A New Model For Induction Motor With Induced Saliencies

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Abstract—**This paper develops a formal and general model for an induction motor with periodic variations of the rotor slot; this induction motor with induced saliencies is commonly used in sensorless control applications. In order to obtain the model, first the analysis of the stator and rotor inductances modified by induced saliencies is carried out; second, the electromagnetic and electomechanical equations in ABC frame are developed and third, the model is represented in the natural orthogonal a-b frames. The obtained model is suitable for control design and compatible with the classical ab model for induction motors without saliencies.**

Index Terms - Induction motor, induced saliencies, sensorless control.

NOMENCLATURE

Variables	
i	: electrical current
Ψ, ψ	: magnetic flux
$\mathcal{F}$	: magnetomotive force
B	: flux densities
α, θ_r	: rotor angle, ($\alpha = \nu\theta_r$)
T_l	: mechanical load torque
T_e	: electrical torque
Parameters	
ν	: number of pole-pairs
R	: electrical resistence
L	: electrical inductance
M	: inertial mass
f	: friction coefficient
μ_0	: the air magnetic permeability
l	: the axial length of the machine
r	: mean radius at the air gap of the machine
Superscripts	:
ABC	: conventional three-axis systems
abh	: orthogonal three-axis systems
ab	: orthogonal by-axis systems
Subscripts	:
s	: refer to the stator frame
r	: refer to the rotor frame
$\dot{(\cdot)}$	: differentiation with respect to time

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I. INTRODUCTION

In the past and present decade, the control for Induction Motors (**IM**) without mechanical shaft sensors has led to researchers and manufacturers to include sensorless vector drives.

The main problem in the sensorless control of induction motors is around zero speed or zero excitation frequency. In order to solve the problem, several works have been developed for modelling the phenomena in the stator and rotor slots; other ones more complexes, include a detailed tensor model of the IM; however, these approaches are something complex for control.

The design of high performance sensorless drives of induction motors include an observer for the rotor/stator flux and one estimation function for the mechanical speed; some problems have been reported and can be summarized as: loss of observability around zero excitation frequency, incorrect flux/torque estimation induced by errors in the estator and rotor resistances, steady-state inestability at low speed, particularly under regeneration. About solving the loss of observability at zero excitation frequency, one possible alternative solution is the Induction Motor with Induced Saliencies **IMIS**.

The saliencies of the **IM** used for sensorless control have been classified as: slot harmonics, designed asymetry (induced saliencies), saturation and dynamic eccentricity. The induced saliencies can be described as changes in the rotor slots geometry.

The sensorless control of the **IMIS**, for position tracking and speed control including both saliencies and input high frequency signals, has been reported by Holtz [1], [2], Degner[3] and Jansen [4] with promising results; Quan[5] has reported the high performance near to zero speed.

In this paper only changes in the rotor slots width are considered, then for obtain the model, the flux variations induced by the saliencies must be took into account; other important assumptions are: a) the variation of the rotor slots width can be represented for a periodical (with a pole pitch) and sinusoidal permeance; b) the squirrel-cage motor is approximated by an equivalent polyphase wound rotor with the same pole number and equivalents turns Nr and resistance Rr; c) the stator and rotor windings may be approximated as sinusoidal distributed windings; d) the stator and rotor steel have a high permeability; and e) the air gap is assumed uniform and the Carter factor is modified to approach the saliencies phenomena.

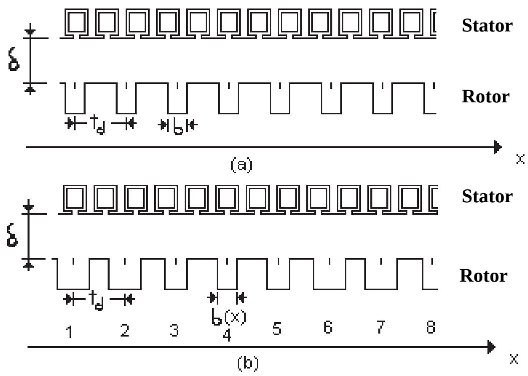

Fig. 1. IM stator and rotor scheme

II. Air gap Correction

In Fig. 1. the slot rotor pitch (td) and the air-gap (δ) are constants; in a) the rotor slot width b is constant and in b) the rotor slot width is position dependent $b(x)$. Although the permeability of the gap region is constant, it is bounded on either side by iron surface which far from being smooth, is indented with slots in the circumferential direction, introducing variations in the air gap permeance[6]. It is possible to suppose that the actual slotted surface can be replaced by an equivalent unslotted surface having the same cross-section but with modified "equivalent" air gap δK_c, where K_c the so called Carter factor, is the relation between the equivalent air gap permeance and the actual air gap permeance.

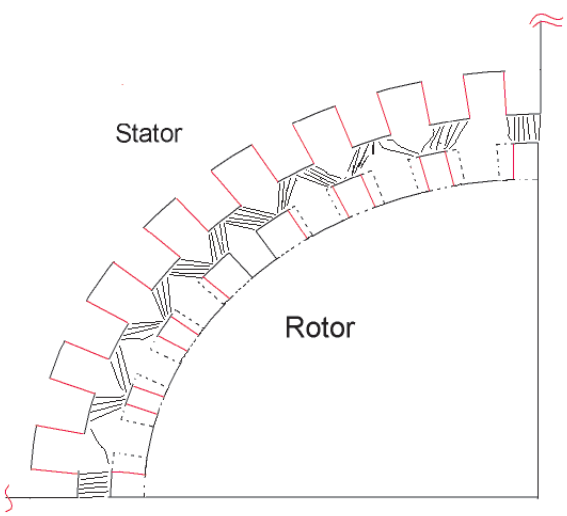

Fig. 2. IMIS stator and rotor scheme.

For the **IMIS**, the periodical variation of the rotor slot width, superpose a new space variation on the air gap flux density (Fig. 2). It is possible to consider a new correction factor for the sinusoidal modulation of the slot wide rotor, called "Corrected Carter Factor" and it is denoted here as K_s.

$$K_s = \frac{1}{\alpha_1 - \alpha_2 \cos(2\nu\theta_r)} \tag{1}$$

where, α_1 and α_2 are constants associated with a maximal and minimal air gap equivalent (Fig. 3). The equivalent air gap is:

$$\delta_s = \delta K_s = \delta \frac{1}{\alpha_1 - \alpha_2 \cos(2\nu\theta_r)} = \frac{1}{\delta_1 - \delta_2 \cos(2\nu\theta_r)} \tag{2}$$

$\delta_1 = \alpha_1/\delta$ and $\delta_2 = \alpha_2/\delta$.

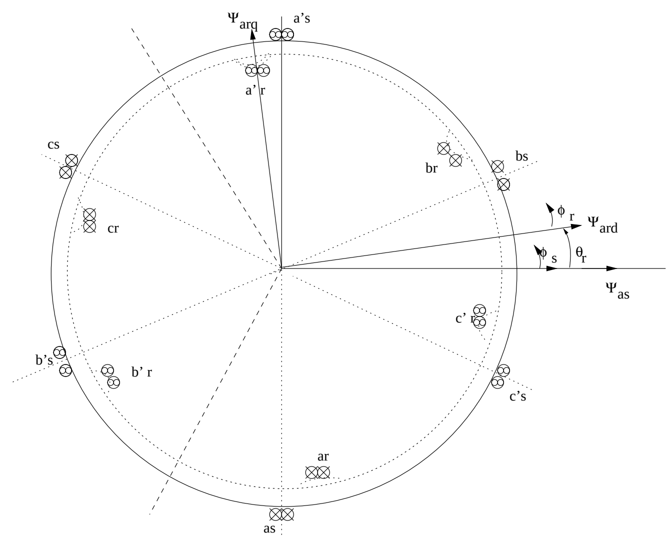

Fig. 3. Scheme of the Induction Motor

Remark: In absence of the rotor slot modulation, $\delta_2 = \alpha_2/\delta = 0$, then $\delta_s = \delta K_s$, so, $K_s = K_c$ and $\delta_s = \delta_c$.

III. Winding Inductances

A. Stator Inductances

The currents in balanced and steady state conditions are

$$\begin{aligned} i_A &= \sqrt{2} I_s \cos(w_a t + \phi_s(0)) \\ i_B &= \sqrt{2} I_s \cos(w_a t - 2\pi/3 + \phi_s(0)) \\ i_C &= \sqrt{2} I_s \cos(w_a t + 2\pi/3 + \phi_s(0)) \end{aligned} \tag{3}$$

Then the magnetomotive forces by the stator windings are respectively:

$$\begin{aligned} \mathfrak{F}_A &= i_A N_s \cos(\nu\phi_s) \\ \mathfrak{F}_B &= i_B N_s \cos(\nu\phi_s - 2\pi/3) \\ \mathfrak{F}_C &= i_C N_s \cos(\nu\phi_s + 2\pi/3) \end{aligned} \tag{4}$$

Using the Amper's Law and the new air gap factor, the phase air gap flux densities are:

$$\begin{aligned} B_A(\nu\phi_s, \theta_r) &= \mu_0 \frac{F_A}{\delta_s(\phi_s, \theta_r)} = \frac{\mu_0 N_s}{\delta_s(\phi_s, \theta_r)} i_A \cos(\nu\phi_s) \\ B_B(\nu\phi_s, \theta_r) &= \frac{\mu_0 N_s}{\delta_s(\phi_s, \theta_r)} i_B \cos(\nu\phi_s - 2\pi/3) \\ B_C(\nu\phi_s, \theta_r) &= \frac{\mu_0 N_s}{\delta_s(\phi_s, \theta_r)} i_C \cos(\nu\phi_s + 2\pi/3) \end{aligned} \tag{5}$$

The phase flux and flux linkages are:

$$\psi_{sx}(\nu\phi_s,\theta_r)=\int_{\phi_s}^{\phi_s+2\pi/2\nu}B_x(\nu\xi,\theta_r)rld\xi \tag{6}$$

$$\begin{aligned}\Psi_{sx}(\nu\phi_s,\theta_r) &= \nu\int N_x(\phi_s)\psi_{sx}(\nu\phi_s,\theta_r)d\phi_s\\ &= \nu\int N_x(\phi_s)\int_{\phi_s}^{\phi_s+2\pi/2\nu}B_x(\nu\xi,\theta_r)\\ &\quad\cdot rld\xi d\phi_s\end{aligned} \tag{7}$$

where the sub index "x" is for the A, B or C windings; "l" and "r" are respectively, the axial length and the mean radius at the air gap of the machine, "ξ" is the integration variable and $N_x(\phi_s)$ represents the sinusoidal distributed winding.

So, to the phase A,

$$\begin{aligned}\Psi_{sA} &= \nu\int N_A(\phi_s)\int_{\phi_s}^{\phi_s+\pi/\nu}B_A(\nu\xi,\theta_r)rld\xi d\phi_s\\ &= -N_s^2\mu_0\nu i_A rl\int_{\pi/\nu}^{2\pi/\nu}\sin(\nu\phi_s)\cdot\\ &\quad\int_{\phi_s}^{\phi_s+\pi/\nu}\cos(\nu\xi)\\ &\quad\cdot\{\delta_1-\delta_2 cos2\nu(\phi_s-\theta_r)\}d\xi d\phi_s\end{aligned} \tag{8}$$

The total phase flux linkages are:

$$\begin{aligned}\Psi_A &= L_{\sigma s}i_A+\Psi_{sA}\\ \Psi_B &= L_{\sigma s}i_B+\Psi_{sB}\\ \Psi_C &= L_{\sigma s}i_C+\Psi_{sC}\end{aligned} \tag{9}$$

Neglecting the phase leakage inductance ($L_{\sigma s}$), the stator inductances are:

$$\begin{aligned}L_{AA} &= \frac{N_s^2}{\nu}\pi\mu_0 rl\{\delta_1-\frac{\delta_2}{2}\cos(2\nu\theta_r)\}\\ L_{BB} &= \frac{N_s^2}{\nu}\pi\mu_0 rl\{\delta_1-\frac{\delta_2}{2}\cos(2\nu\theta_r-\frac{2\pi}{3})\}\\ L_{CC} &= \frac{N_s^2}{\nu}\pi\mu_0 rl\{\delta_1-\frac{\delta_2}{2}\cos(2\nu\theta_r+\frac{2\pi}{3})\}\end{aligned} \tag{10}$$

The mutual inductances are calculated following the same procedure; the mutual flux linkages are:

$$\begin{aligned}\Psi_A\,|_{i_B} &= -\nu\int_{\pi/\nu}^{2\pi/\nu}N_s\sin(\nu\phi_s)\cdot\\ &\quad\int_{\phi_s}^{\phi_s+\pi/\nu}\mu_0 N_s i_B\cos(\nu\xi-\frac{2\pi}{3})rl\\ &\quad\cdot\{\delta_1-\delta_2\cos 2\nu(\xi-\theta_r)\}d\xi d\phi_s\end{aligned}$$

Therefore, the mutual inductances for stator windings are:

$$\begin{aligned}L_{AB} &= -\frac{N_s^2}{2\nu}\pi\mu_0 rl\delta_1-\frac{N_s^2}{2\nu}\pi\mu_0 rl\delta_2\cos(2\nu\theta_r-\frac{2\pi}{3})\\ L_{AC} &= -\frac{N_s^2}{2\nu}\pi\mu_0 rl\delta_1-\frac{N_s^2}{2\nu}\pi\mu_0 rl\delta_2\cos(2\nu\theta_r+\frac{2\pi}{3})\\ L_{BC} &= -\frac{N_s^2}{2\nu}\pi\mu_0 rl\delta_1-\frac{N_s^2}{2\nu}\pi\mu_0 rl\delta_2\cos(2\nu\theta_r)\end{aligned}$$

and $L_{BA}=L_{AB}$, $L_{CA}=L_{AC}$, $L_{CB}=L_{BC}$.

If,

$$L_s = \frac{N_s^2}{\nu}\pi\mu_0 rl\delta_1 \quad ; \Delta L_s = \frac{N_s^2}{2\nu}\pi\mu_0 rl\delta_2$$

then,

$$\begin{aligned}L_{AA} &= \frac{N_s^2}{\nu}\pi\mu_0 rl\delta_1-\frac{N_s^2}{2\nu}\pi\mu_0 rl\delta_2\cos(2\nu\theta_r)\\ &= L_s-\Delta L_s\cos(2\nu\theta_r)\end{aligned}$$

So, for the others inductances in the stator

$$\begin{aligned}L_{BB} &= L_s-\Delta L_s\cos(2\nu\theta_r-2\pi/3)\\ L_{CC} &= L_s-\Delta L_s\cos(2\nu\theta_r+2\pi/3)\\ L_{AB} &= -\frac{L_s}{2}-\Delta L_s\cos(2\nu\theta_r-2\pi/3)\\ L_{AC} &= -\frac{L_s}{2}-\Delta L_s\cos(2\nu\theta_r+2\pi/3)\\ L_{BC} &= -\frac{L_s}{2}-\Delta L_s\cos(2\nu\theta_r)\\ L_{CA} &= L_{AC}\\ L_{CB} &= L_{BC}\end{aligned}$$

finally:

$$Ł_{\mathbf{ss}}=\begin{bmatrix}L_{AA} & L_{AB} & L_{AC}\\ L_{BA} & L_{BB} & L_{BC}\\ L_{CA} & L_{CB} & L_{CC}\end{bmatrix}$$

B. Rotor Inductances

Assuming wound symmetrical currents in the rotor:

$$\begin{aligned}i_a &= \sqrt{2}I_r\cos(w_r t+\phi_r(0))\\ i_b &= \sqrt{2}I_r\cos(w_r t-2\pi/3+\phi_r(0))\\ i_c &= \sqrt{2}I_r\cos(w_r t+2\pi/3+\phi_r(0))\end{aligned} \tag{11}$$

Where w_r is the angular speed in the rotor.

The direct and quadrature axis components of the "a" phase rotoric current are:

$$i_{ad}=i_a\cos(\nu\theta_r) \;\; ; \;\; i_{aq}=-i_a\sin(\nu\theta_r)$$

θ_r is the angle between the "A" stator phase axis and "a" rotor phase axis.

The flux linkage, created by a direct (i_{ad}) and quadrature (i_{aq}) components are respectively:

$$\psi_d^r=i_{ad}L_{ad} \;\; ; \psi_q^r=i_{aq}L_{aq}$$

Assuming non saturation, the total flux linkage for a rotor phase "a" is the addition of the flux linkages ψ_d^r y ψ_q^r,

$$\psi_{ra}=L_{aa}i_a=\psi_d^r\cos(\nu\theta_r)-\psi_q^r\sin(\nu\theta_r)$$

and,

$$\begin{aligned}L_{aa} &= \frac{\psi_d^r\cos(\nu\theta_r)-\psi_q^r\sin(\nu\theta_r)}{i_a}\\ &= \frac{i_{ad}L_{ad}\cos(\nu\theta_r)-i_{aq}L_{aq}\sin(\nu\theta_r)}{i_a}\\ &= \frac{i_a\cos(\nu\theta_r)L_{ad}\cos(\nu\theta_r)+i_a\sin(\nu\theta_r)L_{aq}\sin(\nu\theta_r)}{i_a}\\ &= L_{ad}cos^2(\nu\theta_r)+L_{aq}sen^2(\nu\theta_r)\\ &= L_{ra}-\triangle L_{ra}cos(2\nu\theta_r)\end{aligned}$$

with

$$L_{ra}=\frac{1}{2}(L_{ad}+L_{aq}) \;\; ; \;\; \triangle L_{ra}=\frac{1}{2}(L_{aq}-L_{ad})$$

Similarly, to the other phases,

$$\begin{aligned}L_{bb} &= L_r-\triangle L_r\cos(2\nu\theta_r+2\pi/3)\\ L_{cc} &= L_r-\triangle L_r\cos(2\nu\theta_r+4\pi/3)\end{aligned}$$

The mutual inductances L_{ab}, L_{ac}, and L_{bc} are calculated in the same way,

$$\begin{aligned} L_{ba} &= \frac{\psi_{ba}}{i_a} = -\frac{L_r}{2} - \triangle L_r\cos(2\nu\theta_r + 4\pi/3) \\ L_{ca} &= \frac{\psi_{ca}}{i_a} = -\frac{L_r}{2} - \triangle L_r\cos(2\nu\theta_r + 2\pi/3) \\ L_{bc} &= \frac{\psi_{bc}}{i_a} = -\frac{L_r}{2} - \triangle L_r\cos(2\nu\theta_r) \end{aligned}$$

and

$$Ł_{\mathbf{rr}} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}$$

C. Stator-Rotor Mutual Inductance

In phase “a” the magnetomotive force in the equivalent winding is:

$$\mathfrak{F}_a = N_r i_a cos(\nu\phi_r)$$

Considering the new factor K_s and the initial condition $\phi_s(0) = 0$, the phase “a” flux density distribution is:

$$B_a(\nu\phi_r, \theta_r) = \mu_0 \tfrac{F_a}{\delta_s(\phi_s,\theta_r)} = \tfrac{\mu_0}{\delta_s(\phi_s,\theta_r)} N_r i_a cos(\nu\phi_r) \quad (12)$$

The mutual flux between stator phase “A” and rotor phase “a” is then:

$$\psi_{Aa}(\nu\phi_r, \theta_r) = \int_{\phi_s}^{\phi_s + 2\pi/2\nu} B_a(\nu\xi, \theta_r) rl d\xi \quad (13)$$

Assuming that the windings have a sinusoidal distribution, the total mutual flux linkage is,

$$\begin{aligned} \Psi_{Aa}(\nu\phi_r, \theta_r) &= -\nu\int_{\pi/\nu}^{2\pi/\nu} N_s \sin(\nu\phi_s) \\ &\quad \cdot \int_{\phi_s}^{\phi_s+\pi/\nu} \tfrac{\mu_0}{\delta_s(\phi_s,\theta_r)} N_r i_a \cos(\nu\phi_r) \\ &\quad \cdot rl d\xi d\phi_s \\ &= \tfrac{N_s N_r}{\nu}\mu_0 \pi i_a rl\left(\delta_1 + \tfrac{\delta_2}{2}\right)\cos(\nu\theta_r) \end{aligned} \quad (14)$$

and,

$$L_{Aa} = \tfrac{\Psi_{Aa}}{i_a} = L_{sr}\cos(\nu\theta_r)$$

where,

$$L_{sr} = \frac{N_s N_r}{\nu}\mu_0 \pi rl\left(\delta_1 + \frac{\delta_2}{2}\right)$$

The other mutual inductances are obtained following the same procedure for calculating L_{Aa}.

$$Ł_{\mathbf{sr}} = L_{sr}\begin{bmatrix} \cos(\alpha) & \cos(\alpha + 2\pi/3) & \cos(\alpha - 2\pi/3) \\ \cos(\alpha - 2\pi/3) & \cos(\alpha) & \cos(\alpha + 2\pi/3) \\ \cos(\alpha + 2\pi/3) & \cos(\alpha - 2\pi/3) & \cos(\alpha) \end{bmatrix}$$

$\alpha = \nu\theta_r$

IV. ELECTROMAGNETIC CIRCUITS EQUATIONS

The three-phase induction motor model with distributed windings, in the estator frame, is

$$V_s^{ABC} = \mathbf{R_s} i_s^{ABC} + \dot{\Psi}_s^{ABC} \quad (15)$$

$$V_r^{ABC} = \mathbf{R_r} i_r^{ABC} + \dot{\Psi}_r^{ABC} \quad (16)$$

$\mathbf{R_s}$ and $\mathbf{R_r}$ are diagonal matrices in $\mathcal{R}^3$, V_s^{ABC} and V_r^{ABC} are column vectors describing the stator and rotor voltages respectively.

The flux linkage equations are:

$$\Psi_s^{ABC} = Ł_{\mathbf{ss}} i_s^{ABC} + Ł_{\mathbf{sr}} i_r^{ABC} \quad (17)$$

$$\Psi_r^{ABC} = (Ł_{\mathbf{sr}})^T i_s^{ABC} + Ł_{\mathbf{rr}} i_r^{ABC} \quad (18)$$

The rotor expressions are referred to the stator frame, using the appropriated transformation.

V. ELECTROMECHANICAL EQUATIONS

The electromechanical equations are:

$$\begin{aligned} M\dot{w}_r &= -fw + T_e - T_l \\ \dot{\theta}_r &= w_r \end{aligned} \quad (19)$$

The electrical torque T_e is:

$$\begin{aligned} T_e &= +\tfrac{1}{2}(i_s^{ABC})^T \tfrac{\partial Ł_{\mathbf{ss}}}{\partial\theta_r} i_s^{ABC} \\ &\quad +(i_s^{ABC})^T \tfrac{\partial Ł_{\mathbf{sr}}}{\partial\theta_r} i_r^{ABC} + \tfrac{1}{2}(i_r^{ABC})^T \tfrac{\partial Ł_{\mathbf{rr}}}{\partial\theta_r} i_r^{ABC} \end{aligned} \quad (20)$$

VI. ORTHOGONAL FRAME EQUATIONS

Due to the polyphasic model complexity, is normal to make simplifications using methods of transformation [8].; the basic Concordia transformation replaces the symmetrical three-phase variables[1] into symmetrical bi-phase equivalent variables. It is denominated the ortoghonal or “ab” model[2].

$$x^{abh} = \mathbf{T_c} x^{ABC} = \sqrt{2/3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} x^{ABC}$$

where:

$x^{ABC} \rightarrow$ three-phase variables; and
$x^{abh} \rightarrow$ orthogonal bi-phase variables.

So, the variables transformations can be expressed according to the relationships in Table I.
Consequently, the mathematical expressions of stator and rotor voltages and flux linkages are:

$$V_s^{ab} = \mathbf{R_s} i_s^{ab} + \dot{\Psi}_s^{ab} \quad (21)$$

$$0 = \mathbf{R_r} i_r^{ab} + \dot{\Psi}_r^{ab} \quad (22)$$

$$\Psi_s^{ab} = \mathbf{Lss}(2\nu\theta_r) i_s^{ab} + \mathbf{Lsr}^T(\nu\theta_r) i_r^{ab} \quad (23)$$

$$\Psi_r^{ab} = \mathbf{Lsr}(\nu\theta_r) i_s^{ab} + \mathbf{Lrr}(2\nu\theta_r) i_r^{ab} \quad (24)$$

[1] Defined as a set of equal amplitud sisusoidal which are displaced by 120 degress.

[2] Also referenced as “dqo”, we use the “abh” to distinguish of others references frames

TABLE I

TRANSFORMATION RELATIONSHIPS

Stator	Rotor
$i_s^{abh} = \mathbf{T_c} i_s^{ABC}$	$i_r^{abh} = \mathbf{T_c} i_r^{ABC}$
$v_s^{abh} = \mathbf{T_c} v_s^{ABC}$	$v_r^{abh} = \mathbf{T_c} v_r^{ABC}$
$\Psi_s^{abh} = \mathbf{T_c} \Psi_s^{ABC}$	$\Psi_r^{abh} = \mathbf{T_c} \Psi_r^{ABC}$
$i_s^{ABC} = \mathbf{T_c}^{-1} i_s^{abh}$	$i_r^{ABC} = \mathbf{T_c}^{-1} i_r^{abh}$
$v_s^{ABC} = \mathbf{T_c}^{-1} v_s^{abh}$	$v_r^{ABC} = \mathbf{T_c}^{-1} v_r^{abh}$
$\Psi_s^{ABC} = \mathbf{T_c}^{-1} \Psi_s^{abh}$	$\Psi_r^{ABC} = \mathbf{T_c}^{-1} \Psi_r^{abh}$

where

$$\begin{aligned}
\mathbf{R_s} &= R_s\mathbf{I} \quad \in \Re^2\\
\mathbf{R_r} &= R_r\mathbf{I} \quad \in \Re^2\\
\mathbf{Lss}(2\nu\theta_r) &= \mathbf{T_c}Ł_{ss}\mathbf{T_c}^{-1}\\
&= L_s\mathbf{I} - \Delta L_s \mathbf{U^T}(2\nu\theta_r)\mathbf{I}^-\\
\mathbf{Lrr}(2\nu\theta_r) &= \mathbf{T_c}Ł_{rr}\mathbf{T_c}^{-1}\\
&= L_r\mathbf{I} - \Delta L_r \mathbf{U^T}(2\nu\theta_r)\mathbf{I}^-\\
\mathbf{Lsr}(\nu\theta_r) &= L_{sr}\mathbf{U}(\nu\theta_r)\\
\mathbf{Lsr}^T(\nu\theta_r) &= L_{sr}\mathbf{U^T}(\nu\theta_r)
\end{aligned}$$

The electromechanical equation is given by (19), and the transformed torque equation is:

$$\begin{aligned}
T_e = &\frac{1}{2}(i_s^{abh})^T \frac{\partial \mathbf{L_{ss}}}{\partial\theta_r} i_s^{abh}\\
&+(i_s^{abh})^T \frac{\partial \mathbf{L_{sr}}}{\partial\theta_r} i_r^{abh} + \frac{1}{2}(i_r^{abh})^T \frac{\partial \mathbf{L_{rr}}}{\partial\theta_r} i_r^{abh}
\end{aligned}$$

with

$$\begin{aligned}
\frac{\partial}{\partial\theta_r}(\mathbf{T_c}Ł_{ss}\mathbf{T_c}^{-1}) &= \frac{\partial \mathbf{L_{ss}}}{\partial\theta_r} &= 2\nu\Delta L_s\mathbf{U^T}(2\nu\theta_r)\mathbf{I}^-\mathbf{J}\\
\frac{\partial}{\partial\theta_r}(\mathbf{T_c}Ł_{rr}\mathbf{T_c}^{-1}) &= \frac{\partial \mathbf{L}_{rr}}{\partial\theta_r} &= 2\nu\Delta L_r\mathbf{U^T}(2\nu\theta_r)\mathbf{I}^-\mathbf{J}\\
\frac{\partial}{\partial\theta_r}(\mathbf{T_c}Ł_{sr}\mathbf{T_c}^{-1}) &= \frac{\partial \mathbf{L_{sr}}}{\partial\theta_r} &= \nu L_{sr}\mathbf{U^T}(\nu\theta_r)\mathbf{J}
\end{aligned}$$

$$\begin{aligned}
T_e = &\nu c\Delta L_s (i_s^{ab})^T \mathbf{U}^T(2\nu\theta_r)\mathbf{I}^-\mathbf{J} i_s^{ab}\\
&+\nu c L_{sr}(i_s^{ab})^T\mathbf{U^T}(\nu\theta_\mathbf{r})\mathbf{J} i_r^{ab}\\
&+\nu c\Delta L_r (i_r^{ab})^T \mathbf{U}^T(2\nu\theta_r)\mathbf{I}^-\mathbf{J} i_r^{ab}
\end{aligned} \tag{25}$$

A. State Space Model

Considering the stator current i_s^{ab} and the rotor flux Ψ_r^{ab} as state variables, the model in the natural reference frame is calculated. From (24),

$$i_r^{ab} = \mathbf{Lrr^{-1}}(2\nu\theta_r)\left\{\Psi_r^{ab} - L_{sr}\mathbf{U}(\nu\theta_r)i_s^{ab}\right\} \tag{26}$$

Replacing (26) in (22)

$$\dot{\Psi}_r^{ab} = \mathbf{R_r}\mathbf{Lrr^{-1}}(2\nu\theta_r)[L_{sr}\mathbf{U}(\nu\theta_r)i_s^{ab} - \Psi_r^{ab}] \tag{27}$$

To obtain $\dot{i}_s^{ab}$, replacing (26) in (24) and calculating

$$\dot{i}_s^{ab} = -\mathbf{P}_\sigma(\theta_r, w)i_s^{ab} - \mathbf{Q}_\sigma(\theta_r, w)\Psi_r^{ab} + U_s^{ab} \tag{28}$$

where:

$$\begin{aligned}
\mathbf{P}_\sigma(\theta_r, w) &= \mathbf{P}(\theta_r, w)/Det_\sigma\\
\mathbf{Q}_\sigma(\theta_r, w) &= \mathbf{Q}(\theta_r, w)/Det_\sigma\\
U_s^{ab} &= \sigma^{-\mathbf{1}}(\nu\theta_r)V_s^{ab}
\end{aligned}$$

Again, the electromechanical equation is given by (19), and the transformed torque equation is:

$$\begin{aligned}
T_e = &-\nu c_1 L_{sr}(i_s^{ab})^T\mathbf{J}\mathbf{I}^-\mathbf{U}(2\nu\theta_r)\cdot\\
&[2\Delta L_r\mathbf{U}(2\nu\theta_r) + \Delta L_s\mathbf{I}]i_s^{ab}\\
&-\nu\Delta L_r c_1 L_{sr}{}^{-1}(\Psi_r^{ab})^T\mathbf{J}\mathbf{I}^-\mathbf{U}(2\nu\theta_r)\Psi_r^{ab}\\
&+\nu c_1 (i_s^{ab})^T\cdot\\
&\left\{L_r\mathbf{U^T}(\nu\theta_r)\mathbf{J} + 2\Delta L_r\mathbf{J}\mathbf{I}^-\mathbf{U}(3\nu\theta_r)\right\}\Psi_r^{ab}\\
&+\nu c_1\Delta L_r(\Psi_r^{ab})^T\mathbf{J}\mathbf{I}^-\mathbf{U}(3\nu\theta_r)i_s^{ab}
\end{aligned} \tag{29}$$

B. Equivalence among models

If it is considered the case of the "clasical" **IM**, i.e, without "induced saliencies", the parameters ΔL_s and ΔL_r are adjusted to zero; in this particular situation it is possible to probe that the **IMIS** converge to the **IM** model doing the factors Δ_i in the **IMIS** equal to zero.

$$\begin{aligned}
\dot{i}_s^{ab}\mid_{\Delta L_s=\Delta L_r=0} &= -\gamma i_s^{ab} + \eta\{a\mathbf{I} - \nu w\mathbf{J}\}\mathbf{U^T}(\nu\theta_r)\Psi_r^{ab}\\
&\quad + \frac{V_s^{ab}}{\sigma_J L_s}\\
\dot{\Psi}_r^{ab}\mid_{\Delta L_s=\Delta L_r=0} &= -a\Psi_r^{ab} + b\mathbf{U}(\nu\theta_r)i_s^{ab}\\
T_e\mid_{\Delta L_s=\Delta L_r=0} &= \nu\tfrac{L_{sr}}{L_r}(i_s^{ab})^T\mathbf{U^T}(\nu\theta_r)\mathbf{J}\Psi_r^{ab}\\
\dot{\theta}_r &= w
\end{aligned}$$

Which is the classical ab model found in the induction motors literature[8].

VII. CONCLUSIONS

A new general model to the **IMIS** was developed which is computed in natural orthogonal components. The model is suitable for control design and compatible with the classical ab model for induction motors without saliencies. A similar model for control by signals injection at high frequencies was reported by Jansen et al.[4], but the model for the **IMIS** presented here is more general.

It is observed a complexity in the components of the currents by the presence of the Δ_i variations and their implicit dependence with the rotor position θ_r. Note that the rotor position dependence in the model can not be avoided by rotor variables transformations ($\alpha - \beta$ model).

Current work is to research the stability and observability properties of this model, analyze the induced harmonics in the torque and currents, among others.

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APPENDIX

Definitions and properties

$$\mathbf{U}(\cdot) = \begin{bmatrix} cos(\cdot) & sin(\cdot) \\ -sin(\cdot) & cos(\cdot) \end{bmatrix}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{I}^- = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad ; \quad \mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\mathbf{U^T}(\cdot) = \mathbf{U}(-(\cdot))$$

$$\mathbf{I}^-\mathbf{I}^- = \mathbf{I} \quad ; \quad \mathbf{JJ} = \mathbf{I}^- \quad ; \quad \mathbf{I}^- = \mathbf{I}^{-^{-1}}$$

$$\mathbf{JI}^- = -\mathbf{I}^-\mathbf{J} = \mathbf{I}^-\mathbf{J^T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

To the **IM**,

$$\begin{aligned} \sigma_J &\doteq 1 - \frac{L_{sr}^2}{L_s L_r} \\ a &\doteq R_r/L_r \\ \gamma &\doteq \frac{R_s}{L_s\sigma_J} + \frac{R_r}{L_r^2}\frac{L_{sr}^2}{L_s\sigma_J} \\ \eta &\doteq \frac{L_{sr}}{L_s L_r \sigma_J} \\ b &= aL_{sr} \end{aligned}$$

To the **IMIS**,

$$\begin{aligned} L_r &\equiv \tfrac{L_{rd}+L_{rq}}{2} \\ \Delta L_r &\equiv \tfrac{L_{rq}-L_{rd}}{2} \\ L_{rd} &\equiv L_r - \Delta L_r \\ L_{rq} &\equiv L_r + \Delta L_r \\ L_s &\equiv \tfrac{L_{sd}+L_{sq}}{2} \\ \Delta L_s &\equiv \tfrac{L_{sq}-L_{sd}}{2} \\ L_{sd} &\equiv L_s - \Delta L_s \\ L_{sq} &\equiv L_s + \Delta L_s \\ L_{rd}L_{rq} &= L_r^2 - L_{rq}^2 \\ L_{sd}L_{sq} &= L_s^2 - L_{sq}^2 \\ c_1 &= L_{sr}/(L_{rd}L_{rq}) \\ L_L &= L_s - c_1 L_{sr} L_r \end{aligned}$$

$$L_L = L_s - c_1 L_{sr} L_r = L_s - \frac{L_{sr}}{L_{rd}L_{rq}} L_{sr} L_r$$

$$\begin{aligned} P(\theta_r, w) = &\{R_s + c_1^2(L_r^2 + \Delta L_r^2)R_r\}\mathbf{I} \\ &+2L_r\Delta L_r \mathbf{U}^T(4\nu\theta_r)\mathbf{I}^- \\ &-2\nu w(\Delta L_s \mathbf{I} + 2c_1 L_{sr}\Delta L_r \mathbf{U}^T(2\nu\theta_r)) \\ &\cdot\mathbf{U}^T(2\nu\theta_r)\mathbf{JI}^- \end{aligned}$$

$$\begin{aligned} Q(\theta_r, w) = &-R_r c_1^2(L_{sr}^{-1})(L_r^2 + \Delta L_r^2)\mathbf{U^T}(\nu\theta_r) \\ &-2R_r c_1^2(L_{sr})^{-1} L_r \Delta L_r \mathbf{U^T}(3\nu\theta_r)\mathbf{I}^- \\ &+\nu w[c_1 L_r \mathbf{U^T}(\nu\theta_r)\mathbf{J} \\ &+3c_1\Delta L_r \mathbf{U^T}(3\nu\theta_r)\mathbf{JI}^-] \end{aligned}$$

$$\begin{aligned} \boldsymbol{\sigma}^{-1}(\nu\theta_r) = &\frac{1}{Det_\sigma}\Big\{L_L\mathbf{I} + \Delta L_s \mathbf{U^T}(2\nu\theta_r)\mathbf{I}^- \\ &+c_1 L_{sr}\Delta L_r \mathbf{U^T}(4\nu\theta_r)\mathbf{I}^-\Big\} \end{aligned}$$

$$\begin{aligned} Det_\sigma = &(L_L)^2 - \Delta L_s^2 - c_1^2 L_{sr}^2 \Delta L_r^2 \\ &-2c_1 L_{sr}\Delta L_s \Delta L_r cos(2\nu\theta_r) \end{aligned}$$

$$\mathbf{Lrr^{-1}}(2\nu\theta_r) = \frac{1}{L_{rd}L_{rq}}(L_r\mathbf{I} + \Delta L_r \mathbf{U^T}(2\nu\theta_r)\mathbf{I}^-)$$