ADAPTIVE CONTROL OF NONLINEAR SYSTEMS USING A RECURRENT NEURAL NETWORK

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1.- INTRODUCTION

Artificial Neural Networks (ANNs) have been shown to approximate non-linear mappings to any arbitrary degree of accuracy [1, 2]. It is this particular property of ANNs, which make them ideal candidates for modelling non-linear systems. ANNs can be categorised into two classes, namely feedforward networks and recurrent networks. Most current research in the application of neural networks to control has been carried out using feedforward networks. The approach used here is to formulate the problem in discrete time and is similar to the NARMA approach, see Narendra and Parthasarathy [3] and Cheng and Billings [4]. This method requires as inputs to the network a number of past values for each physical input and output of the system, where the number of past inputs and outputs needed are determined by trial and error.

An alternative to the feedforward network is the recurrent neural network first introduced by Hopfield [5] in the context of associative memories for pattern recognition. Recurrent neural networks have two salient features that distinguish them from feedforward networks. One is their node characteristics, which involve nonlinear differential equations, while feedforward networks have only static nonlinear node characteristics. The other major distinction is topology, in recurrent networks there are both feedforward and feedback connections, in other words the network is fully connected.

Along with the development of neural networks, there has been a tremendous development in the area of non-linear control using differential geometry [6]. Using the geometric approach, a number of control schemes, e.g. [7], [8] have been developed. These schemes require a state-space model of the system being controlled. So far most research has concentrated on using models developed from first principles. These models in order to retain accuracy are very complex, and not of much use for control. A simplified model, on the other hand is not a faithful representation of the system.

In this paper a control scheme which linearizes the system is discussed. The idea here is to integrate recurrent neural networks and the linearizing control scheme proposed by Kravaris and Chung [9]. A straightforward approach would have been to identify the non-linear plant using a recurrent neural network, and then synthesise the control law using this network. However, this particular methodology is eschewed here, for this would mean tedious calculations of the various Lie derivatives of the network and the exact cancellation of non-linear terms. Rather than go through a process of first identifying the plant and then evaluating the various parameters for linearizing the plant, a more interesting scheme would be one where the network designs the linearizing laws for the system. This means that the network provides us with the linearizing parameters as outputs, rather than the outputs of the system.

The paper is organised as follows : section 2 deals with the definition of Lie derivatives and the concept of relative degree. The section 3 discusses the linearizing state feedback and the Globally Linearizing Control (GLC) structure. Section 4 shows that a recurrent neural network can improve the performance of the GLC structure and reduce the calculations, this new structure is called the Adaptive Globally Linearizing Control (AGLC). Finally, section 5 presents the simulation results for a single link manipulator controlled with the AGLC structure.

2. - MATHEMATICAL PRELIMINARIES

Before proceeding any further, some definitions from differential geometry and Lie algebra are required.

2.1.- Lie Derivatives

Consider a control affine single input - single output nonlinear system

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$$x = f(x) + g(x).u$$
$$y = h(x) \tag{1}$$

where $x \in R^n$, $u, y \in R$, h(x) is a C^∞ scalar field on R^n and f(x), g(x) are C^∞ vector fields on R^n. For this system the Lie derivative of h(x) with respect to f(x) is given by

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x)$$

Further Lie derivatives can also be obtained, first along the vector field f(x) and then along the vector field g(x)

$$L_g L_f h(x) = \sum_{i=1}^{n} \frac{\partial (L_f h(x))}{\partial x_i} g_i(x)$$

or recursively along the vector field f(x)

$$L_f^k h(x) = \sum_{i=1}^{n} \frac{\partial (L_f^{k-1} h(x))}{\partial x_i} f_i(x)$$

with $L_f^0 h(x) = h(x)$.

2.2.- Relative Degree

The relative degree is an invariant characteristic of non-linear dynamic systems. It is defined as the number of times one has to differentiate the output y(t) with respect to time in order to have the input, u(t), explicitly appearing. Also, it is the number of times that the input, u(t), has to be integrated in order to affect the output y(t).

The system (1) is said to have *relative degree* r if

1.- $L_g L_f^k h(x) = 0$ for all x and all k < r - 1.

2.- $L_g L_f^{r-1} h(x) \neq 0$.

Thus for a system with relative degree r

$$\frac{d^k y}{dt^k} = L_f^k h(x), \quad k = 0,\ldots,r-1 \tag{2a}$$

$$\frac{d^r y}{dt^r} = L_f^r h(x) + L_g L_f^{r-1} h(x).u \tag{2b}$$

3.- INPUT - OUTPUT LINEARIZATION

Consider the input

$$u = \frac{-L_f^r h(x) + v}{L_g L_f^{r-1} h(x)} \tag{3}$$

with v an external input. Replacing (3) into (2b) yields the linearized equation

$$\frac{d^r y}{dt^r} = v$$

The input (3) is called the linearizing state feedback or linearizing control, because it produces a linear system between the new input v and the system output y.

In general, if the nonlinear system (1) has relative degree r, then there is always a state feedback that makes the input - output (v - y) behaviour of the closed loop system linear. This feedback is

$$u = \frac{-(L_f^r h(x) + \beta_1 . L_f^{r-1} h(x) + \ldots + \beta_{r-1} . L_f h(x) + \beta_r h(x)) + v}{L_g L_f^{r-1} h(x)} \tag{4}$$

Replacing (4) in (2b)

$$\frac{d^r y}{dt^r} = v - \beta_1 . L_f^{r-1} h(x) - \ldots - \beta_{r-1} . L_f h(x) - \beta_r h(x)$$

and using (2a), the input - output (v-y) behaviour of the closed loop system is then governed by

$$\frac{d^r y}{dt^r} + \beta_1 . \frac{d^{r-1} y}{dt^{r-1}} + \ldots + \beta_{r-1} . \frac{dy}{dt} + \beta_r . y = v \tag{5}$$

Note that in order to obtain the equation (5) the state feedback (4) needs to **cancel exactly** the nonlinear terms $L_f^r h(x)$ and $L_g L_f^{r-1} h(x)$. This is the main drawback of the linearizing control, the exact cancellation of nonlinear terms.

The parameters $\beta_1, \ldots, \beta_r$ are based on the desired input - output characteristic, this means that the v-y system can have arbitrarily placed poles. After linearizing the system (1), one can use an external PI loop

$$v = k_p\left[y_d - y\right] + k_i \int_0^t \left[y_d - y\right].dt \qquad (6)$$

to force the output y(t) to track a desired trajectory $y_d(t)$. This control structure was proposed by Kravaris and Chung [9] and it is called the Globally Linearizing Control (GLC) structure, see Fig.1.

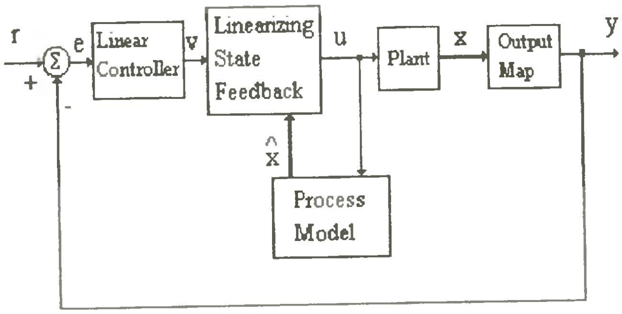

Fig.1.- Globally Linearizing Control (GLC) structure.

The design procedure using the GLC structure is simple : compute the linearizing state feedback (4) from the system model and then tune the PI controller (6).

4.- ADAPTIVE GLOBALLY LINEARIZING CONTROL

There are two drawbacks of the globally linearizing control in practical implementations. Firstly, the GLC is based on exact cancellation of nonlinear terms, if there is any uncertainty in the plant model the cancellation is not exact and the resulting input-output (v-y) equation is not linear. Secondly, it is necessary to estimate the state variables of the plant to calculate the state feedback (4), for nonlinear systems, there is no general observer theory.

In this work a Recurrent Neural Network (RNN) is proposed to calculate adaptively the state feedback (4). There are three advantages with this approach : (a) there is no need to identify the plant and to calculate the Lie derivatives for the state feedback, (b) it is not necessary to observe the plant to estimate the state vector and (c) the cancellation of the nonlinear terms is adaptive. The resulting control structure, which we call the *Adaptive Globally Linearizing Control (AGLC) structure* is depicted in Fig.2.

4.1.- Adaptive State Feedback

The state feedback (4) can be written as

$$u = \alpha + \beta . v \qquad (7)$$

with the coefficients,

$$\alpha = \frac{-(L_f^r h(x) + \beta_1 . L_f^{r-1} h(x) + \ldots + \beta_{r-1} . L_f h(x) + \beta_r h(x))}{L_g L_f^{r-1} h(x)}$$

$$\beta = \frac{1}{L_g L_f^{r-1} h(x)}$$

To evaluate these parameters α and β we need to determine the Lie derivatives L_f^k h(x) for k = 1 to r. This makes the scheme very cumbersome for large values of r (or systems where the relative degree is large). To overcome this problem, the Recurrent Neural Network (8) is trained to provide the two linearizing parameters α and β.

$$\begin{aligned} \tau . \dot{\chi} &= -\chi + W.\sigma(\chi) + \Gamma . u \\ \alpha &= \chi_1 \\ \beta &= \chi_2 \end{aligned} \qquad (8)$$

with,

$$\chi \in R^N , N > n$$

$$u, y \in R$$

$$[W]_{N x N} ; [\Gamma]_{N x 1}$$

The network equations (8) are equivalent to the set of differential equations

$$\begin{aligned} \tau . \dot{\chi}_i &= -\chi_i + \sum_{j=1}^{N} \omega_{ij} . \sigma(\chi_j) + \gamma_i . u \\ i &= 1, \ldots, N \\ \alpha &= \chi_1 \\ \beta &= \chi_2 \end{aligned} \qquad (9)$$

To reiterate this method has the following advantages :

(i) There is no need to identify the plant and to calculate the Lie derivatives for the state feedback.

(ii) It is not necessary to observe the plant to estimate the state vector.

(iii) The cancellation of the nonlinear terms is adaptive.

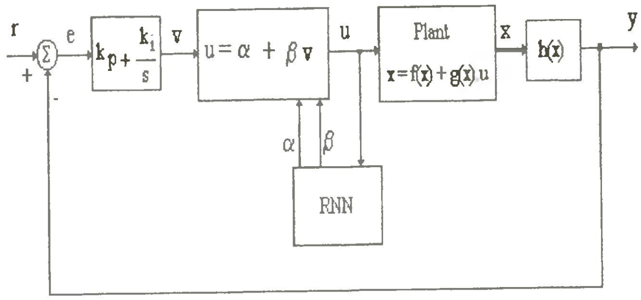

Fig.2.- Adaptive Globally Linearizing Control (AGLC) structure.

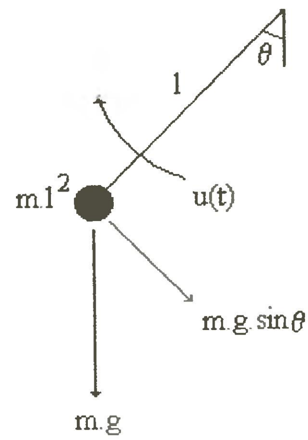

Fig.3.- Non-linear plant : single link manipulator.

5.- SIMULATION RESULTS

The Adaptive Globally Linearizing Control (AGLC) structure was simulated using an integration step T_i=0.01s. The goal was to train the recurrent neural network embedded in the AGLC structure to match the desired global dynamics,

$$\frac{y_d}{r} = \frac{1}{(s+1)^2}$$

with the PI controller

$$v = \left[k_p + \frac{k_i}{s} \right] . e$$

where $k_p = 1.0$ and $k_i = 1.0$.

5.1.- The Plant

The nonlinear plant, shown in Fig.3, is a single link manipulator described by

$$m.l^2.\ddot{\theta}(t) + v.\dot{\theta}(t) + m.g.l.\sin\theta(t) = u(t)$$

where the length, mass and friction coefficients are l=1 m, m = 2.0 kg and v = 1.0 kg.m^2/s, respectively [10].

The corresponding state representation is

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -9.8\sin x_1 - 0.5x_2 + 0.5u$$
$$y = x_1$$

with $x_1(0) = x_2(0) = 0$ and $y = \theta(t)$.

5.2.- Training

A recurrent neural network (8) with N = 5 neurons, $\tau = 1.0$ and $\sigma(x) = \tanh(x)$ was trained repetitively over a fixed time interval $[0, t_f]$. The matrices W and Γ were adjusted with the chemotaxis algorithm [11] to minimise the discrete version of the performance index (10). The reference input was a step r = 0.5, t_f = 20 s and the initial conditions for the neuron states were selected at random. All the initial conditions of the AGLC structure were reset to the same values at the beginning of each trial or training cycle.

$$J = \sqrt{\frac{1}{t_f}\int_0^{t_f} e^2(t).dt} = \sqrt{\frac{1}{t_f}\int_0^{t_f} \left[y(t) - y_d(t)\right]^2 .dt} \quad (10)$$

The Fig.4 shows the desired output and the plant output after training the neural network, the final performance index was J = 0.0015. The Fig.5 presents the state feedback coefficients α and β. The Fig.6 shows the linear controller output v and the linearizing control u.

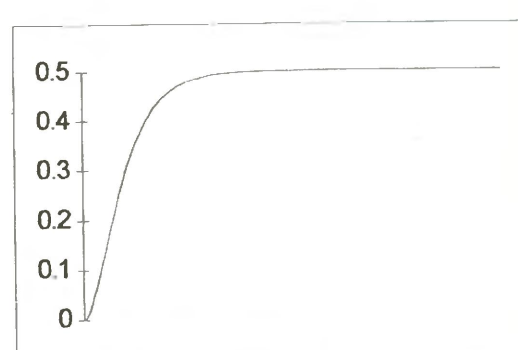

Fig.4.- Desired output y_d and plant output y in the AGLC structure for a step reference r = 0.5, k_p = 1.0 and k_i = 1.0.

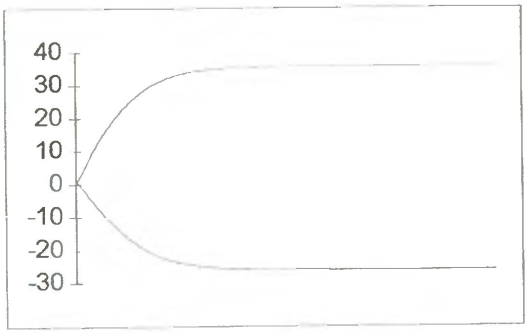

Fig.5.- State feedback coefficients α (bottom) and β.

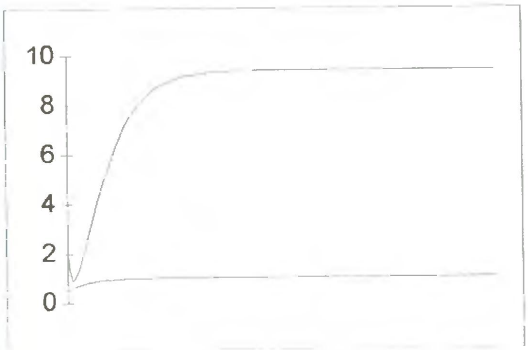

Fig.6.- Linear controller output v (bottom) and the linearizing state feedback u.

5.3.- Testing

The system was tested using another step inputs and adjusting the parameters of the linear controller. The Fig.7 shows the unit step response with $k_p = 0.5$ and $k_i = 0.7$. the performance index was J = 0.0127. The Fig.8 presents the step response for $r = \pi / 2$ with $k_p = 0.5$ and $k_i = 0.5$, the performance index was J= 0.0210.

The Fig.9 shows the response for the step r = 0.5, when the mass of the single link manipulator was changed from 2.0 kg to 2.5 kg. The gains of the PI controller were $k_p = 1.0$ and $k_i = 1.0$ and the performance index was J = 0.0111.

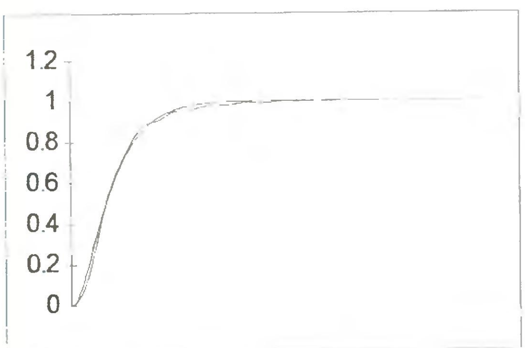

Fig.7.- Desired output y_d and plant output y in the AGLC structure for a step reference r = 1.0, $k_p = 0.5$ and $k_i = 0.7$.

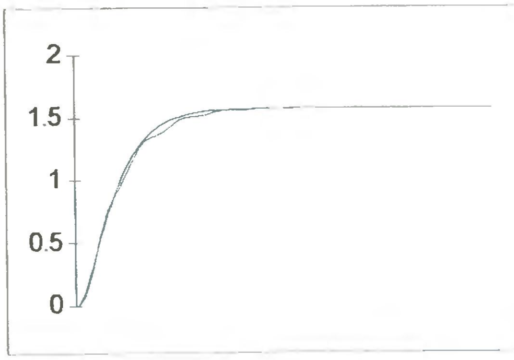

Fig.8.- Desired output y_d and plant output y in the AGLC structure for a step reference $r = \pi / 2$, $k_p = 0.5$ and $k_i = 0.5$.

The Fig.10 presents the response for the step r = 0.5, when a disturbance of magnitude 0.1 was applied to the reference during the interval Δt = [10.0, 12.0] s. The gains of the PI controller were $k_p = 1.0$ and $k_i = 1.0$ and the performance index was J=0.0267.

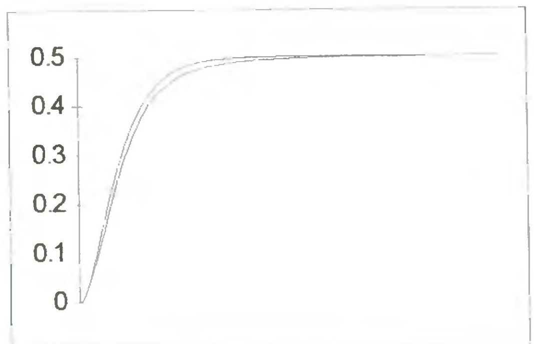

Fig.9.- Desired output y_d and plant output y in the AGLC structure for a step reference r = 0.5, $k_p = 1.0$ and $k_i = 1.0$. The mass of the plant was changed from 2.0 kg to 2.5 kg.

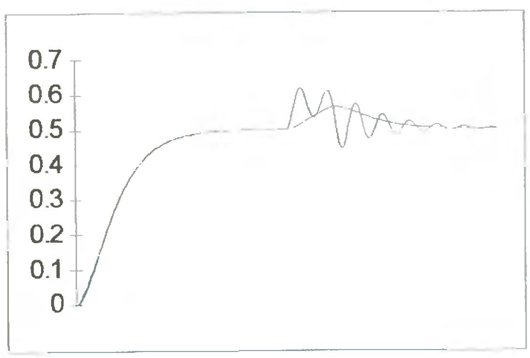

Fig.10.- Desired output y_d and plant output y in the AGLC structure for a step reference r = 0.5, $k_p = 1.0$ and $k_i = 1.0$. A disturbance of magnitude 0.1 was introduced in the reference during the interval Δt=[10.0, 12.0] s.

6.- CONCLUSIONS

In the GLC structure, it is necessary to identify the nonlinear plant to obtain the linearizing state feedback. The identified model is used to calculate the Lie derivatives and observe the states of the plant. Unfortunately in practical implementations the main drawback of the linearizing control is that it is based on exact cancellation of nonlinear terms, if there is any uncertainty in the identified model, the resulting input - output equation is not linear.

In the proposed AGLC approach the linearizing state feedback coefficients are calculated directly by the neural network and the network can be trained to improve the cancellation of the nonlinear terms and verify any performance index. Also, there is no need to calculate the Lie derivatives or observe the plant.

During the training stage of the AGLC the input to the nonlinear plant is changed repetitively to improve a performance index over a fixed time interval. The training algorithm does not depend on the plant equations or the functional form of the desired response.

7.- REFERENCES

1 CYBENKO, G.: *Approximation by superpositions of a sigmoidal function, Technical Report*, University of Illinois Urbana-Champaign Department of Electrical and Computer Engineering, 1988.

2 FUNAHASHI, K.I.: *On the approximate realization of continuous mappings by neural networks*, *Neural Networks*, 1989, 2, 183-192.

3 NARENDRA, K.S. and PARTHASARATHY, K.: *Identification and control of dynamical systems using neural networks*, IEEE Transactions on Neural Networks, 1990, 1, 4 - 26.

4 CHEN, S. and BILLINGS, S. A.: *Neural networks for nonlinear dynamic system modelling and identification*. Int. J. Control, 1992, 56, 319 - 346.

5 HOPFIELD, J.J.: *Neurons with graded response have collective computational properties like those of two state neurons*. Proc. Natl. Acad. Sci. USA, 1984, 81, 3088-3092.

6 ISIDORI, A.: *Nonlinear Control Systems*. Springer-Verlag, 1989.

7 MARINO, R.: *Feedback linearization techniques in robotics and power systems*. In FLIESS, M. and M. HAZEWINKEL, Editors, Algebraic and Geometric Methods in Nonlinear Control Theory, D. Reidel Publishing Company, 1986, 523 - 543.

8 MARINO, R. , PERESADA, S. and VALIGI, P. *Adaptive input - output linearizing control of induction motors*, IEEE Trans. on AC, 1993, 38, 208 -221.

9 KRAVARIS, C. and CHUNG, C.B.: *Nonlinear state feedback synthesis by global input/output linearization*, AIChE Journal, 1987, 33, 592-603.

10 JIN, L., NIKIFORUK, P. N. and GUPTA, M.: *Direct adaptive output tracking control using multilayered neural networks*. IEE Proc. - D, 1993, 140, 393 - 398.

11 BREMERMANN, H. J. and ANDERSON, R. W.: *An alternative to Back Propagation : a simple rule of synaptic modification for neural net training and memory*. Report of the Center for Pure and Applied Mathematics, PAM - 483, University of California, Berkeley, 1990.

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