaves as an RL circuit so its current lags the voltage.

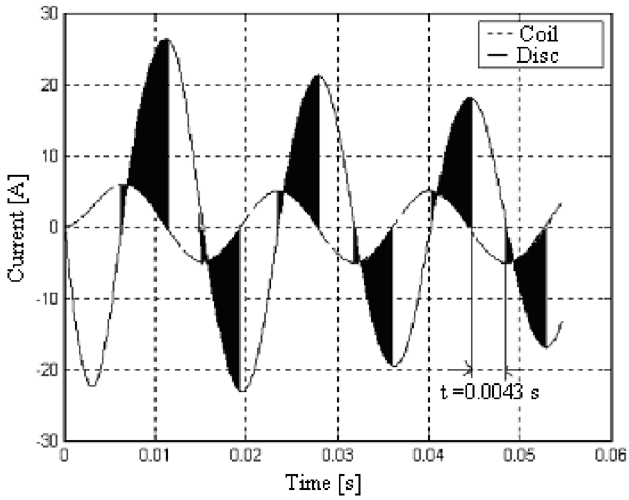

Fig. 3. Disc and coil currents

The system is the interaction of two subsystems located at the coil and the disc, which act as two magnets created by AC currents so their poles will also be alternately. In Fig. 3, dark zones correspond to the time intervals that the currents through the disc and coil turn in the same direction and therefore they are attracted. The opposite phenomenon is shows in the white zones.

It is also observed that in just one attraction cycle of the disc current, an attraction-repulsion cycle is generated. This is the reason for which, the instantaneous force that acts on the disc, oscillates to twice the frequency of the current in the coil (Figure 4). This result can be explained mathematically as the product of two sine waves, one lagging the other, as shown in (21), notice that the constant term (an average value) this related with the lag angle of the currents.

$$\sin\left(\omega t+\phi\right)=\frac{1}{2}\cos(\phi)-\frac{1}{2}\cos(\omega t+\phi) \quad (21)$$

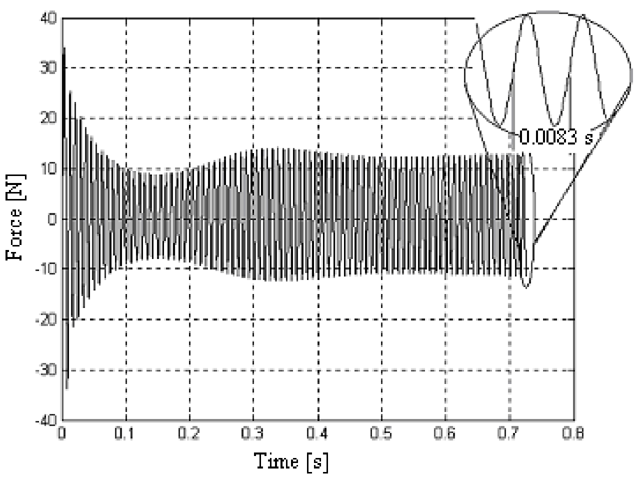

Fig. 4. Instantaneous force

Although the instantaneous force has positive and negative values (Fig. 4), exists an average force (Fig. 5) that lift up the disc. The average value of the force is directly related to the lag angle between the voltage in the disc and its current.

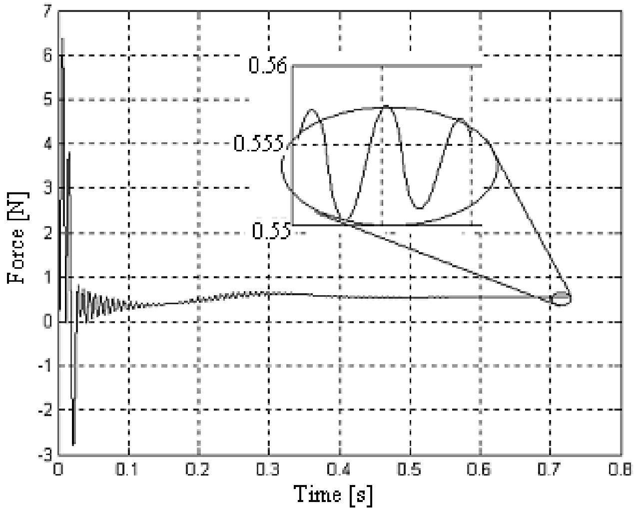

Fig. 5. Average force

The signal corresponding to the height of the disc (Fig. 6) presents oscillations at two different frequencies. The first, at 17.083 rad/s (2.7189 Hz), fades, and the second one, at 757.01 rad/s (120.48 Hz), remains in a stationary state.

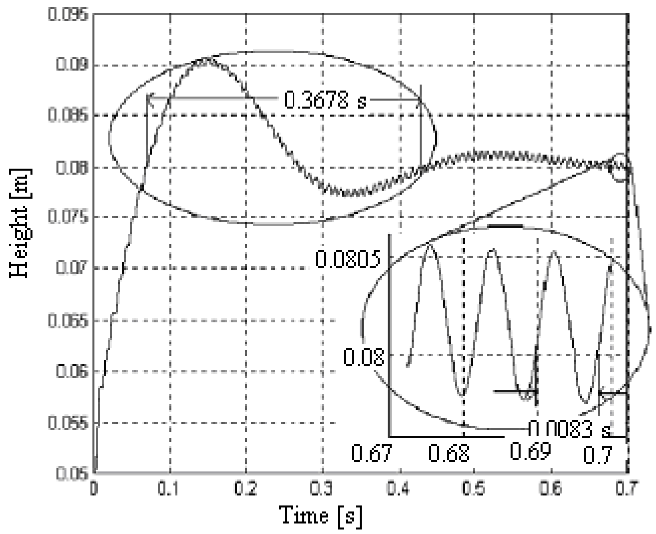

Fig. 6. Disc height

Fig. 7 shows a comparison between the height of the disc at 60 Hz and at 350 Hz. At 60 Hz the angle of impedance of the disc is 2.8017 degrees while at 350 Hz it is 15.93 degrees.

At 350 Hz the disk gets higher in a stationary state (0.088 meters) than at 60 Hz (0.080 meters), but it oscillates more during the transient time. Height changes with respect to frequency because of the changes in impedance, and changes in transient time and oscillations are due to inductive effects that increase as the frequency does, causing the damping factor to decrease.

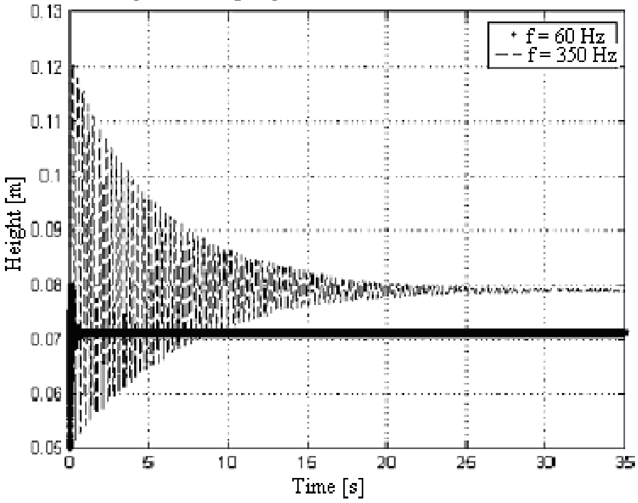

Fig. 7. Comparison of the height of the disc at 60 Hz and 350 Hz

IV. Conclusion

In this paper we have presented a mathematical model for a levitation system using repellent forces, described by parameters that are easy to obtain since they correspond to the geometry of the materials and their physical constants.

Also we verified that the mathematical description in the space state is effective, since it offers an easy way to describe non-linear systems and use computational tools to simulate and solve.

The levitation mechanism for eddy currents, presents interesting dynamics for evaluation of linear and non-linear controllers.

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