Non Linear Dynamics of an Electromagnetic Suspension/Levitation System

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Abstract—**This paper presents a mathematical model and the performance evaluation of a kind of electromagnetic levitation/suspension system, where the static and dynamic magnetic fields are linked through conductor bodies. The complex and nonlinear dynamic of the system, allows it to illustrate in a practical form, different aspects of automatic control like mathematical modelling, nonlinear analysis, simulating, controllers design, evaluation and validation of linear and nonlinear controllers. Also, the system allows to illustrate the basic principles of operation of the big developments in power applications, force generation and energy conversion. The performance evaluation is started from a model obtained by direct evaluation of mechanical forces of electromagnetic origin and virtual work method.**

Index terms—**Electromagnetic Suspension, Electrodynamic Suspension, Electrostatic, Magnetostatic, Magnetodynamic, Nonlinear multivariable systems.**

I. INTRODUCTION

LEVITATION techniques, have allowed the realization of big developments on power, force generating and energy conversion applications [1].

Electromagnetic suspension (EMS) [2][3][4] and electrodynamic suspension (EDS) [6][7][8] are two categories to obtain mechanical contactless developments. On these, controlled magnetic fields are induced by means of the use of coils and in some cases superconductive coils.

In EMS, levitation is obtained by attractive magnetic force applied to adjacent sides of an air gap [2] whereas in EDS, levitation is obtained by the repulsive force between an incident magnetic field and an induced one on a movable body [6][8].

With the mechanism shown in figure 1 it is possible to illustrate and experiment operation principles in EMS and EDS, controlling the electrical current that excites conductor magnets.

This suspension/levitation mechanism can meet experimental requirements for courses on automatic control, modern control engineering, experimentation of principles, axioms and models related to electrostatic, magnetostatic and magneto-dynamic [10].

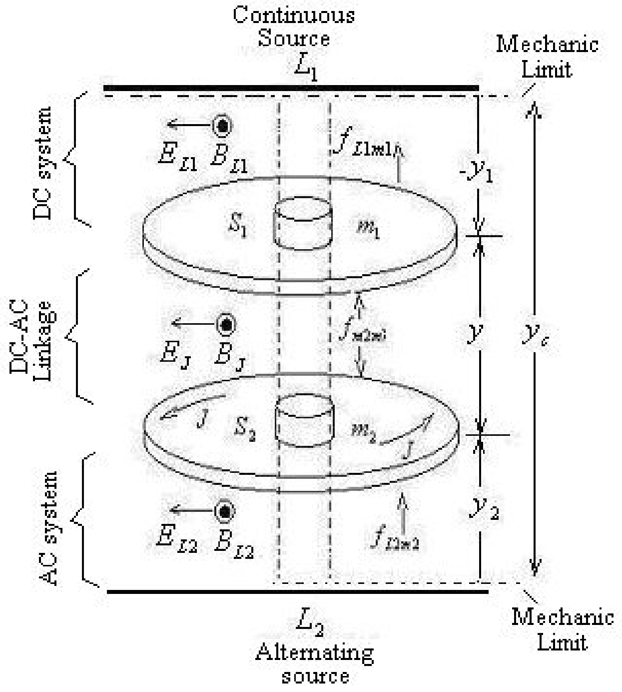

Figure 1. Suspension / Levitation system driven by electrical current

Complex dynamic and multivariable of the system allow nonlinear analysis of multivariable systems with structure of uncertainty [12], feedback linearization and the evaluation of algorithms based on intelligent techniques [16].

The system can be represented in the companion form or controllability canonical form [11]: $\dot{x} = f(x, p) + g(x)u$.

In order to deduce the dynamics of the system represented in state space, a direct evaluation of mechanical and electromotive forces of electromagnetic origin and the virtual work method for electro-mechanical complex systems modelling [13] are used.

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II. SUSPENSION /LEVITATION SYSTEM MODELLING

Any change in the energy of the system shown in figure 1, must satisfy the conservation equation:

$$\begin{pmatrix} Input \\ electrical \\ Energy \end{pmatrix} + \begin{pmatrix} Input \\ Mechanical \\ Energy \end{pmatrix} = \begin{pmatrix} Accumulated \\ Energy \end{pmatrix} - \begin{pmatrix} Dissipated \\ Energy \end{pmatrix} \quad (1)$$

Applying the basic principles of energy conservation, the direct evaluation of Maxwell's equations and the virtual work method [13] it is possible to calculate present forces on the suspension/levitation mechanism and to obtain a model for the system, covering the following basic goals:

- Nonlinear dynamic analysis for multivariable systems with an uncertainty structure.
- Design and evaluation of linear and nonlinear controllers.

In order to derive a nominal model, the following assumptions are considered:

- Uniform magnetic induction through the air gap.
- Absence of magnetic dispersion flows.
- Losses caused by Eddy currents and hysteresis are neglected.
- For the AC system, iron core concentrates magnetic flow and limits the degrees of freedom of the disc and contributes with a small viscous friction C_2.
- For DC system, the core non-conductor, it limits the degrees of freedom of the disc and contributes with a small viscous friction C_1.
- Relative permeabilities are constant.
- Magnetic induction B_J is uniform and is given by effect of Foucault's currents in the aluminium disc.
- Bodies with mass m_1 and m_2 are conductors, isotropic, linear and homogeneous [10]:
  $$\vec{D} = \varepsilon \vec{E}; \quad \vec{B} = \mu \vec{H}; \quad \vec{J} = \sigma \vec{E}$$
- Aluminium disc is non-magnetizable, so, the effects of the present energies in the DC subsystem are considered negligible.

DC suspension

In the system driven by direct current in figure 1, when a current I_1 flows through the coil L_1 sets a magnetic field B_{L1} between the core and the movable body through the air gap. Magnetic flux lines form closed-paths and current I_1 controls the amount of flux through a given surface [10].

When current I_1 flows through the coil, it creates a static magnetic field B_{L1} in the air gap, which polarizes the movable ferromagnetic body with mass m_1. Polarization of the movable body is opposite to the coil polarization, so a force f_{L1m1} that attracts the ferromagnetic body with mass m_1 toward the coil is created.

In [2] the following expression was obtained for the attractive magnetic:

$$f_{L1m1} = F(I_1, y_1) = \frac{S_1 I_1^2}{2\mu_o} G_1(y_1) \quad (2)$$

$$G_1(y_1) = 0.00098\, e^{-55\, y_1} - 0.00071\, e^{-406.1\, y_1} + 0.00012\, e^{-378.6\, y_1}$$

Where:

S_1	Surface of the iron disc
μ_0	Magnetic permeability of free space
I_1	Direct current
y_1	Air gap

$G_1(y_1)$ is a nonlinear expression for the magnetic flux intensity as a function of the air gap, obtained in an experimental way.

Electrical component is modelled as a RL circuit. R_1 is the coil resistance and L_1 is coil inductance. The relation between voltage $V_{CD} = u_1$ and current I_1 through the coil is:

$$u_1 = R_1 I_1 + L_1 \frac{d(I_1)}{dt} \quad (3)$$

AC levitation

In the system driven by alternating current in figure 1, when the varying-in-time magnetic field $\vec{B}_{L2}$ flows through a plane surface S_2 with conductivity σ, according to Ampere's law, a density of current $\vec{J} = \sigma \vec{E}_{L2}$ is produced in the conductor. $\vec{E}_{L2}$ is the electrical field given by Faraday-Henry's law,

$\nabla x \vec{E}_i = \frac{1}{\sigma}\nabla x \vec{J} = -\frac{\partial \vec{B}_i}{\partial t}$, which says that on the conductor body a magnetic field is induced and is opposed the incident magnetic field $\vec{B}_{L2}$.

The phase difference between $\vec{B}_{L2}$ given by the coil and current $\vec{J}$ that flows through the aluminium disc with mass m_2, creates a repulsive force f_{L2m2} that will lift the disc to a natural equilibrium point [7][9]. This repulsive force is expressed with the experimental Lorent's equation:

$$\vec{f}_{L2m2} = \vec{J}\, x\, \vec{B}_{L2} .$$

By direct evaluation of the mechanical forces of electromagnetic origin, in [6] the following expression for the repulsive magnetic force was obtained:

$$f_{L2m2} = -\frac{\mu \cdot N(D-Q)R^2 . i_i i_2}{3\left(R^2 + {y_2}^2\right)^{3/2}} \quad (4)$$

Where:

D	Aluminium disc outer radius
Q	Aluminium disc inner radius
μ	Core permeability
N	Number of turns
R	Coil mean radius
i_i	Induced current in the aluminium disc
i_2	Coil current

Considering the solenoid like a circuit *RL*, the expression for the excitation current is obtained from:

$$u_2 = R_2 i_2 + L_2 \dot{i}_2 \quad (5)$$

$$u_2 = V_o sen\, \omega t$$

The induced voltage on the disc is [6]:

$$V_{iD} = \frac{\pi . Q^2 \mu \cdot NR^2}{2}\left[\frac{u_2}{L_2\left(R^2+y_2^2\right)^{3/2}} - \frac{R_2 . i_2}{L_2\left(R^2+y_2^2\right)^{3/2}} - \frac{3 y_2 . i_2}{\left(R^2+y_2^2\right)^{5/2}}\dot{y}_2\right] \quad (6)$$

An expression in order to obtain the induced current is:

$$V_{iD} = R_D \cdot i_i + L_D \dot{i}_i \quad (7)$$

Where:

$R_D = \frac{2\pi \cdot \rho}{h_2 \cdot Ln\left(D/Q\right)}$: Aluminium disc resistance.

$L_D = \frac{\mu \cdot \pi \cdot Q^2}{D+Q}$: Aluminium disc inductance.

ρ = Aluminium resistivity.

h_2 = aluminium disc thickness.

Electromagnetic interaction DC-AC

In the interaction DC-AC shown in figure 1, the resultant magnetic field is the average of the static magnetic field given by the DC system and the dynamic magnetic field given by the AC system.

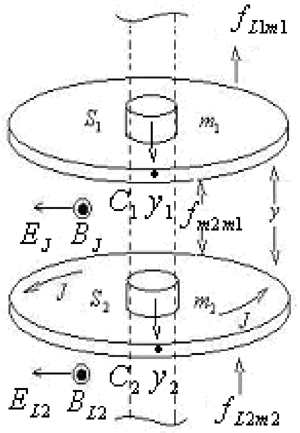

Figure 2. Scheme for the electromagnetic link

Since the aluminium disc is magnetically smooth, the static magnetic field does not polarize it and therefore the interaction force is subjected to a time-varying magnetic flux.

To obtain the interaction force f_{m2m1} , the virtual work method [7] is applied.

On the figure 3, according to Biot-Savart's law [10], the magnetic field B_J produced at a point over the Z axis is:

$$B_J(y) = \frac{\mu . d_2^2 . i_i}{2\left(\sqrt{d_2^2 + y^2}\right)^3}$$

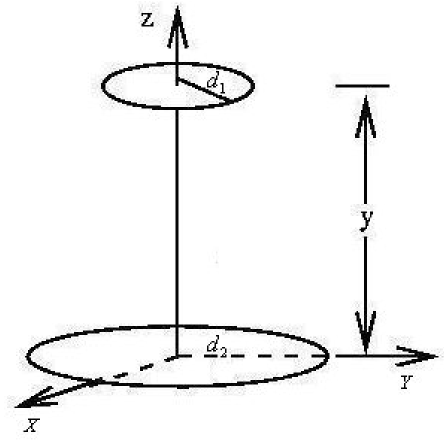

Figure 3. Diagram to obtain the magnetic field B_J

Ignoring flux dispersion, the link of magnetic flux between the discs is:

$$\phi = \pi . d_1^2 \ B_T(y) = \frac{\mu \pi d_1^2 d_2^2 . i_i}{2\left(\sqrt{d_2^2 + y^2}\right)^3}$$

Therefore, the mutual inductance is given as:

$$L(y)) = \frac{\phi}{i_i} = \frac{\mu \pi d_1^2 d_2^2}{2\left(\sqrt{d_2^2 + y^2}\right)^3}$$

The linkage co-energy is [7]:

$$W'(i_i, y) = \frac{1}{2} i_i^2 L(y)$$

And the interaction magnetic force f_{m2m1} is:

$$f_{m2m1} = \frac{\partial W'}{\partial y} = -\frac{3\mu.\pi.d_1^2 d_2^2 i_i^2 y}{2\left(d_2^2 + y^2\right)^{5/2}} \qquad (8)$$

Where:

$$d_1 = d_2 = D - Q$$

$$y = y_c - (y_2 + y_1)$$

So far, the ports of interchange of electrical energy have been obtained, including forces of electromagnetic origin f_{L1m1}, f_{L2m2}, f_{m2m1}. To complete the energy balance (equation 1) with the modelling theory for electromagnetic systems for the virtual work method [13], is necessary to deduce the mechanical ports defined by the variable force.

In the figure 2, by the Newton's second law, the movement equations that characterize the suspension/levitation system are:

$$m_1 \ddot{y}_1 = f_{L1m1} - f_{m2m1} - C_1 \dot{y}_1 - m_1 g \qquad (9)$$

$$m_2 \ddot{y}_2 = f_{L2m2} + f_{m2m1} - C_2 \dot{y}_2 - m_2 g \qquad (10)$$

The state vector for the system and its representation in space state are:

$$\dot{x} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]^T = \left[I_1, y_1, \dot{y}_1, i_2, y_2, \dot{y}_2, i_i \right]^T$$

$$\dot{x}_1 = -\frac{R_1}{L_1} x_1 + \frac{1}{L_1} u_1 \quad (11)$$

$$\dot{x}_2 = x_3 \qquad (12)$$

$$\dot{x}_3 = \frac{1}{m_1} f_{L1m1} + \frac{1}{m_1} f_{m2m1} - \frac{C_1}{m_1} x_3 - g \qquad (13)$$

$$\dot{x}_4 = -\frac{R_2}{L_2} x_4 + \frac{u_2}{L_2} \qquad (14)$$

$$\dot{x}_5 = x_6 \qquad (15)$$

$$\dot{x}_6 = \frac{1}{m_2} f_{L2m2} - \frac{1}{m_2} f_{m2m1} - \frac{C_2}{m_2} x_6 - g \qquad (16)$$

$$\dot{x}_7 = -\frac{R_D}{L_D} x_7 + \frac{V_{iD}}{L_D} \quad (17)$$

With:

$$V_{iD} = \frac{\pi . Q^2 \mu \cdot N R^2}{2} \left[\frac{u_2}{L_2 \left(R^2 + x_5^2\right)^{3/2}} - \frac{R_2 . x_4}{L_2 \left(R^2 + x_5^2\right)^{3/2}} - \frac{3 x_5 x_4}{\left(R^2 + x_5^2\right)} x_6 \right]$$

The induced current depends on the states x_4, x_5 and x_6 According to Faraday-Henry and Ampere-Maxwell laws.

III. PERFORMANCE EVALUATION

Mathematical analysis.

With the geometric approach [12][14][15], the suspension/levitation system, can be represented in the companion form [11]:

$$\dot{x}(t) = f(x(t)) + \sum_{i=1}^{2} g_i((x(t)) u_i \quad (18)$$

$$y(x(t)) = \beta(x(t)) \qquad (19)$$

Where:

$$f(x)=\begin{bmatrix} -\frac{R_1}{L_1}x_1 \\ x_3 \\ \frac{1}{m_1}f_{L1m1}+\frac{1}{m_1}f_{m2m1}-\frac{C_1}{m_1}x_3-g \\ -\frac{R_2}{L_2}x_4 \\ x_6 \\ \frac{1}{m_2}f_{L2m2}-\frac{1}{m_2}f_{m2m1}-\frac{C_2}{m_2}x_6-g \\ -\frac{R_D}{L_D}x_7+\frac{V_{iD}}{L_D} \end{bmatrix}$$

$$g_1=\left[\frac{1}{L_1}\ 0\ 0\ 0\ \ 0\ 0\ \ 0\right]^T u_1,$$

$$g_2=\left[0\ 0\ 0\ \frac{1}{L_2}\ 0\ \ 0\ 0\right]^T u_2$$

$$y=\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}\begin{bmatrix}x_2\\ x_5\end{bmatrix} \qquad (20)$$

Where β_1,β_2 they are the gains of the sensors.

The representation in the form (18), allows the multivariable análisis with uncertainty structure [12].

It can be demonstrated [14][15] that the equations of the mechanism of suspension/levitation in form (18), satisfy the conditions of decoupling and exact linearization with stability, which transform the system into a parallel connection of single-input single-output systems. That is the natural continuation of the efforts made to study decoupling of linear systems.

Numerical analysis.

Table 1, shows the parameters equivalent to the geometry of the suspension/Levitation system.

With the method of numerical solution OD45 (Dormand-Prince) and toolbox Simulink of Matlab, the simulations were made of the system represented with expressions (11)-(17).

Figure 4 shows the diagram of simulation for the system.

TABLE I

PARAMETERS OF THE MAGNETIC LEVITATION /SUSPENSION SYSTEM

Symbol	Quantity	Value (SI)
μ_0	Permeability	$4\pi \times 10^{-7}$ Wb/(A·m)
D	External radius disc	0.062125 m
Q	Internal radius disc	0.0131 m
$h_{1,2}$	Thickness disc	0.0016 m
$S_{1,2}$	Disc surface	0.00266 m^2
N	Number of Turns	500
R	Radius coil	0.0600 m
m_1	Mass	0.0630 Kg
m_2	Mass	0.0565 Kg
g	Gravity	9.8 m/s^2
μ	Relative permeability	5500 Wb/(A·m)
L_1	Coil inductance	0.418 H
L_2	Coil inductance	0.470 H
R_1	Coil Resistance	20 Ω
R_2	Coil Resistance	10.7 Ω
ρ	Aluminum resistivity	2.75e-8 Ω·m
A_{core}	Core area	0.00121 m^2
R_D	Disc Resistance	6.9380e-5 Ω
L_D	Disc inductance	9.0062e-9 H
f	Source frequency	60 Hz
V_0	Voltage peak	$120V_{AC}$
$C_{1,2}$	Static friction	0.61[1]

[1] Friction coefficient , aluminum on smooth steel.

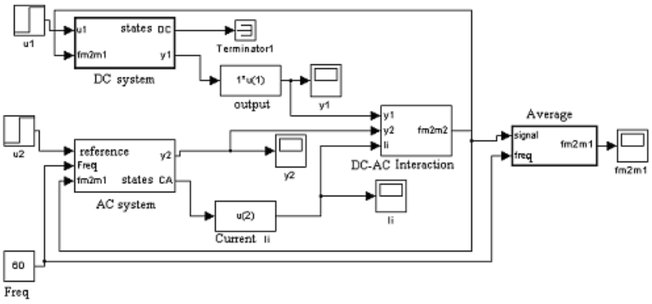

Figure 4. Simulation diagram for the Suspension/levitation system

The following results were obtained, according to the experimental results:

I. By Earnshaw's theorem [5], because of the nature of static fields, it is impossible to set a static equilibrium point for the ferromagnetic disc (figure 5). This unstable condition can be dealt with feedback control of the excitation current I_1 [14].

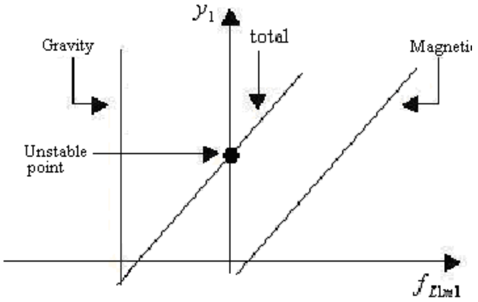

Figure 5. The EMS is unstable

Figure 6 shows the behavior of the state associated with the position of the ferromagnetic disc.

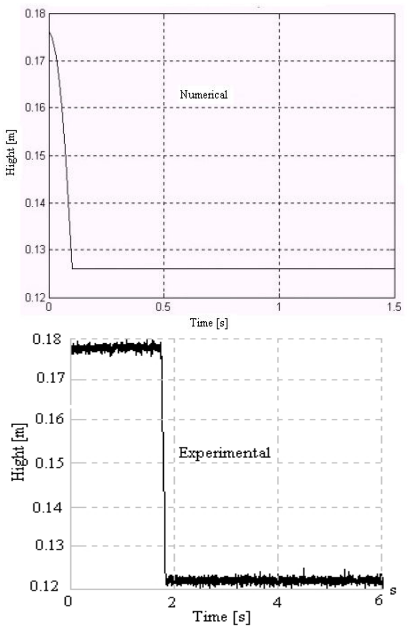

Figure 6. Behavior of the states x_2 ans x_5 .

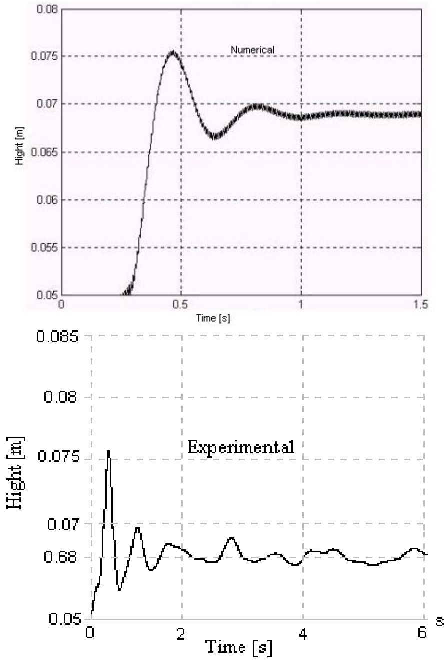

Figure 7. Behavior of the state x_5 .

II. A stability analysis done to the system AC, shows that the levitation is stable (see figure 8), but underdamped [7][9], as in figure 7.

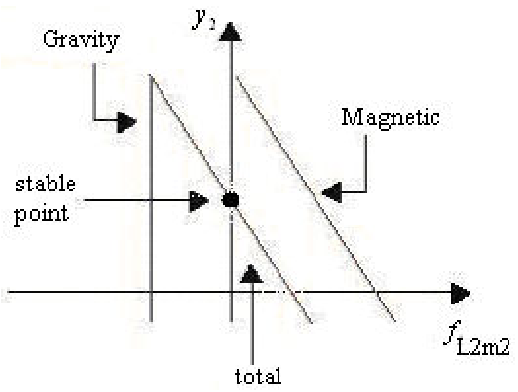

Figura 8. The EDS is stable

Experimentation shows that the dynamics of the vertical suspension for electrodynamically levitated bodies are underdamped and the state associated with the bodies position presents oscillations in stationary state [6][7][9], shown in figure 7.

III. In the CD-AC link it was verified that the main source of energy it is provided by the AC system, which induces a current in the iron disc associated with state x_2 .

If the aluminum disc is on a balanced position by effect of the variant field, in absence of the static magnetic field, when approaching the iron disc, the aluminum disc is attracted by the force average of electromagnetic induction, shown in figure 9.

The same behavior is obtained in presence of the static field.

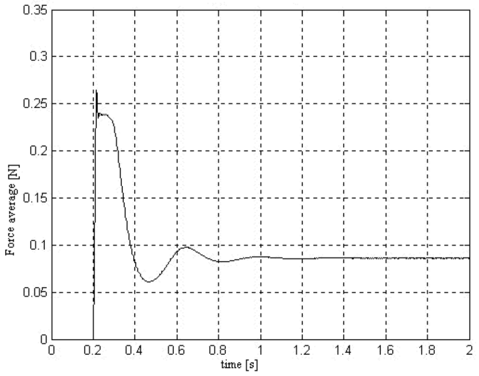

Figure 9. Link Force average f_{m2m1} .

IV. CONCLUSIONS

The main source of inaccuracy in the analytical deduction of the forces of electromagnetic origin $f_{L1m1}, f_{L2m2}, f_{m2m1}$, is to have ignored the losses in the core, the losses due to hysteresis and the eddy currents. These normally produce heating in the metallic pieces which reduce the system performance [1].

Ideally the magnetic field uniform means that the field is the same in all surface of magnetic contact. This allows to deal the system like damped parameters and therefore arriving easily at a representation of the system in the space of states.

An interesting work is not to consider the magnetic field uniform, account magnetic dispersion flows and conduction losses.

This demands a distributed treatment, which can be done with the Finite Element Method (FEM) [1], that provides excellent results for electromagnetic analysis and design.

The parameters considered like constants in the modeling, tending to change with the thermal elevation. This problem is considered like parametric uncertainty [12], which typically can be dealt with the robust control [15], representing the nominal system in the companion form [11]

The complex dynamics of the suspension/levitation system is appropriate for the application of the control based on heuristic techniques [16].

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