POWER SYSTEMS DIRECT TRANSIENT STABILITY ASSESSMENT USING STRUCTURE PRESERVING MODELS: AN OVERVIEW

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ABSTRACT

This article is a brief overview of some of the research work in the past few years on Transient Stability assessment of Power Systems by the so-called *second method of Lyapunov. The paper concentrates specifically on the structure preserving models, which have been the basis for further improvements on system stability analysis using Lyapunov Functions—also known as Transient Energy Functions (TEF)—to obtain reasonable estimates of the region of attraction for the post-fault stable equilibrium point. The most significant achievements are briefly discussed and a small five-bus Power System is used as an example for the application of these techniques. A critique of this approach and some research ideas are presented.*

1.- Introduction

The Transient Stability analysis of Power Systems has been usually performed by solving a set of nonlinear differential equations using diverse numerical integration techniques, e.g. the trapezoidal rule. The main problem with this approach is that due to the complexity and size of these systems, it takes rather large amounts of CPU time to simulate the equations and the different faults that could occur in the circuit. This is a major drawback for on-line stability assessment and the main reason why direct Lyapunov analysis is so appealing to the Power System Engineers.

The classical stability analysis has considered the loads to be constant impedances, allowing their inclusion in the admittance matrix to speed up the computation of the solution trajectories. Some other load models—that have proved more accurate in predicting the stability of the real system—have also been used during the years [1], allowing most of them the reduction of the system to generator buses. Although the structure of the system is lost in the process, there is a significant reduction in CPU time.

Initially, constant impedance load models were used to analyze the stability of the system by direct Lyapunov methods [2], but in order to apply classical Lyapunov stability theory the system had to be assumed lossless to avoid having path dependent integrals in the TEF. From the point of view of the transmission lines and generators this is a reasonable assumption; however, this is unjustifiable for the loads due to their large resistance. Several researchers have continued using these reduced system models for direct Transient Stability assessment [3], avoiding approximations by numerically calculating the path dependent integrals. The problem with most of these techniques is the lack of formal theoretical justification, using only engineering criteria to validate the procedure, i.e. how good the predictions of clearing times are as compared to time simulations.

Bergen and Hill in [4] and [5] present a new approach for modelling the load, making possible a thorough Lyapunov stability analysis. Although this model is by no means complete, because does not consider the reactive part of the load that accounts for voltage variations, it facilitates the theoretical analysis of the power system. Narasimhamurthi and Musavi [6] introduce the reactive part of the load into the system model by considering the reactive power flow equations as a set of nonlinear constraints for the state space differential equations. Tsolas et. al. [7] improve the technique by presenting a new TEF to account for the field variations of generators during transients.

The nonlinear constraints raise the problem of having to deal with an ill-posed or degenerate system, as described by DeMarco in [8], due to the singularity of the Jacobian of the power flow equations close to voltage collapse. DeMarco in his work applies singular perturbation techniques to overcome this difficulty.

This paper describes in detail the basic principles used in direct Transient Stability assessment with structure preserving models, and proves the main theorems behind the Bergen-Hill model.

2.- General Procedure

The Transient Stability analysis in Power Systems consist in solving three sets of differential equations: pre-fault, fault, and post-fault. The pre-fault study is usually done by phasor analysis, i.e. obtain a reasonable stable solution for the nonlinear power flow equations, since under certain approximations the system can be considered to be in quasi-steady state. This solution is then used as the initial condition of the system state variables $\mathbf{x}_F(0)$ for the fault simulation.

The fault analysis, which accounts for the behavior of the system before the fault is cleared by switching operation, is performed by solving a set of nonlinear differential equations that neglect the fast transients in the transmission lines treating them as constant impedances—lumped parameters. The state variables $\mathbf{x}_F(t)$ in this case come from the generators and loads, depending on how the system is modelled. The duration of this simulation does not take long due to the usually fast response of the system breakers, and its is definitely a completely unstable condition for the system, assuming that the fault being analyzed is considered "large", e.g. a three phase fault in an important transmission line. This simulation produces a set of trajectories for $\mathbf{x}_F(t)$ and, depending on the clearing time t_c, yields the new state of the system $\mathbf{x}_F(t_c)$ used as initial condition for the final analysis.

The post-fault study is where the classical and direct methods differ. The classical Transient Stability consists on solving a new set of nonlinear differential equations corresponding to the new structure of the system, which was changed due to switching. By looking at the trajectories, we can conclude whether the system is asymptotically stable over a sufficiently long time interval, or not. On the other hand, the direct methods try to answer whether the initial state of the new system $\mathbf{x}_F(t_c)$ is inside the stability region of the stable equilibrium point $\mathbf{x}_s$. The latter can be resolved using Lyapunov's theory, which basically consists in the following:

a) Find the stable equilibrium point (s.e.p.) $\mathbf{x}_s$ of the post-fault system by solving a set of nonlinear power flow equations. This equilibrium point is not always unique, but we are only interested in the high voltage solution.

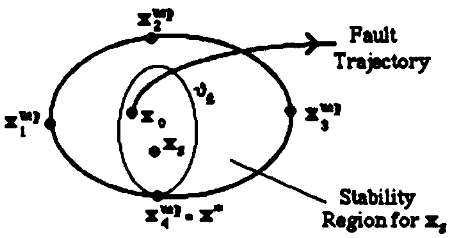

Figure 1: Region of attraction of the s.e.p. $\mathbf{x}_s$.

b) Find the unstable equilibrium points (u.e.p.), and consider that a set of u.e.p.'s lie on the boundary of the stability region $A(\mathbf{x}_s)$ for $\mathbf{x}_s$, namely $\partial A(\mathbf{x}_s)$. This assumption is regarded by some authors as a "folk theorem" [9], but it seems to be reasonable if one thinks of the stability region as an energy valley with the s.e.p. in the lowest point, and the contour as formed by the u.e.p.'s or energy peaks surrounding it (figure 1). To find all the u.e.p.'s one could solve several times the power flow equations, an enormous task specially in large systems due to the vast number of unstable solutions [17]. Because of this problem the u.e.p.'s are usually approximated, based on the observation that transient instability occurs when a single generator accelerates or separates from the rest of the system. In the infinite bus-generator model this approximation can be expressed as

$$\delta_i^{unst.} = \pi - \delta_i^{stab.} \quad \forall\, i \in \text{Generators}$$

For large systems this is inaccurate; other techniques have been devised for this case.

c) Define a locally positive definite energy function $\vartheta(\mathbf{x})$ in $A(\mathbf{x}_s)$, with a locally negative definite derivative of time $\dot{\vartheta}(\mathbf{x})$. This function is used to evaluate the energy of the u.e.p. $\mathbf{x}^*$ closest to $\mathbf{x}_s$, i.e.

$$\vartheta_\ell = \vartheta(\mathbf{x}^*)$$

This is a conservative estimate of the stability region, as it can be seen in figure 1. Some authors have suggested a different technique that consists on estimating the point—the controlling u.e.p.—where the fault trajectory leaves the region of attraction $A(\mathbf{x}_s)$. In this paper we are just going to present the Bergen-Hill technique to approximate ϑ_ℓ, which avoids having to calculate the u.e.p.'s.

3.- State Space Model

The state space description of the model is based on the following assumptions:

- Generators (m): Constant voltage source (1p.u.) behind the transient reactance. This reactance is treated as part of the Transmission System. The mechanical system is simulated by Newton's equation (1).

$$M_g \ddot{\delta}_g + D_g \dot{\delta}_g = P_M^0 \qquad (1)$$

- Transmission System (l+m): Pure reactance (lossless) as shown in (2). Contains the m generator reactances and the l original transmission lines.

$$P_{rs} = b_{rs} \sin(\delta_r - \delta_s) \qquad (2)$$

- Loads (n_0): Frequency dependent active load (3), with no voltage dependance. The system is assumed to have loads on *every* bus.

$$P_L = P_L^0 + D_L \dot{\delta}_L \qquad (3)$$

The total number of buses of the system is $n = m + n_0$, and the last bus angle is used as a reference ($\delta_n = 0$). In [4] the buses are numbered so that the generators are first ($i = 1, 2, \ldots, m$) and then the loads ($i = m+1, m+2, \ldots, n$), while in [5] the loads are listed first ($i = 1, 2, \ldots, n-m$) and then the generators ($i = n-m+1, n-m+2, \ldots, n$). This change of reference is of no significant effect for the theorems proven later.

Consequently, the post-fault system model for $i = 1, 2, \ldots, n$ is:

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i + \sum_{\substack{j=1 \\ i \neq j}}^{n} b_{ij} \sin(\delta_i - \delta_j) = P_{Mi}^0 - P_{Di}^0 = P_i^0 \qquad (4)$$

where:

$M_i = 0 \quad i \in \text{Loads}$
$M_i > 0 \quad i \in \text{Generators}$
$D_i > 0 \quad \forall\, i$
$P_{Mi}^0 = 0 \quad i \in \text{Loads}$
$P_{Di}^0 = 0 \quad i \in \text{Generators}$

Considering that $\dot{\delta}_i = \omega_i$, and adding (4) for all i we obtain

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} b_{ij} \sin(\delta_i - \delta_j) = 0 \quad \text{(lossless)}$$

$$\sum_{i=1}^{n} M_i \dot{\omega}_i + \sum_{i=1}^{n} D_i \omega_i = \sum_{i=1}^{n} P_i^0 \qquad (5)$$

Depending on the fault and clearing strategies, sometimes the power balanced is lost, making the post-fault equilibrium point of (5) different from zero. In order to force this equilibrium to zero, i.e. $\omega'^0 = 0$, the following transformation has to be carried out, based on the original equilibrium point:

$$\omega^0 = \sum_{i=1}^{n} P_i^0 \Big/ \sum_{i=1}^{n} D_i$$

$$\Rightarrow \quad \omega_i' := \omega_i - \omega^0 \qquad P_i' := P_i - D_i \omega^0$$

Hereafter, the prime superscripts are dropped to simplify the notation.

Equations (4) can be transformed into the classical state space representation by defining the internodal angles α_i:

$$\alpha_i := \delta_i - \delta_n \quad \Rightarrow \quad \alpha = T\delta = [I_{n-1} \mid -e_{n-1}] \begin{bmatrix} \delta_1 \\ \dots \\ \delta_{n-1} \\ -- \\ \delta_n \end{bmatrix}$$

Notice that T has full row rank. Now choose a partition of T so that the augmented system bus angles are divided in generator (m) and load $(n_0=n-m)$ angles,

$$\alpha = [T_g \mid T_L] \begin{bmatrix} \delta_g \\ -- \\ \delta_L \end{bmatrix} \; ([4]) \qquad \alpha = [T_L \mid T_g] \begin{bmatrix} \delta_L \\ -- \\ \delta_g \end{bmatrix} \; ([5])$$

$$\dot{\alpha} = T_L \, \omega_L + T_g \, \omega_g \tag{6}$$

The vector of line angle differences σ $(l+m)$ can be define in terms of α by means of the Bus incidence matrix A [11], and also in terms of the tree branch angles θ $(n-1)$ of a system cutset by using the Basic Cutset incidence matrix Q [11], as shown in (7). These two matrices are full row rank by definition.

$$\sigma = A^T \alpha \tag{7a}$$

$$\sigma = Q^T \theta \tag{7b}$$

The line angles σ_i are identical to the branch angles θ_i, if the line i is contained in the cutset C_s, i.e. $\sigma_i = \theta_i \; \forall i \in C_s$.

Equation (4) in matrix form is

$$\begin{bmatrix} M_g & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\omega}_g \\ \dot{\omega}_L \end{bmatrix} + \begin{bmatrix} D_g & 0 \\ 0 & D_L \end{bmatrix} \begin{bmatrix} \omega_g \\ \omega_L \end{bmatrix} + \begin{bmatrix} T_g^T \\ T_L^T \end{bmatrix} [f(\alpha) - P^0] \tag{8}$$

Where M_g, D_g, and D_L are obviously positive definite matrices, and $f(\alpha)$ represents the power flow equations.

$$P^0 = \begin{bmatrix} P_M^0 \\ P_D^0 \end{bmatrix}$$

$$f(\alpha) = \underset{i=1..n-1}{\text{vector}} \left\{ b_{in} \sin(\alpha_i) + \sum_{\substack{k=1 \\ k \neq i}}^{n-1} b_{ik} \sin(\alpha_i - \alpha_k) \right\} \tag{9}$$

Using (6), equation (8) can be transformed into the state space representation of the system (10).

$$\dot{\alpha} = T_g \omega_g - T_L D_L^{-1} T_L^T [f(\alpha) - P^0] \tag{10a}$$

$$\dot{\omega}_g = -M_g^{-1} D_g \omega_g - M_g^{-1} T_g^T [f(\alpha) - P^0] \tag{10b}$$

Notice that the equilibrium points $(0, \alpha^e)$ are the solutions to the power flow problem, i.e.

$$f(\alpha^e) = P^0$$

4.- Stable and Unstable Equilibrium (Linear Model)

To determine the stable and unstable equilibrium points, and the regions where they lie, one can linearize the system and either find the eigenvalues of the state matrix or define an energy function to apply standard Lyapunov stability theory [10].

Bergen and Hill utilize the energy function to prove asymptotic stability and complete instability. Their studies rigorously prove, as shown below, that the s.e.p.'s of the simplified power system lie in the polytope

$$\Omega = \{\alpha \in \mathcal{R}^{n-1} \mid \sigma = A^T\alpha, \; |\sigma_k| < \frac{\pi}{2} \; \forall k = 1, .., l\},$$

and the u.e.p.'s are contained in the polytope

$$\Lambda = \{\alpha \in \mathcal{R}^{n-1} \mid \sigma = A^T\alpha, \frac{\pi}{2} < |\sigma_k| < \frac{3\pi}{2} \; \forall k \in C_s\},$$

where C_s is a set of lines that form part of a fundamental cutset of the augmented system.

Linearizing the state space equations (10) around the equilibrium point $(0, \alpha^e)$, i.e. $\Delta\alpha = \alpha - \alpha^e$, $\Delta\omega_g = \omega_g - \omega_g^e = \omega_g$, we obtain

$$\Delta\dot{\alpha} = T_g \omega_g - T_L D_L^{-1} T_L^T F(\alpha^e) \Delta\alpha \tag{11a}$$

$$\dot{\omega}_g = -M_g^{-1} D_g \omega_g - M_g^{-1} T_g^T F(\alpha^e) \Delta\alpha \tag{11b}$$

Where $F(\alpha^e)$ is the symmetric Jacobian of the the power flow equations evaluated at the equilibrium point α^e,

$$F(\alpha) = \left[\frac{\partial f_i}{\partial \alpha_j}(\alpha) \right]_{(n-1) \times (n-1)}$$

$$\frac{\partial f_i}{\partial \alpha_j}(\alpha) = \begin{cases} b_{in} \cos(\alpha_i) + \sum\limits_{\substack{k=1 \\ k \neq j}}^{n-1} b_{ik} \cos(\alpha_i - \alpha_k) & i = j \\ -b_{ij} \cos(\alpha_i - \alpha_j) & i \neq j \end{cases}$$

This is equivalent to

$$F(\alpha) = A \, G(A^T\alpha) \, A^T = A \, G(\sigma) \, A^T$$

$$G(\sigma) = \underset{k=1..l}{\text{diag}} \{b_k \cos(\sigma_k)\} = \underset{k=1..l}{\text{diag}} \{g_k(\sigma_k)\} = \text{diag}\{g(\sigma)\}$$

Notice that, as long as $\sigma_k^e \neq \frac{\pi}{2} \forall k$, the matrix $F(\alpha^e)$ is full rank, since it is diagonal dominant and none of the rows is identically zero ($b_k > 0$, $\cos(\sigma_k^e) \neq 0$)

If $\alpha^e \in \Omega$, the matrix $G(\sigma^e)$ is positive definite, because all terms in the diagonal are positive ($b_k > 0$, $\cos(\sigma_k^e) > 0$). On the other hand, A^T is full column rank; therefore, $F(\alpha^e)$ is positive definite.

$$y^T F(\alpha^e) y = y^T A G(\sigma^e) A^T y = (A^T y)^T G(\sigma^e) A^T y = v^T G(\sigma^e) v > 0$$

For $\alpha^e \in \Lambda$, the matrix $G(\sigma^e)$ is negative definite for a proper choice of α^e ($\sigma_k^e = 0 \; \forall k \notin C_s$, $\sigma_k^e \neq 0 \; \forall k \in C_s$), since all terms in the diagonal are negative or zero ($b_k > 0$, $\cos(\sigma_k^e) \leq 0$). Hence, $F(\alpha^e)$ is negative definite for this choice of α^e.

$$y^T F(\alpha^e) y = y^T A G(\sigma^e) A^T y = (A^T y)^T G(\sigma^e) A^T y = v^T G(\sigma^e) v < 0$$

Based on these observations we can now state and prove the following theorems:

Theorem 1.- If $\alpha^e \in \Omega$, the equilibrium point $(0, \alpha^e)$ is asymptotically stable.

Proof: Define the Lyapunov function:

$$\vartheta(\omega_g, \Delta\alpha) := \vartheta(\Delta x) = \frac{1}{2}\omega_g^T M_g \omega_g + \frac{1}{2}\Delta\alpha^T F(\alpha^e)\Delta\alpha \quad (12)$$

Clearly this function is positive definite, since $M_{g_i} > 0 \; \forall i$ and $F(\alpha^e)$ positive definite $\forall \alpha^e \in \Omega$. On the other hand,

$$\dot{\vartheta}(\omega_g, \Delta\alpha) = \dot{\vartheta}(\Delta x) = \left[\frac{\partial\vartheta}{\partial \Delta x}\right]\Delta\dot{x}$$

$$= \Delta\alpha^T F(\alpha^e) T_g \omega_g - \Delta\alpha^T F(\alpha^e) T_L D_L^{-1} T_L^T F(\alpha^e)\Delta\alpha$$
$$\omega_g^T D_g \omega_g - \omega_g^T T_g^T F(\alpha^e)\Delta\alpha$$
$$= -\Delta\alpha^T F(\alpha^e) T_L D_L^{-1} T_L^T F(\alpha^e)\Delta\alpha - \omega_g^T D_g \omega_g$$
$$= -(T_L^T F(\alpha^e)\Delta\alpha)^T D_L^{-1} T_L^T F(\alpha^e)\Delta\alpha - \omega_g^T D_g \omega_g$$
$$= -z^T D_L^{-1} z - \omega_g^T D_g \omega_g$$

Observe that $D_i > 0 \;\; \forall i$; therefore, D_L^{-1} and D_g are positive definite. Then,

$$-\dot{\vartheta}(\omega_g, \Delta\alpha) \geq 0 \qquad \forall \Delta\alpha, \omega_g$$

Furthermore, $\dot{\vartheta}(\omega_g, \Delta\alpha) = 0$ and $\dot{\omega}_g = 0$ imply $\omega_g = 0$ and

$$\begin{matrix} T_L^T F(\alpha^e)\Delta\alpha = 0 \\ T_g^T F(\alpha^e)\Delta\alpha = 0 \end{matrix} \Rightarrow T^T F(\alpha^e)\Delta\alpha = 0$$

$$\Rightarrow \quad \Delta\alpha = 0 \qquad (T^T,\; F(\alpha^e) \text{ full rank})$$

Hence, by LaSalle's theorem [10] the equilibrium point is asymptotically stable. ∎

Theorem 2.- If $\alpha^e \in \Lambda$, the equilibrium point $(0, \alpha^e)$ is completely unstable.

Proof: Choose the Lyapunov function to be:

$$\vartheta'(\omega_g, \Delta\alpha) = -\frac{1}{2}\omega_g^T M_g \omega_g - \frac{1}{2}\Delta\alpha^T F(\alpha^e)\Delta\alpha$$

Since $F(\alpha^e)$ is negative definite for proper values of the sate variables arbitrarily close to the equilibrium, i.e. $(0, \varepsilon)$, the energy function $\vartheta'(\omega_g, \Delta\alpha)$ takes positive values. On the other hand,

$$\dot{\vartheta}'(\omega_g, \Delta\alpha) = (T_L^T F(\alpha^e)\Delta\alpha)^T D_L^{-1} T_L^T F(\alpha^e)\Delta\alpha + \omega_g^T D_g \omega_g$$
$$= z^T D_L^{-1} z + \omega_g^T D_g \omega_g$$

Where D_L^{-1} and D_g are positive definite. Then,

$$\dot{\vartheta}'(\omega_g, \Delta\alpha) \geq 0 \qquad \forall \Delta\alpha, \omega_g$$

Finally, $\dot{\vartheta}'(\omega_g, \Delta\alpha) = 0$ and $\dot{\omega}_g = 0$ imply $\omega_g = 0$ and

$$\begin{matrix} T_L^T F(\alpha^e)\Delta\alpha = 0 \\ T_g^T F(\alpha^e)\Delta\alpha = 0 \end{matrix} \Rightarrow T^T F(\alpha^e)\Delta\alpha = 0$$

$$\Rightarrow \quad \Delta\alpha = 0 \qquad (T^T,\; F(\alpha^e) \text{ full rank})$$

Therefore, by the first instability theorem [10] the equilibrium point is completely unstable. ∎

5.- A General Lyapunov Function

The state space representation of the augmented system can be rewritten in the multivariable Luré form shown in equation (13). The transfer function for the linear portion is shown in (14).

$$\dot{x} = Ax - B\phi(y) \qquad (13)$$
$$y = C^T x$$

$$G(s) = C^T(sI - A)^{-1}B \qquad (14)$$

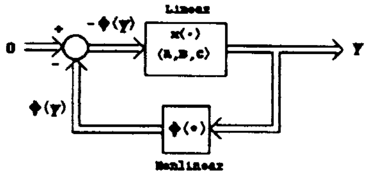

Figure 2: Multivariable Luré Form

Rewriting (10) in matrix form we have that

$$\begin{bmatrix}\dot{\omega}_g \\ \dot{\alpha}\end{bmatrix} = \begin{bmatrix}-M_g^{-1}D_g & 0 \\ T_g & 0\end{bmatrix}\begin{bmatrix}\omega_g \\ \alpha\end{bmatrix} - \begin{bmatrix}M_g^{-1}T_g^T \\ T_L D_L^{-1} T_L^T\end{bmatrix}[f(\alpha) - P^0]$$

$$\alpha = [0 \;\; I_{n-1}]\begin{bmatrix}\omega_g \\ \alpha\end{bmatrix}$$

Which give us

$$A = \begin{bmatrix}-M_g^{-1}D_g & 0 \\ T_g & 0\end{bmatrix} \qquad B = \begin{bmatrix}M_g^{-1}T_g^T \\ T_L D_L^{-1} T_L^T\end{bmatrix} \qquad C = \begin{bmatrix}0 \\ I_{n-1}\end{bmatrix}$$

$$\phi(\alpha) = f(\alpha) - P^0$$

Then it follows from (14) that

$$G(s) = [0 \;\; I_{n-1}]\begin{bmatrix}sI_m + M_g^{-1}D_g & 0 \\ -T_g & sI_{n-1}\end{bmatrix}^{-1}\begin{bmatrix}M_g^{-1}T_g^T \\ T_L D_L^{-1} T_L^T\end{bmatrix}$$

$$G(s) = \frac{1}{s}[T_g(sI_m + M_g^{-1}D_g)^{-1}M_g^{-1}T_g^T + T_L D_L^{-1} T_L^T] \qquad (15)$$

Now we can use Anderson's theorem [12], which is a multivariable version of the Kalman-Yacubovitch lemma stated in [10]. This theorem starts by defining the transfer function

$$Z(s) := (p + qs)G(s),$$

where $p \geq 0$, $q > 0$, and $s = -p/q$ is not a pole of $G(s)$. If (A, B, C) is a minimal state space representation of $G(s)$, and $Z(s)$ is positive real, there exist real matrices P, Q, and W, with P positive definite symmetric, such that

$$PA + A^T P = -QQ^T \qquad (16a)$$
$$PB = pC + qA^T C - QW^T \qquad (16b)$$
$$W^T W = q(C^T B + B^T C) \qquad (16c)$$

From (15),

$$Z(s) = \frac{p+qs}{s}[T_g(sI_m + M_g^{-1}D_g)^{-1}M_g^{-1}T_g^T + T_L D_L^{-1} T_L^T]$$

Where $Z(s)$ positive real if $\mathcal{Re}\{Z(j\omega)\} > 0$. A sufficient condition for a positive real $Z(s)$ is then:

$$q \geq p \, \frac{M_{gi}}{D_{gi}} \qquad \forall \; i = 1,2,..,m$$

Hence, p=0 and q>0 guarantees $\mathcal{Re}\{\mathbf{Z}(j\omega)\}>0$.

With this assumption, and replacing $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ in (16), it can be shown that the following matrix equations hold:

$$\mathbf{P}=\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2^T \\ \mathbf{P}_2 & \mathbf{P}_3 \end{bmatrix} \qquad \mathbf{Q}=\begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix} \qquad \mathbf{W}=\begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{bmatrix}$$

$$(16a) \Rightarrow \qquad \mathbf{Q}_{21}=\mathbf{0} \quad \mathbf{Q}_{22}=\mathbf{0}$$

$$\mathbf{P}_2\mathbf{M}_g^{-1}\mathbf{D}_g=\mathbf{P}_3\mathbf{T}_g \qquad (17)$$

$$\mathbf{P}_1\mathbf{M}_g^{-1}\mathbf{D}_g+\mathbf{D}_g\mathbf{M}_g^{-1}\mathbf{P}_1-\mathbf{P}_2^T\mathbf{T}_g-\mathbf{T}_g^T\mathbf{P}_2=\mathbf{Q}_{11}\mathbf{Q}_{11}^T+\mathbf{Q}_{12}\mathbf{Q}_{12}^T$$

$$(16c) \Rightarrow \quad \mathbf{W}_1^T\mathbf{W}_1+\mathbf{W}_2^T\mathbf{W}_2=2q\mathbf{T}_L\mathbf{D}_L^{-1}\mathbf{T}_L^T$$

$$\Rightarrow \quad \mathbf{W}_1=\sqrt{2q}\mathbf{D}_L^{-1/2}\mathbf{T}_L^T, \quad \mathbf{W}_2=\mathbf{0}$$

$$\Rightarrow \quad \mathbf{QW} = \begin{bmatrix} \sqrt{2q}\mathbf{Q}_{11}\mathbf{D}_L^{-1/2}\mathbf{T}_L^T \\ \mathbf{0} \end{bmatrix} \qquad (18)$$

Equations (16b) and (18) yield

$$\mathbf{P}_1\mathbf{M}_g^{-1}\mathbf{T}_g+\mathbf{P}_2^T\mathbf{T}_L\mathbf{D}_L^{-1}\mathbf{T}_L^T=q\mathbf{T}_g^T-\sqrt{2q}\mathbf{Q}_{11}\mathbf{D}_L^{-1/2}\mathbf{T}_L^T \qquad (19)$$

$$\mathbf{P}_2\mathbf{M}_g^{-1}\mathbf{T}_g + \mathbf{P}_1^T\mathbf{T}_L\mathbf{D}_L^{-1}\mathbf{T}_L^T = \mathbf{0}$$

From (17) and (19), and with $\mathbf{T}$ full rank, it can be proved that $\mathbf{P}_2=\mathbf{0}$ and $\mathbf{P}_3=\mathbf{0}$, and

$$\mathbf{P}_1\mathbf{M}_g^{-1}\mathbf{D}_g+\mathbf{D}_g\mathbf{M}_g^{-1}\mathbf{P}_1=\mathbf{Q}_{11}\mathbf{Q}_{11}^T+\mathbf{Q}_{12}\mathbf{Q}_{12}^T \qquad (20a)$$

$$\mathbf{P}_1\mathbf{M}_g^{-1}\mathbf{T}_g^T=q\mathbf{T}_g^T-\sqrt{2q}\mathbf{Q}_{11}\mathbf{D}_L^{-1/2}\mathbf{T}_L^T \qquad (20b)$$

Now, replacing $\mathbf{T}_g$ and $\mathbf{T}_L$ by their original form in (20b) we have

$$\mathbf{P}_1\mathbf{M}_g^{-1}\left[\begin{array}{c|c} 0 & \mathbf{I}_{m-1} \\ -\bullet_{n-m}^T & -\bullet_{m-1}^T \end{array}\right] = q\left[\begin{array}{c|c} 0 & \mathbf{I}_{m-1} \\ -\bullet_{n-m}^T & -\bullet_{m-1}^T \end{array}\right] - \sqrt{2q}\mathbf{Q}_{11}\mathbf{D}_L^{-1/2}\left[\mathbf{I}_{n-m} \mid 0\right]$$

Which is equivalent to

$$\mathbf{P}_1\mathbf{M}_g^{-1}\begin{bmatrix} 0 \\ -\bullet_{n-m}^T \end{bmatrix} = q\begin{bmatrix} 0 \\ -\bullet_{n-m}^T \end{bmatrix} - \sqrt{2q}\mathbf{Q}_{11}\mathbf{D}_L^{-1/2}$$

$$\mathbf{P}_1\mathbf{M}_g^{-1}\begin{bmatrix} \mathbf{I}_{m-1} \\ -\bullet_{m-1}^T \end{bmatrix} = q\begin{bmatrix} \mathbf{I}_{m-1} \\ -\bullet_{m-1}^T \end{bmatrix} \qquad (21)$$

One possible solution for (21) is

$$\mathbf{P}_1 = q\mathbf{M}_g + \mu\mathbf{M}_g\mathbf{1}_{mm}\mathbf{M}_g \qquad (22)$$

Where μ is a scalar constant. This m by m symmetric matrix is positive definite if μ is chosen so that $\mathbf{P}_1$ is diagonal dominant, i.e.

$$0 \geq \mu > \mu_0 = \frac{-q}{\sum_{i=1}^{m} M_{gi}}$$

Finally, the Lyapunov function for system (13) is a multivariable version of the one used in Popov's criterion [10], i.e.

$$\vartheta(\mathbf{x}) = \frac{1}{2}\,\mathbf{x}^T\mathbf{P}\mathbf{x} + q\int_0^{\mathbf{y}=\mathbf{C}^T\mathbf{x}} \phi(\xi)^T d\xi \qquad (23)$$

With the following time derivative

$$\dot{\vartheta}(\mathbf{x}) = \left[\frac{\partial\vartheta}{\partial\mathbf{x}}\right]\dot{\mathbf{x}} = \mathbf{x}^T\mathbf{P}\dot{\mathbf{x}} + q\phi^T\mathbf{C}^T\dot{\mathbf{x}}$$

$$= -\frac{1}{2}(\mathbf{x}^T\mathbf{Q}-\phi^T\mathbf{W}^T)(\mathbf{Q}^T\mathbf{x}-\mathbf{W}\phi)-p\mathbf{x}^T\mathbf{C}\phi \qquad (24)$$

From equations (23) and (24), and assuming that $\phi(\mathbf{y})$ lies in the sector $(0,\infty)$ and $\phi(\mathbf{0})=\mathbf{0}$ and $\mathbf{Z}(s)$ positive real, the equilibrium point of (13) is asymptotically stable. This comes directly from the observations that $\vartheta(\mathbf{x})$ is positive definite, $\dot{\vartheta}(\mathbf{x})\leq 0$, and $\dot{\vartheta}(\mathbf{x})=0$ and $\dot{\mathbf{x}}=\mathbf{0}$ iff $\mathbf{x}=\mathbf{0}$, under the previous assumptions.

For the specific case of the the power system, the energy function (23) becomes:

$$\vartheta(\omega_g,\alpha) = \frac{q}{2}\,\omega_g^T\mathbf{M}_g\omega_g + \frac{\mu}{2}\,\omega_g^T\mathbf{M}_g\mathbf{1}_{mm}\mathbf{M}_g\omega_g + q\int_{\alpha^s}^{\alpha}[\mathbf{f}(\xi)-\mathbf{P}^0]^T d\xi \qquad (25)$$

If this TEF is evaluated at the point where the fault has been cleared, and it lies inside the stability region, i.e. $\vartheta(\omega_{g_F}(t_C),\alpha_F(t_C))<\vartheta_\ell$, the system is asymptotically stable. Hence, a good estimate of the region of attraction is a requirement to adequately assess the post-fault stability of the system.

6.- Stability Region Estimate

One of the key ideas in direct transient stability assessment of power systems is to find a good estimate of the region of attraction for the post-fault s.e.p. Bergen and Hill [4] devised a technique that avoids having to calculate the post-fault u.e.p.'s in order to find the minimum ϑ_ℓ of the TEF. They use an special case of the Lyapunov function (25), with q=1 and μ=0.

$$\vartheta(\omega_g,\alpha) = \frac{1}{2}\,\omega_g^T\mathbf{M}_g\omega_g + \int_{\alpha^s}^{\alpha}[\mathbf{f}(\xi) - \mathbf{P}^0]^T d\xi$$

$$\vartheta(\omega_g,\alpha) = \frac{1}{2}\,\omega_g^T\mathbf{M}_g\omega_g + W(\alpha,\alpha^s) \qquad (26)$$

Where $W(\alpha,\alpha^s)$ can be evaluated in close form by assuming symmetry of the power flow Jacobian $\mathbf{F}(\alpha)$, which guarantees a path independent integral.

$$W(\alpha,\alpha^s) = \int_{\alpha^s}^{\alpha}[\mathbf{f}(\xi)-\mathbf{P}^0]^T d\xi$$

$$= \int_{\alpha^s}^{\alpha}[\mathbf{f}(\xi)-\mathbf{f}(\alpha^s)]^T d\xi$$

$$= \int_{\alpha^s}^{\alpha}[\mathbf{g}(\mathbf{A}^T\xi)-\mathbf{g}(\mathbf{A}^T\alpha^s)]^T\mathbf{A}^T d\xi$$

$$= \int_{\sigma^s}^{\sigma}[\mathbf{g}(\mathbf{u})-\mathbf{g}(\sigma^s)]^T d\mathbf{u}$$

$$= \sum_{k=1}^{l} b_k \int_{\sigma_k^s}^{\sigma_k} [\sin(u) - \sin(\sigma_k^s)]\,du$$

$$= \sum_{k=1}^{l} b_k \, h(\sigma_k, \sigma_k^s)$$

$$\Rightarrow h(\sigma_k, \sigma_k^s) = \cos(\sigma_k^s) - \cos(\sigma_k) + (\sigma_k^s - \sigma_k)\sin(\sigma_k^s)$$

Figure 3 shows different plots of $h(\sigma_k, \sigma_k^s)$ for $\sigma \in \Gamma^\ell(\sigma^s) = \{\sigma \in \Re^l \mid \sigma_k \in (\sigma_k^l, \sigma_k^u), \sigma_k^l = -\pi - \sigma_k^s, \sigma_k^u = \pi - \sigma_k^s, \forall k = 1, \ldots, l\}$

Observe that the energy function (12) used to prove the stability or instability of the equilibrium points in the linearize model, is just a "linear" version of (26).

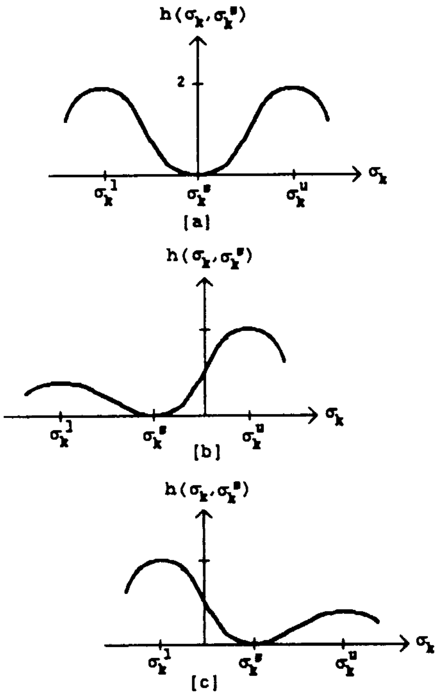

Figure 3: $h(\sigma_k, \sigma_k^s)$ for [a] $\sigma_k^s = 0$, [b] $-\pi/2 < \sigma_k^s < 0$, and [c] $0 > \sigma_k^s > \pi/2$

Now we can prove that $\Gamma^\ell(\sigma^s)$ is the stability region of the post-fault stable equilibrium point $(0, \alpha^s)$.

Theorem 3.- The equilibrium point $(0, \alpha^s)$ is asymptotically stable in the polytope $\Gamma^\ell(\mathbf{A}^T\alpha^s)$. Furthermore, all the u.e.p.'s lie on the closure $\partial\Gamma^\ell(\mathbf{A}^T\alpha^s)$

Proof: This is based on LaSalle's theorem [10]. Observe that (26) is positive definite in $\Gamma^\ell(\mathbf{A}^T\alpha^s)$, since $\mathbf{M}_g$ is positive definite and every $h(\sigma_k, \sigma_k^s)$ is positive definite in the polytope $\Gamma^\ell(\sigma^s)$. On the other hand,

$$\dot{\vartheta}(\omega_g, \alpha) = \left[\frac{\partial\vartheta}{\partial\omega_g} \quad \frac{\partial\vartheta}{\partial\alpha}\right] \begin{bmatrix} \dot{\omega}_g \\ \dot{\alpha} \end{bmatrix}$$

$$= -\omega_g^T D_g \omega_g - [\mathbf{f}(\alpha) - \mathbf{P}^0]^T \mathbf{T}_L D_L^{-1} \mathbf{T}_L^T [\mathbf{f}(\alpha) - \mathbf{P}^0] \leq 0$$

Moreover, $\dot{\vartheta}(\omega_g, \alpha) = 0$ implies

$$\begin{cases} \text{(i)} \ \omega_g = 0 \\ \text{(ii)} \ \mathbf{T}_L^T[\mathbf{f}(\alpha) - \mathbf{P}^0] = 0; \ \dot{\omega}_g = 0 \\ \quad \Rightarrow \mathbf{T}_g^T[\mathbf{f}(\alpha) - \mathbf{P}^0] = 0 \quad \text{from (10)} \\ \quad \Rightarrow \mathbf{T}^T[\mathbf{f}(\alpha) - \mathbf{P}^0] = 0 \quad \mathbf{T}^T \text{ full col.rank} \\ \quad \Rightarrow \mathbf{f}(\alpha) = \mathbf{P}^0 \Rightarrow \alpha = \alpha^e \end{cases}$$

Finally, in figure 3 we can see that $h(\sigma_k, \sigma_k^s)$ has unstable equilibrium points lying on σ_k^l and σ_k^u, i.e. maximum potential energy. Thus, all the system u.e.p.'s are in the closure $\partial\Gamma^\ell(\mathbf{A}^T\alpha^s)$. ■

Based on the Lyapunov function (26), Bergen and Hill propose a method to estimate the value of the energy function of the closest u.e.p. $(0, \alpha^*)$, i.e.

$$\vartheta_\ell = W(\alpha^*, \alpha^s) = \underset{\forall\alpha=\alpha^{unst}}{\text{minimum}}\{W(\alpha, \alpha^s)\},$$

without having to calculate the u.e.p.'s $(0, \alpha^{unst})$.

For a system where $\mathbf{P}^0 = 0$, there are one s.e.p. $(0, 0)$ and several u.e.p.'s $(0, \alpha^{unst} = [0, \pm\pi])$. One can define a series of cutsets $\{C_i\}$ that include some of the *saturated* lines with angles $\sigma_i = \pm\pi$, and for each one of this cutsets we can evaluate the energy function

$$\vartheta(0, \alpha^{unst}) = W(\alpha^{unst}, \alpha^s) = 2 \sum_{k \in C_i} b_k$$

Obviously, the value of ϑ_ℓ can be estimated from the most *vulnerable cutset*, i.e.

$$\vartheta_\ell = \underset{\forall i}{\text{minimum}} \left\{ 2 \sum_{k \in C_i} b_k \right\}$$

This idea can be extended to the general condition $\mathbf{P}^0 \neq 0$. In this case associated to every cutset C_i we define a direction; the lines with angles σ_i positively oriented with respect to the direction of C_i form the subset C_i^+, while the rest conform C_i^-. Then, the following *cutset vulnerability indices* are defined:

$$v_i^+ := \sum_{k \in C_i^+} b_k \, h(\sigma_k^u, \sigma_k^s) + \sum_{k \in C_i^-} b_k \, h(\sigma_k^l, \sigma_k^s)$$

$$v_i^- := \sum_{k \in C_i^+} b_k \, h(\sigma_k^l, \sigma_k^s) + \sum_{k \in C_i^-} b_k \, h(\sigma_k^u, \sigma_k^s)$$

Hence, the estimate for ϑ_ℓ becomes

$$\vartheta_\ell = \underset{\forall i}{\text{minimum}} \left\{ \min(v_i^+, v_i^-) \right\}$$

7.- Reactive Power and Flux Decay

In order to have an appropriate representation of the system, it is important to consider the reactive part of the load as well as the flux decay in the generators, to account for voltage variation during the transient analysis—a significant characteristic of power systems during fault conditions. This can be accomplished by using the following models for each one of the elements in the system:

- Generators (m): Neglecting the effect of saliency, we can represent the generator by the following equations:

$$P_{gt} = \frac{V_g V_t}{X_d'} \sin(\delta_g - \delta_t) = b_{gt} V_g V_t \sin(\delta_g - \delta_t) \qquad (27a)$$

$$Q_{gt} = \frac{V_g^2}{X_d'} - \frac{V_g V_t}{X_d'} \cos(\delta_g - \delta_t) = -b_{gg} V_g^2 - b_{gt} V_g V_t \cos(\delta_g - \delta_t) \qquad (27b)$$

$$\dot{V}_g = \frac{1}{T_{do}'} \left\{ E_f - V_g - \frac{X_d - X_d'}{X_d'} [V_g - V_t \cos(\delta_g - \delta_t)] \right\}$$

$$= -\left(\frac{X_d - X_d'}{T_{do}'} \right) V_g^{-1} \left\{ \frac{-E_f V_g + V_g^2}{X_d - X_d'} + \frac{V_g^2}{X_d'} - \frac{V_g V_t}{X_d'} \cos(\delta_g - \delta_t) \right\}$$

$$= -\left(\frac{X_d - X_d'}{T_{do}'} \right) V_g^{-1} \, [q_g(V_g) + Q_{gt}] \qquad (27c)$$

$$M_g \ddot{\delta}_g + D_g \dot{\delta}_g = P_M^0 \qquad (27d)$$

Where $V_g \angle \delta_g$ is the internal q-axis voltage of the generator, $V_t \angle \delta_t$ is the terminal voltage, and X_d and X_d' are the d-axis synchronous and transient reactances, respectively.

- Transmission System (l+m): Neglecting the resistance in the transmission lines, the active and reactive flow in every transmission line can be represented by equations (28). Notice that equations (27a-b) have the form of (28); hence, the transient reactance of the generator can be included in the transmission system.

$$P_{rs} = b_{rs} V_r V_s \sin(\delta_r - \delta_s) \qquad (28a)$$

$$Q_{rs} = -b_{rr} V_r^2 - b_{rs} V_r V_s \cos(\delta_r - \delta_s) \qquad (28b)$$

- Loads (n_0): Assuming frequency dependence and voltage dependence of the reactive power, the loads can be modelled by equations (29).

$$P_L = P_L^0 + D_L \dot{\delta}_L \qquad (29a)$$

$$q_L(V_L) = \begin{cases} Q_L^0 + Q_L^1 V_L + \ldots + Q_L^P V_L^P & \text{(expan.)} \\ Q_L^0 \left(\frac{V_L}{V_L^0} \right)^2 & \text{(const. reac.)} \end{cases} \qquad (29b)$$

In all these models the active power flow equations and the differential equations for ω_g and δ have not significantly changed; therefore, the state space representation of these variables remains the same, although the bus voltages have to be considered as variables. The inclusion of generator flux decay in the system modelling yields additional state space differential equations, while the reactive power flow can be treated as a set of nonlinear constraints. Thus, the state space equations for the power system become

$$\dot{\alpha} = T_g \omega_g - T_L D_L^{-1} T_L^T [f(\alpha, V) - P^0] \qquad (30a)$$

$$\dot{\omega}_g = -M_g^{-1} D_g \omega_g - M_g^{-1} T_g^T [f(\alpha, V) - P^0] \qquad (30b)$$

$$\dot{V}_g = -D_x h_g(\alpha, V) \qquad (30c)$$

$$h_L(\alpha, V) = 0 \qquad (30d)$$

Where, assuming $\alpha_n = 0$,

$$D_x = \operatorname*{diag}_{k=1..m} \left\{ \frac{X_{d_k} - X_{d_k}'}{T_{do_k}'} \right\}$$

$$f(\alpha, V) = \operatorname*{vector}_{i=1..n-1} \left\{ \sum_{k=1}^{n} b_{ik} V_i V_k \sin(\alpha_i - \alpha_k) \right\}$$

$$h_g(\alpha, V) = \operatorname*{diag}_{i=n-m+1..n} \{V_i^{-1}\} \operatorname*{vector}_{i=n-m+1..n} \left\{ q_{g_i}(V_i) - \sum_{k=1}^{n} b_{ik} V_i V_k \cos(\alpha_i - \alpha_k) \right\}$$

$$= [V_g]^{-1} \, [q_g(V_g) + Q_g(\alpha, V)]$$

$$h_L(\alpha, V) = \operatorname*{diag}_{i=1..n-m} \{V_i^{-1}\} \operatorname*{vector}_{i=1..n-m} \left\{ q_{L_i}(V_i) - \sum_{k=1}^{n} b_{ik} V_i V_k \cos(\alpha_i - \alpha_k) \right\}$$

$$= [V_L]^{-1} \, [q_L(V_L) + Q_L(\alpha, V)]$$

Notice that the diagonal matrices $[V_g]$ and $[V_L]$ are positive definite due to strictly positive bus voltages.

A proper Lyapunov function or TEF for this state space representation is defined in [8] as follows:

$$\vartheta(\omega_g, \alpha, V) = \frac{1}{2} \omega_g^T M_g \omega_g - P^{0^T} [\alpha - \alpha^s] - \frac{1}{2} \sum_{i=1}^{n} \sum_{k=1}^{n} b_{ik} V_i V_k \cos(\alpha_i - \alpha_k) + \frac{1}{2} \sum_{i=1}^{n} \sum_{k=1}^{n} b_{ik} V_i^s V_k^s \cos(\alpha_i^s - \alpha_k^s) + \sum_{k=1}^{n} \int_{V_k^s}^{V_k} \frac{q_k(u)}{u} du \qquad (31)$$

In [15], DeMarco and Overbye show that the state space equations (30) can be rewritten as

$$\begin{bmatrix} \dot{\omega}_g \\ \dot{\alpha} \\ \dot{V}_g \\ 0 \end{bmatrix} = K \, \nabla \vartheta(\omega_g, \alpha, V) \qquad (32)$$

Where K is a constant matrix. They also indicate that

$$\dot{\vartheta}(\omega_g,\alpha,\mathbf{V}) = \frac{1}{2}\,\nabla\vartheta^T(\omega_g,\alpha,\mathbf{V})\,(\mathbf{K}+\mathbf{K}^T)\,\nabla\vartheta(\omega_g,\alpha,\mathbf{V}) \quad (33)$$

Where $(\mathbf{K}+\mathbf{K}^T)$ is a diagonal negative semi-definite matrix, which makes $\dot{\vartheta}(\omega_g,\alpha,\mathbf{V}) \le 0$. Furthermore, form (32) it is obvious that the linearization of the system about an equilibrium point is stable if the *symmetric* Hessian $\nabla^2\vartheta(\omega_g,\alpha,\mathbf{V})$ is positive definite, which implies that the TEF itself is locally positive definite about the s.e.p. From these observations we can conclude that the proposed TEF is a Lyapunov function, and that the s.e.p. for (30) is asymptotically stable.

From equation (31) we can calculate ϑ_ℓ by finding all the u.e.p.'s of the state space equations at the equilibrium point using the technique described in [16,17], i.e.

$$\vartheta_\ell = \underset{\forall uep}{\mathrm{minimum}}\{\vartheta(0,\alpha^{uep},\mathbf{V}^{uep})\}$$

Estimating the u.e.p.'s is a more difficult task in this case due to the voltage dependence of the state space equations. In [6], Narasimhamurthi and Musavi minimize an approximate potential energy function to find an estimate of the u.e.p.'s. DeMarco and Overbye are presently working in developing a technique to find the closest u.e.p. by minimizing a constrained energy function, without having to solve the power flow equations.

DeMarco demonstrates in [8] that the set of differential equations with algebraic constraints (30), present potential shortcomings in direct Lyapunov stability analysis, due to the ill-conditioning of the power flow Jacobian close to voltage collapse. To avoid this problem he uses singular perturbation techniques to remove the constraints resulting from static load models. By singularly perturbing the algebraic constraints with a small positive number ε, equation (30d) becomes

$$\varepsilon\,\dot{\mathbf{V}}_L = -\,\mathbf{h}_L(\alpha,\mathbf{V})$$

This transforms the nonlinear constraints into additional differential equations of the state space model, allowing the use of standard Lyapunov theorems to find the region of attraction for the post-fault s.e.p. Notice that the s.e.p. of the perturbed system is exactly the same as in the original system (30). TEF (31) can also be used as an energy function for this new set of equations, since (32) and (33) and the observation about the Hessian of the TEF still hold for the singularly perturbed system.

These differential equations yield a stiff system difficult to solve by numerical integration, due to the small value of ε; furthermore, they do not significantly alter the values of the load bus voltages. The advantages of the singular perturbed system are that (i) a complete Lyapunov analysis can be performed for ill-conditioned power systems, and (ii) that an expected exit time from the region of attraction can be calculated and used as a security measure to assess vulnerability to voltage collapse [8,13,15].

8.- Example

In order to show some of the applications of the direct transient analysis described above, a five-bus system [14] depicted in figure 4 was used as an example. All the simulations and calculations were done with the help of SOLVER-Q [18], a general purpose equation handler for the IBM-PC. The simulated event is a balanced three phase fault at bus 2, which is assumed to clear completely at time t_c. The post-fault s.e.p. is then the same as the equilibrium point prior to the fault.

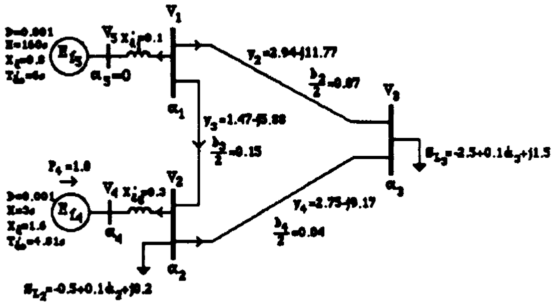

Figure 4: Five-bus system extracted from [14]

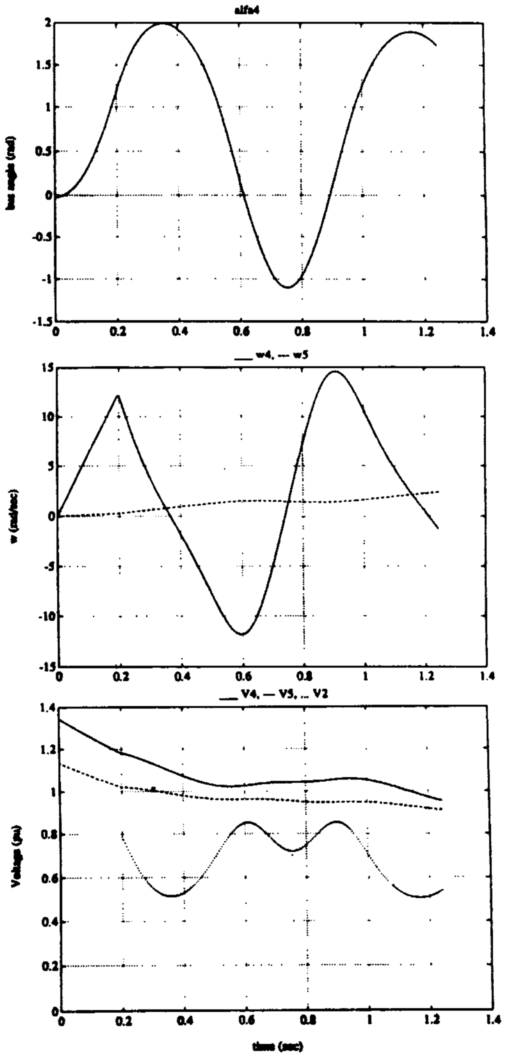

Figure 5: Constant impedance load model. Stable case: t_c=0.2s

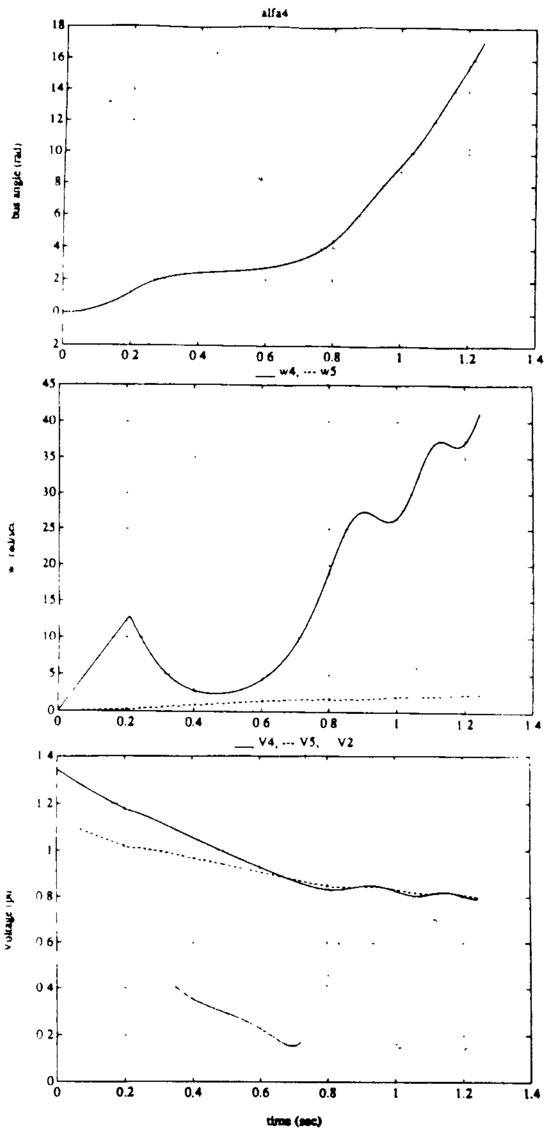

Figure 6: Constant impedance load model. Unstable case: t_c=0.21s

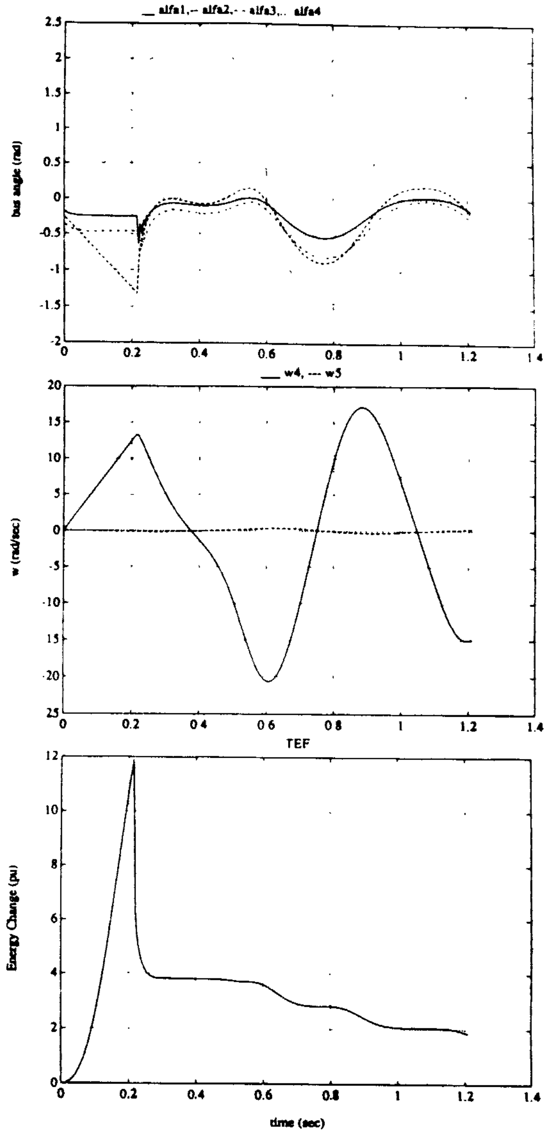

Figure 7: Frequency dependent load model with no reactive power. Stable case: t_c=0.215s

8.1.- Time simulation

Figures 5 and 6 depict the trajectories for the stable and unstable conditions of the sample power system, with the loads simulated as constant impedances. These plots were obtained by the trapezoidal rule of integration for two different clearing times. Observe that the critical clearing time for this load model is t_{cc}=0.205s.

The Bergen-Hill state space model yields the trajectories depicted in figures 7 (stable) and 8 (unstable). The critical clearing time for the frequency dependent load model (t_{cc}=0.2175s) is larger that the one obtained with constant impedance loads, which is a reasonable result considering that the voltage variations are not taken into account in the Bergen-Hill model.

When reactive power of the load is considered—figures 9 (stable) and 10 (unstable)—the critical clearing time is reduced considerably (t_{cc}=0.0495s). These changes in critical clearing time regarding the kind of load model used in time simulations have been widely studied, and the interested reader is referred to [1] for a complete discussion concerning this matter. We are more interested in the effect this phenomenon has in the region of attraction of the post-fault s.e.p., which we expect to shrunk considerably when the reactive power is taken into consideration.

In figures 7 thru 10 we have plotted the TEF for different load models and clearing times. Observe that the energy function decreases asymptotically toward zero for the stable cases, while in the unstable cases the TEF rapidly becomes negative. Although the energy function passes thru zero in the unstable conditions, this does not mean that the system becomes stable, since the state variables time derivatives are not zero—a requirement of LaSalle's theorem for asymptotic stability [10].

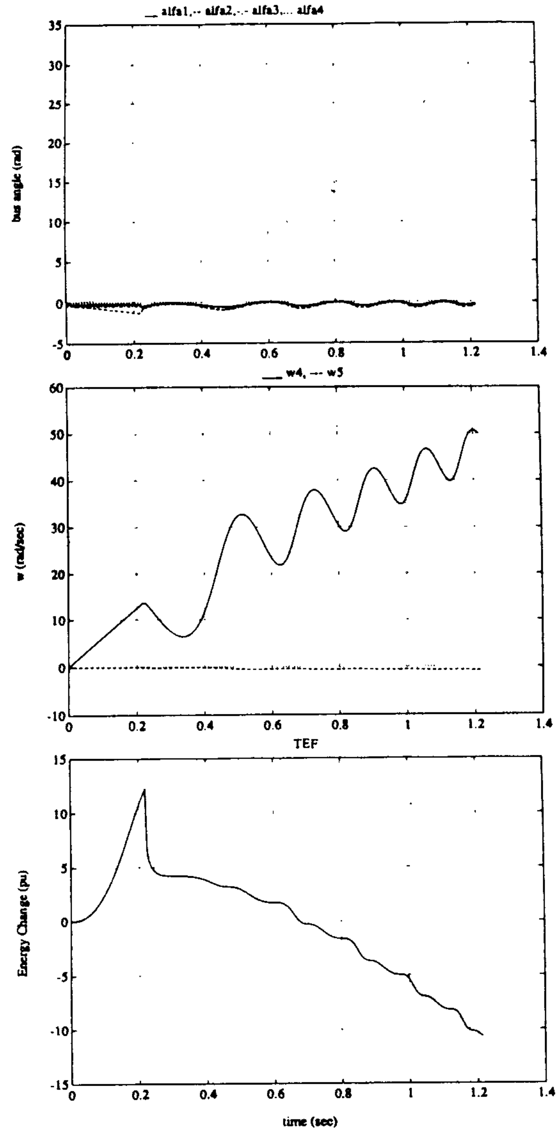

Figure 8: Frequency dependent load model with no reactive power. Unstable case: t_c=0.22s

It is also interesting to highlight in figures 9 and 10 the large dip in the voltages after the fault is applied—specially in bus 3—due to the lack of reactive power support in the system. This suggests that the system is operating close to voltage collapse. For this reason the reactive part of the load was simulated as a constant reactor to avoid problems with the nonlinear constraints in the state space model, since this kind of load model reduces the reactive power demands on the system. Obviously a constant reactive power load model would yield an ill-posed system when the fault is applied under these operating conditions.

8.2.- Energy analysis

Table 1 shows the value of the bus angles α and ϑ_ℓ at the stable equilibrium point and closest unstable equilibrium point for the Bergen-Hill system model. This u.e.p. was calculated by finding all the 37 u.e.p.'s for the sample system, and corresponds to the case where the second generator separates form the rest of the system, i.e. line 5 is saturated. The exact u.e.p.'s were calculated by the Newton-Raphson method with optimal multiplier described in [16], using as initial guesses the angles $\alpha_i{}^s \pm \pi$.

Table 2 shows the calculated Bergen-Hill cutset vulnerability indices for all possible cutsets in the system. Notice that the value of ϑ_ℓ found previously is exactly the same as the minimum index in table 2, which corresponds to the cutset with line 5 saturated.

Using ϑ_ℓ as the estimate of the stability region for the post-fault s.e.p , one can calculate the critical clearing time by evaluating the TEF along the fault trajectory. From figures 7 or 8 t_{cc}'=0.1225s, which is clearly a conservative estimation of the actual value calculated by time simulation, as it was mentioned before.

	α^s	α^*	ϑ_ℓ
α_1	-0.20314	-0.20314	3.8274
α_2	-0.25545	-0.25545	
α_3	-0.34476	-0.34476	
α_4	0.04924	2.58145	
α_5	0.	0.	

Table 1: Stable and closest unstable equilibrium points for the Bergen-Hill model

C_i	{1}	{5}	{2,3}	{3,4}	{2,4}
$v_i{}^+$	14.12	3.827	29.28	28.62	34.34
$v_i{}^-$	26.68	10.11	41.84	31.76	50.05

Table 2: Bergen-Hill cutset vulnerability indices for sample system

The complete state space model (30) give us just two equilibrium points, as shown in table 3. Notice the rather small value of the TEF at the only u.e.p. for the sample system; this tell us that the system is relatively close to voltage collapse [15]. Furthermore, the small region of attraction of the s.e.p. give us complete instability for a fault at bus 2, since the value of the energy function during the fault trajectory—figures 9 and 10—is much higher than ϑ_ℓ.

	α^s	α^*	ϑ_ℓ
α_1	-0.16726	-0.23336	
α_2	-0.21802	-0.30652	
α_3	-0.30653	-0.43434	
α_4	0.00342	-0.00595	
α_5	0.	0.	0.00523
V_1	1.05000	0.87770	
V_2	1.05000	0.88000	
V_3	0.97406	0.80881	
V_4	1.30083	1.15148	
V_5	1.14414	0.98539	

Table 3: Stable and unstable equilibrium points for complete system model

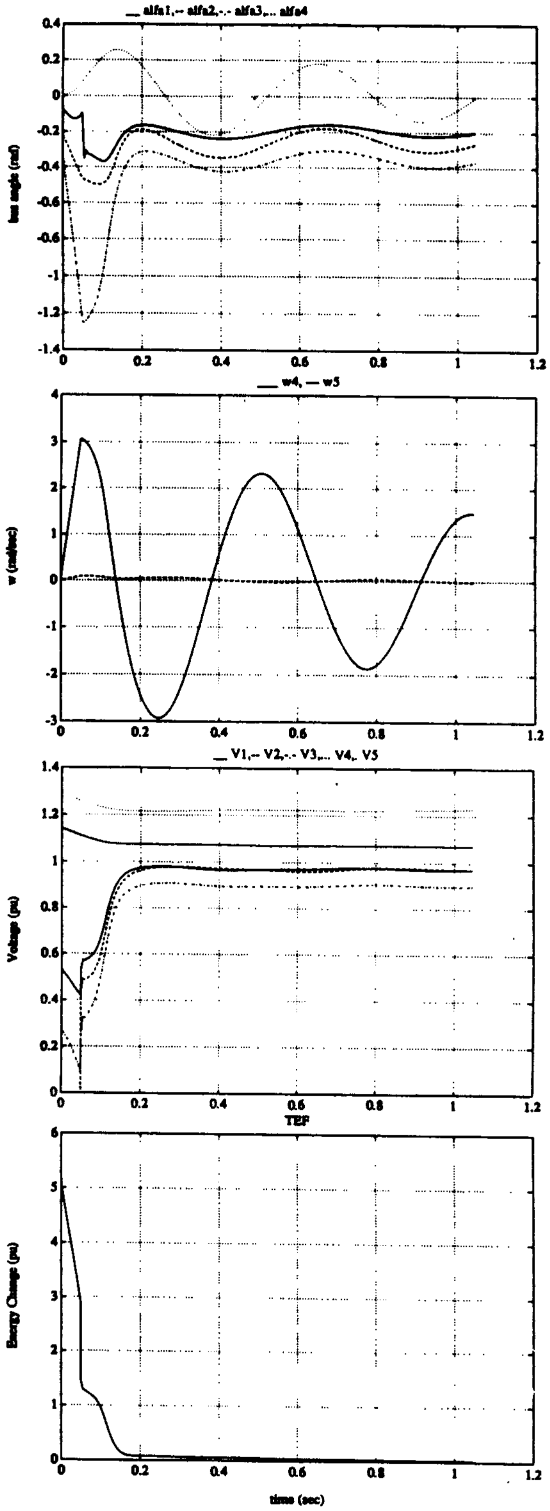

Figure 9: Frequency dependent load model including reactive power. Stable case: t_c=0.049s

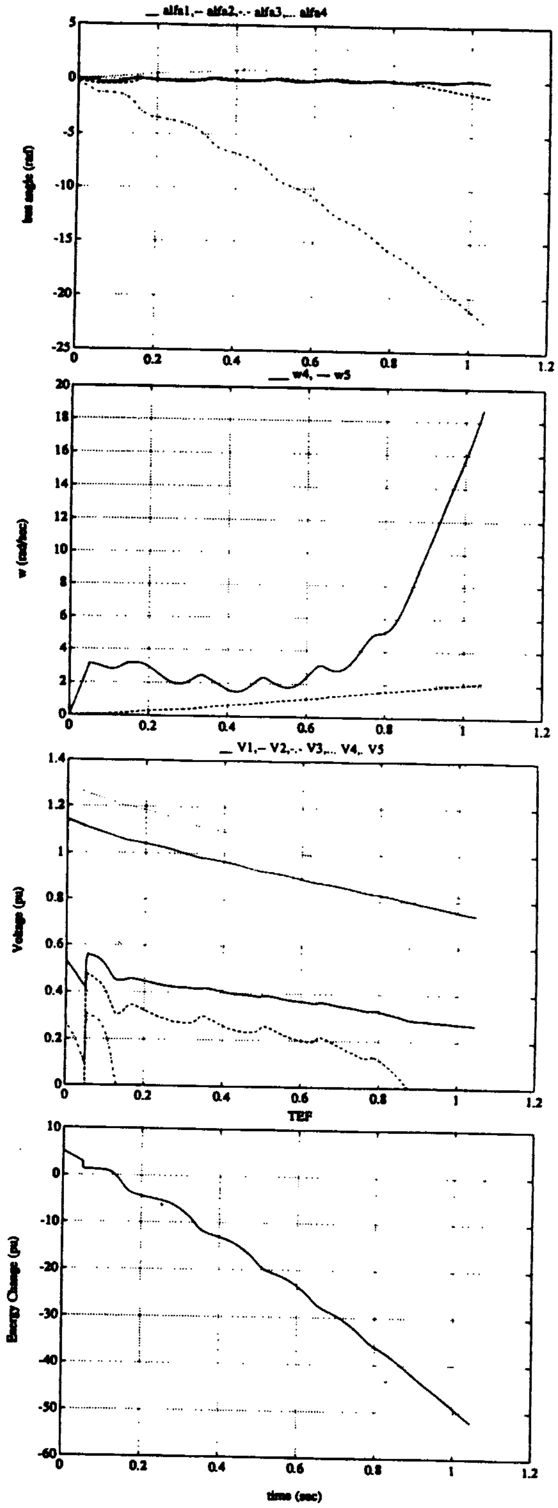

Figure 10: Frequency dependent load model including reactive power. Unstable case: t_c=0.05s

DeMarco and Overbye describe in [15] a method to assess vulnerability to voltage collapse based on the behavior of the energy function observed in our sample system, i.e. the closer to voltage collapse the smaller the value of the energy function at the corresponding u.e.p. This also ratifies the fact already observed that increments in power demand reduce the stability region.

Finally, another interesting fact—also mentioned in [17]—is that the number of u.e.p.'s increases when power demand is reduced. Moreover, these u.e.p.'s appear to come and go in pairs as the load is changed in the system, a phenomenon also noticed by DeMarco and Overbye. Table 4 shows all the u.e.p.'s and TEF values calculated for the sample system when there is only an active power demand of 0.01 p.u. at bus 2. Notice that the equilibrium points for state equations (30) could have negative voltage solutions for the fictitious internal generator buses, due to the characteristics of the field equation (27c); these equilibrium points are ruled out of the analysis.

	α^s	α^u_1	α^u_2	α^u_3
α_1	-0.00021	-0.00209	-0.01930	-0.02340
α_2	-0.00039	-0.00508	-0.12583	-0.31382
α_3	-0.00029	-0.00322	-1.62827	-1.65652
α_4	0.00220	2.18614	0.20841	1.13900
α_5	0.	0.	0.	0.
V_1	1.00000	0.27973	0.06678	0.05796
V_2	1.00000	0.21671	0.04790	0.02049
V_3	1.00528	0.25346	0.00000	0.00000
V_4	0.92848	0.01361	0.15274	0.11794
V_5	0.97179	0.34155	0.15521	0.14749
ϑ	0.	0.54870	0.63327	0.63650

Table 4: Stable and unstable equilibrium points for sample system with reduced load (P_{L_2}=0.01 p.u.)

9.- Comments and Conclusions

Bergen and Hill rigorously prove some of the intuitive ideas that have been used in the direct assessment of transient stability in power systems. They also present a interesting and different idea on how to obtain an estimate of the stability region for the post-fault stable equilibrium point. Although in their work they do not consider the direction of the trajectories of the faulted system to obtain a better estimate of the closure of the stability region, as a first approximation the vulnerability indices are useful to detect the most sensible parts of the system.

The Bergen-Hill structure preserving model definitely presents advantages over previous models used for direct stability assessment, but it is still a rough approximation of the actual power system. System models developed later, have successfully incorporated reactive flow and the flux variations in the generator to account for voltage reductions during Transient Stability studies.

None of the models herein are adequate for representing purely resistive loads and its voltage dependance. Some research is been carried out to try to incorporate this kind of load models into the structure preserving model. For the reduced system models this problem has been resolved by finding the path dependent integrals in the energy function using trapezoidal integration, but this approach does not allow for a thoroughly theoretical justification by classical Lyapunov analysis.

One important system component that should be taken into consideration is the generator's voltage control loop, or Automatic Voltage Regulator, which has been shown to reduce the stability region due to its fast response. However, some mathematical tricks can be played to approximate the voltage control phenomena at generator buses by defining additional differential equations for these voltages.

It would be also interesting to include in the direct stability assessment HVDC systems, due to todays widely spread use of DC links as an integrated part of the power network.

Finally, more research has to be done in developing efficient ways to calculate or estimate the u.e.p.' for the structure preserving model, specifically when reactive power and generator flux decay is taken into consideration. A complete enumeration of the u.e.p.'s is inadequate for large power systems.

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