Estimation of Uncertainty Intervals in the Identification of Electromechanical Modes from Ambient Data

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***Abstract*— This paper discusses the estimation of uncertainty intervals associated with the electromechanical modes identified from ambient data resulting from random load switching throughout the day in power systems. A connection between the second order statistical properties, including confidence intervals, of the identified electromechanical modes and the variance of the parameters of a selected linear model is demonstrated. The results of the presented method are compared with respect to the ones obtained from a Monte-Carlo type of simulation, showing its effectiveness in reducing the number of trials, which would be beneficial for on-line power system monitoring, as it can decrease the number of samples, thus ensuring that the system dynamics would not change significantly over the monitoring time window, and yielding more dependable results. The 2-area benchmark system, with different orders of the system identification model used (ARMA), is utilized to demonstrate the effectiveness of the proposed methodology.**

***Index Terms*— Power system oscillations, modal analysis, power system monitoring, system identification, prediction error methods.**

I. Introduction

On-line power system monitoring based on system identification techniques is of great help for providing insightful information regarding the stability condition of the system under study, as well as for validating off-line models and data. These techniques have been used to identify poorly damped electromechanical modes of a power system using simulations or field measurements, and are particularly important nowadays that phasor-measurement units (PMU) are being more widely deployed and utilized.

Identification techniques and models are either based on the deterministic transient response of the system to a large disturbance or random ambient noise. The transient response of a power system is normally accompanied with ringdown tests and major disturbances, such as adding/removing loads, severe faults and tripping generators. For instance, the well-known Prony method, which employs a deterministic model, has been widely used in power systems to analyze this kind of response [1], [2], [3], [4]. On the other hand, ambient noise, which is a low quality signal, is the natural response of a power system due to small-magnitude, random load switchings; thus, stochastic models such as auto-regressive (AR) models [5], auto-regressive moving-average (ARMA) models [6], and stochastic state-space models [7] have been used in this case.

Identified electromechanical modes are usually represented by a mean value and the corresponding confidence interval, which are estimated by means of Monte-Carlo type of simulations or experiments [6], [7]. However, the drawback in this approach is that it requires repeating the experiments, which in turn can violate the stationarity assumption of the measured signal over a long time window; furthermore, the system dynamics may undergo significant system changes while acquiring the required measurements, such as changing/adding/removing generator units. Therefore, one would like to carry out the experiments in a time window as short as possible, which is the issue that motivates the work presented in the current paper. The authors in [8] introduce a bootstrap method to give confidence interval estimates for the electromechanical modes, and its performance is studied by comparing the results with respect to the ones obtained by means of Monte-Carlo type of simulations. The bootstrap method, however, requires resampling the measured data to estimate the parameters of the system model (e.g. ARMA) for the new data set; this method is computationally expensive, since the resampling process is repeated for large number of trials in order to estimate the confidence intervals.

The electromechanical modes are the roots of the characteristic equation corresponding to the selected model (e.g. AR or ARMA); hence, there is a nonlinear relationship between the model parameters and modes, and between their corresponding variances. Therefore, the theory of the variance of parameters, which is well-understood and developed [9], may be used in this case to determine the variance of these modes. Reference [10] describes a technique that has been used in civil engineering for identification of structures to establish a connection between the variance of parameters and the variance of modal parameters. The application of this particular technique to estimate confidence intervals of electromechanical modes identified from ambient data is discussed here, showing that only one set of data may be used to estimate

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the mode uncertainties, thus reducing the required number of samples.

The rest of the paper is structured as follows: Section II presents the background on estimating the covariance of parameters of a linear parametric model within the context of prediction error methods (PEM) [11]; this information is then employed to estimate the confidence interval of the identified electromechanical modes. The results of applying the proposed technique for the 2-area benchmark system are presented in Section III. Finally, Section IV summarizes the main contributions of this paper.

II. Covariances

A linear parametric model such as AR or ARMA representing the power system is given by:

$$y(k) = H(q)\ e(k) \tag{1}$$

where $y(k)$ is the measured output, such as power through a line; $e(k)$ models the disturbances, i.e. the underlying load switching in a power system; $H(q) = B(q)/A(q)$ is a rational transfer function with unknown parameters θ, defined below; and q is the shifting operator defined by $q^{-1}y[k] = y[k-1]$. One-step-ahead prediction $\hat{y}(k|\theta)$ uses the observations available up to time $k-1$ to predict $y(k)$, thus:

$$\hat{y}(k|\theta) = [1 - H^{-1}(q)]\ y(k) \tag{2}$$

This notation is adopted from [9], and is used here to emphasize its dependence on the parameter vector θ. For instance, for an ARMA(p,d) model described as:

$$y(k) = -\sum_{i=1}^{p} a_i\ y(k-i)\ +\ \sum_{i=0}^{d} b_i\ e(k-i) \tag{3}$$

vector $\theta = [a_1\ ...\ a_p\ b_0\ b_1\ ...\ b_d]^T$ may be computed, within the context of PEMs, by minimizing an objective function such as:

$$\begin{aligned} \hat{\theta} &= \text{arg min}\quad V_N(\theta) \\ V_N(\theta) &= \frac{1}{N}\sum_{k=1}^{N}\frac{1}{2}\epsilon^2(k,\theta) \\ \epsilon(k,\theta) &= y(k) - \hat{y}(k|\theta) \end{aligned} \tag{4}$$

where the function $V_N(\theta)$ denotes the loss which results from the model in the fitting process; “arg min” means the minimization argument of the function $V_N(\theta)$; N is the number of samples; and $\epsilon(k,\theta)$ represents the residuals. This requires an iterative search for θ that yields the minimum of loss function $V_N(\theta)$. Equation (5) represents a nonlinear optimization problem, and thus may lead to local minima.

In this work, only the coefficients of polynomial $A(q)$ are of interest, since it is aimed at extracting the modal content of the signal, which are the roots of $A(q)$. Hence, one may consider applying other techniques such as the Yule-Walker method that only estimates the parameters of $A(q)$, thus employing more simplified and robust numerical techniques. It is also possible to model $y[k]$ in (3) with a high-order AR, rather than using an ARMA, as a result of Kolmogorov’s theorem. This leads to an objective function that can be solved by means of well-known least square methods [12]. A high-order AR, however, leads to extraneous modes close to the system modes, which could be difficult to distinguish from the true modes. Furthermore, an AR model may result in biased estimates if residuals are not white. It is also important to mention that when the signal-to-noise ratio (SNR) is low, the model structure is an ARMA rather than a pure AR. The least square modified Yule-Walker method, which has been employed in [6] to extract the modal content of the ambient noise, presents superior performance compared to the original Yule-Walker method as reported in [13].

The theory of variance of identified model parameters is well-understood and developed in the context of PEMs [9]. Hence, this theory, as explained below, is used here to estimate the variance of identified modes.

At the solution point $\hat{\theta}$, the differentiation of $V_N(\theta)$ with respect to θ has to be zero, i.e.

$$V_N^{'}(\hat{\theta}) = 0 \tag{5}$$

Thus, an iterative algorithm, such as Newton, is used to solve for $\hat{\theta}$ by means of the Taylor series expansion of (5) around a given point θ^* close to $\hat{\theta}$ [11]:

$$0 \approx V_N^{'}(\theta^*) + V_N^{''}(\theta^*)(\hat{\theta} - \theta^*) \tag{6}$$

or

$$(\hat{\theta} - \theta^*) = -\left[V_N^{''}(\theta^*)\right]^{-1} V_N^{'}(\theta^*) \tag{7}$$

This requires the first derivative (gradient) and second derivative (Hessian); thus:

$$V_N^{'}(\theta^*) = -\frac{1}{N}\sum_{k=1}^{N}\psi(k,\theta^*)\ \epsilon(k,\theta^*) \tag{8}$$

$$V_N^{''}(\theta^*) = \frac{1}{N}\sum_{k=1}^{N}\psi(k,\theta^*)\ \psi^T(k,\theta^*) + \frac{1}{N}\sum_{k=1}^{N}\psi^{'}(k,\theta^*)\ \ \epsilon(k,\theta^*) \tag{9}$$

where

$$\psi(k,\theta^*) = -\frac{d}{d\theta}\ \epsilon(k,\theta)\Big|_{\theta=\theta^*} = \frac{d}{d\theta}\ \hat{y}(k|\theta)\Big|_{\theta=\theta^*} \tag{10}$$

Close to the solution $\hat{\theta}$, the predicted errors $\epsilon(k,\theta)$ are independent; thus,

$$V_N^{''}(\hat{\theta}) \approx \frac{1}{N}\sum_{k=1}^{N}\psi(k,\hat{\theta})\ \psi^T(k,\hat{\theta}) \tag{11}$$

A. Covariance of Parameters

It is known that $\sqrt{N}(\hat{\theta} - \theta^*)$ is asymptotically Gaussian distributed with zero mean and a covariance matrix P, i.e. $\mathcal{N}(0,P)$ [9]. Therefore, an estimate of P from available data can be obtained as follows:

$$\begin{aligned} \hat{P} &= \hat{\lambda}_0\ \left(V_N^{''}(\hat{\theta})\right)^{-1} \\ \hat{\lambda}_0 &= \frac{1}{N}\sum_{k=1}^{N}\epsilon^2(k,\hat{\theta}) \end{aligned} \tag{12}$$

where $\hat{\lambda}_0$ is an estimate of the variance of the errors. Then, the covariance of parameter estimates, i.e. $P_{\hat{\theta}} = E[(\hat{\theta} - \theta^*)(\hat{\theta} - \theta^*)^T]$, can be approximated as:

$$P_{\hat{\theta}} \approx \frac{1}{N} \hat{P} \tag{13}$$

The modes of a system are the roots of the characteristic equation, and hence are only dependent on the AR part of an ARMA(p,d). Thus, the covariance matrix $P_{\hat{\theta}}$ is partitioned so that the rows and columns corresponding to the AR and the MA parts are separate as follows:

$$P_{\hat{\theta}} = \begin{bmatrix} P_{\hat{\theta}_{AR}} & P_{\hat{\theta}_{ARMA}} \\ P_{\hat{\theta}_{ARMA}} & P_{\hat{\theta}_{MA}} \end{bmatrix} \tag{14}$$

A relationship between $P_{\hat{\theta}_{AR}}$ and the covariance of modes is established below.

B. Covariance of Modes

System modes cab be related to θ_{AR}, which are the coefficients of the characteristic equation, as follows:

$$\Phi = \gamma(\theta_{AR}) \tag{15}$$

where $\gamma(\theta_{AR})$ is a nonlinear function, and Φ denotes a vector containing the modal parameters. For instance, the real part α and the frequency f of the modes can be used to define:

$$\Phi = [\alpha_1, f_1, \alpha_2, f_2, \ldots, \alpha_p, f_p]^T \in \Re^{2p} \tag{16}$$

In order to obtain the mean and variance of the modes, the expected value operator may be applied to a Taylor series expansion of the function γ about an operating point ($\hat{\Phi}$, $\hat{\theta}_{AR}$), thus:

$$\Phi \approx \hat{\Phi} + J(\hat{\theta}_{AR}) \left(\theta_{AR} - \hat{\theta}_{AR}\right) \tag{17}$$

$$J(\hat{\theta}_{AR}) = \frac{\partial \gamma(\theta_{AR})}{\partial \theta_{AR}} \Bigg|_{\theta_{AR} = \hat{\theta}_{AR}} \tag{18}$$

where $J(\hat{\theta}_{AR}) \in \Re^{2p \times p}$.

Rearranging (17) and applying the second moment operator (covariance) yields:

$$\begin{aligned} \text{Cov}\,\Phi &= E\left[(\Phi - \hat{\Phi})\,(\Phi - \hat{\Phi})^T\right] \\ &= J(\hat{\theta}_{AR})\, P(\hat{\theta}_{AR})\, J^T(\hat{\theta}_{AR}) \end{aligned} \tag{19}$$

This clearly shows the connection between the covariance of estimates $P(\hat{\theta}_{AR})$ and the covariance of modes Cov Φ. Therefore, in (19), $P(\hat{\theta}_{AR})$ can be estimated using (13), and a numeric Jacobian $J(\hat{\theta}_{AR})$ can be obtained as follows:

$$J_{ij}(\hat{\theta}_{AR}) = \frac{\gamma_i(\hat{\theta}_{AR} + \Delta\theta_j) - \gamma_i(\hat{\theta}_{AR} - \Delta\theta_j)}{2h} \tag{20}$$

where $\Delta\theta_j = [0\ \cdots\ 0\ \underbrace{h}_{j}\ 0\ \cdots\ 0]$, with h being a small number.

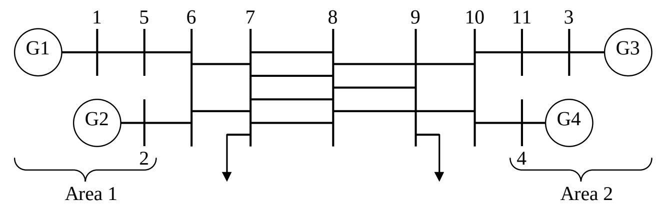

Fig. 1. Two-area benchmark system.

III. Test cases

The proposed method for estimating the standard deviation of the identified modes is tested with the 2-area benchmark system. First, a Monte-Carlo type of simulation with 150 independent simulations is performed. For these trials, 1% of the loads are represented as Gaussian noise; 4-minutes data blocks of a generator output power are recorded in each simulation, and white Gaussian noise was added to the output signals as measurement noise, so that the SNR is 20 db. The signals were passed through a Chebyshev low-pass filter with a cut-off frequency of 2 Hz, and then resampled at 10 Hz rate. The preprocessed data blocks along with a PEM are employed to estimate the parameters of an ARMA(p,d) model representing the power system transfer function.

A single line diagram of the test system is shown in Fig. 1 [14]. The generators are modeled using subtransient models and simple exciters equipped with PSSs. The corresponding static and dynamic data is given in [13]. The total base loading level is 2734 MW and 200 MVar, and loads are modeled as constant PQ loads.

Areas 1 and 2 are connected through tie-lines, and an inter-area mode with a frequency of about 0.75 Hz is observed. Furthermore, the individual machines in each area also contribute to a local mode in the same area with frequencies of about 1.2 Hz and 1.4 Hz in Areas 1 and 2, respectively. Therefore, an inter-area rotor angle mode and two local modes are observed in this test case.

An ARMA(p,d) model with different p's and d's is employed to model the measured signal in every simulation, resulting in the mode estimates shown in Figs. 2 and 3. These show the estimated electromechanical mode corresponding to each model for 150 trials, together with the "true" mode, $-0.1228 \pm j4.7824$, obtained from a linearized model (LM) of the power system. From the system identification point of view, an ARMA model set is more likely to adequately represent the true system; hence, the estimated parameters would be asymptotically unbiased [9].

The estimated standard deviation of both the real part and frequency of the inter-area mode obtained using (19) is depicted in Figs. 4 and 5. Observe that the estimates track the results corresponding to the Monte-Carlo simulation, and the mean of the estimates can provide reasonably good accuracy with a significantly reduced number of trials. For instance, in Fig. 5, the convergence speed of the standard deviation of estimates is nearly 3 to 4 times faster than the Monte-Carlo method, which yields significant reduction in the monitoring time.

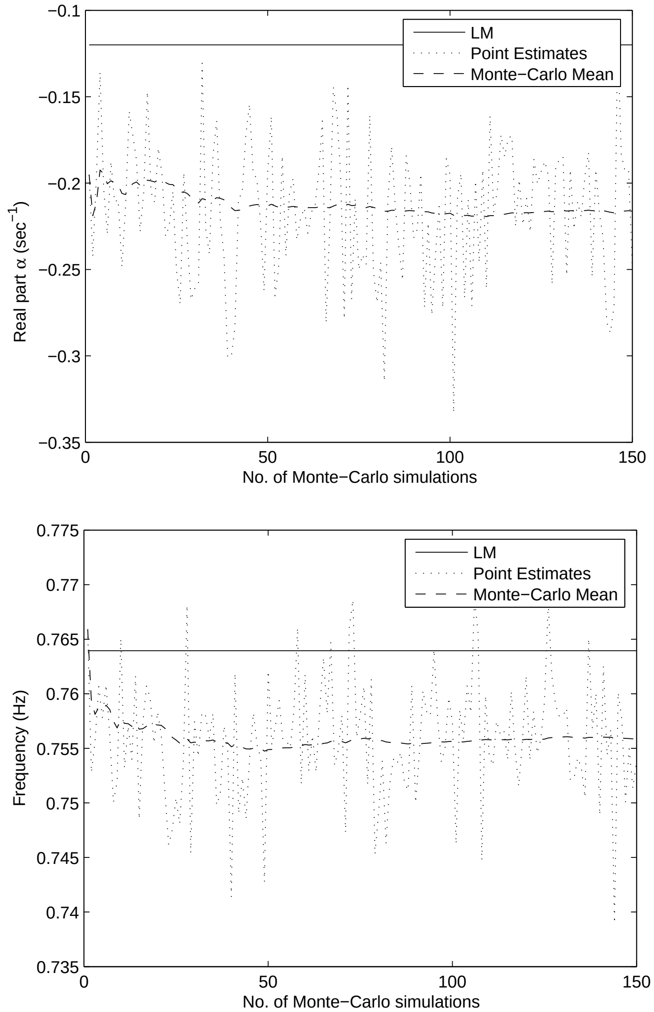

Fig. 2. Real part and frequency of the identified inter-area mode $-0.1228 \pm j4.7824$ for the 2-area benchmark system; ARMA(15,0), i.e. AR(15).

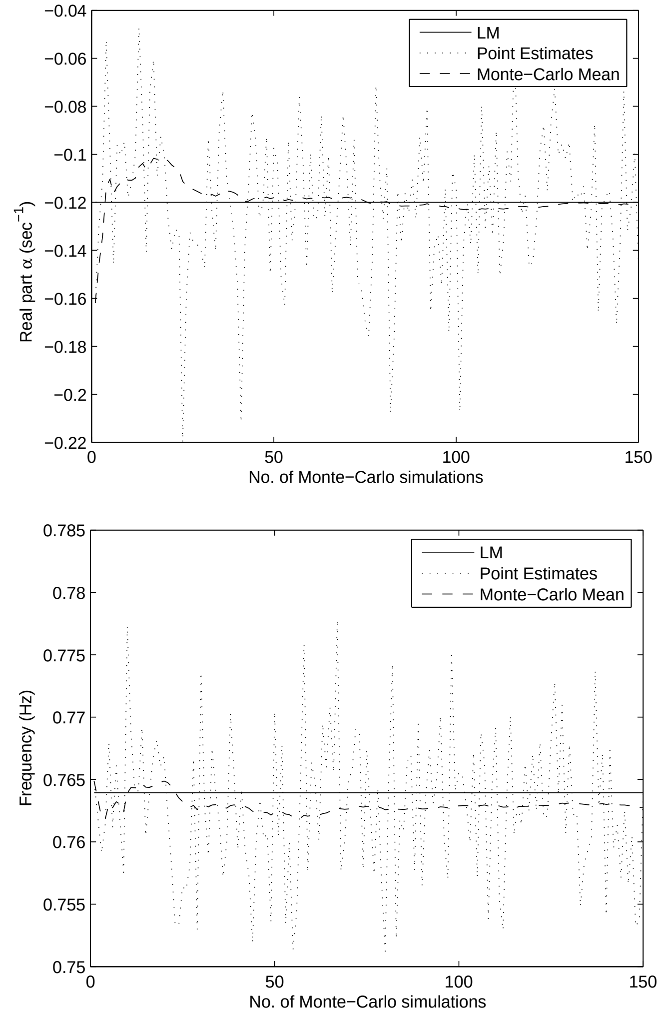

Fig. 3. Real part and frequency of the identified inter-area mode $-0.1228 \pm j4.7824$ for the 2-area benchmark system; ARMA(10,10).

Notice that the uncertainty associated with the real part of the mode is relatively large when compared with the one for the frequency, e.g. the standard deviation of the real part of the mode depicted in Fig. 5 is about 25% of the actual real part, wheras it is only about 0.6% for the frequency. This is due to the fact that obtaining accurate estimates of mode damping in power systems using system identification is more difficult [5], [6], [7].

IV. Conclusions

A novel procedure to calculate the second order statistical properties of identified electromechanical modes from ambient noise in power systems is presented and justified. This method is based on a technique that employs the Taylor series expansion to establish a connection between the variance of model parameters and the variance of eigenvalues. The variance of parameters are briefly reviewed and shown to be estimated by using only one data block, i.e. one set of measurements; this information is then employed to estimate the uncertainty associated with the identified electromechanical modes.

The proposed technique can be used to avoid Monte-Carlo type of analyses, thus resulting in a significant reduction in computational time. Hence, this method, in principle, would facilitate the use of ambient noise in applications requiring on-line modal analysis, such as system control or real-time security monitoring, since it does not require any artificial disturbances such as adding/tripping generators.

References

[1] J. F. Hauer, “The use of Prony analysis to determine modal content and equivalent models for measured power system response,” in *IEEE/PES Symposium on Application of Eigenanalysis and Frequency Domain Methods for System Dynamic Performance*, July 1989, pp. 105–115.

[2] J. R. Smith, F. Fatehi, C. S. Woods, J. F. Hauer, and D. J. Trudnowski, “Transfer function identification in power system applications,” *IEEE Trans. Power Systems*, vol. 8, no. 3, pp. 1282–1290, August 1993.

[3] J. F. Hauer, “Application of Prony analysis to the determination of modal content and equivalent models for measured power system response,” *IEEE Trans. Power Systems*, vol. 6, no. 3, pp. 1062–1068, August 1991.

[4] B. J. Bujanowski, J. W. Pierre, S. M. Hietpas, T. L. Sharp, and D. A. Pierre, “A comparison of several system identification methods with application to power systems,” in *IEEE Proceedings of the 36th Midwest Symposium on Circuits and Systems*, vol. 1, August 1993, pp. 64–67.

[5] J. W. Peirre, D. J. Trudnowski, and M. Donnelly, “Initial results in electromechanical mode identification from ambient data,” *IEEE Trans. Power Systems*, vol. 12, no. 3, pp. 1245–1251, August 1997.

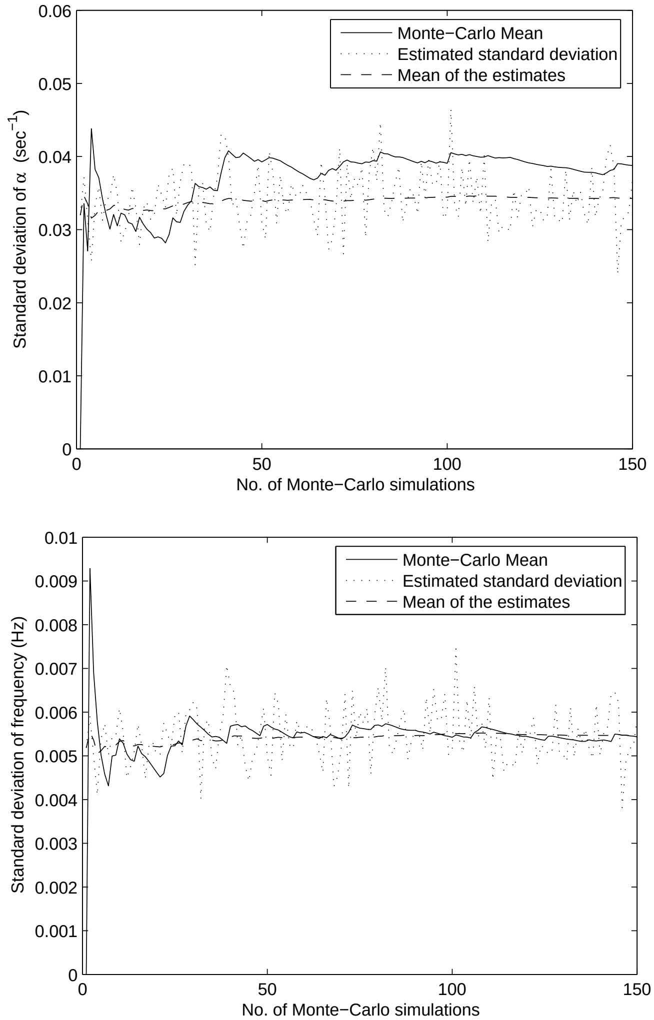

Fig. 4. Standard deviation of the real part and the frequency of the identified inter-area mode $-0.1228 \pm j4.7824$ for the 2-area benchmark system using Monte-Carlo and equation (19); ARMA(15,0), i.e. AR(15).

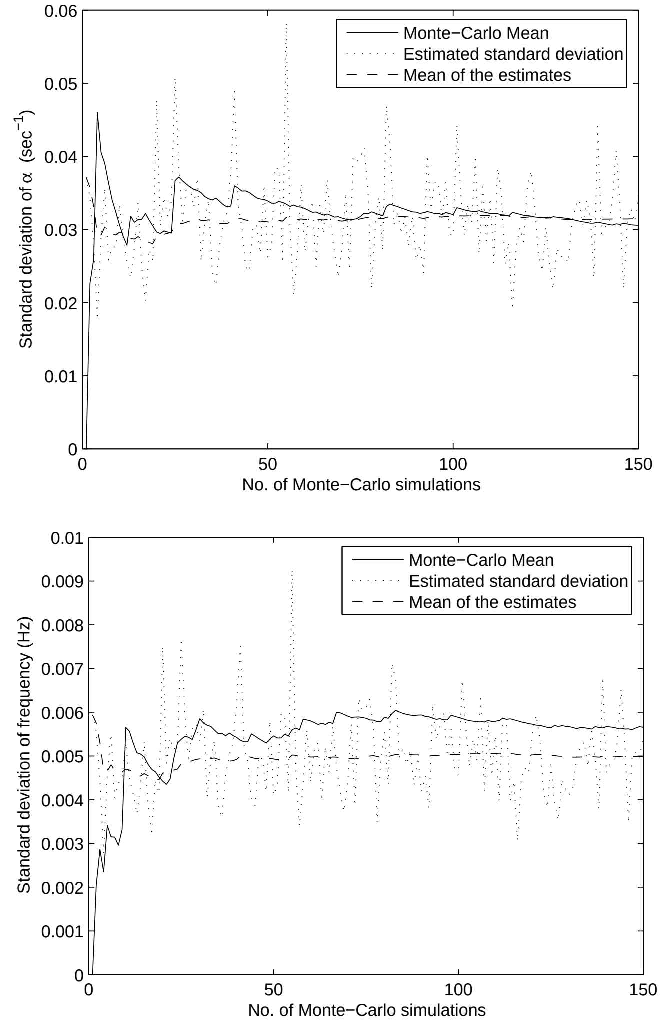

Fig. 5. Standard deviation of the real part and the frequency of the identified inter-area mode $-0.1228 \pm j4.7824$ for the 2-area benchmark system using Monte-Carlo and equation (19); ARMA(10,10).

[6] R. W. Wies, J. W. Pierre, and D. J. Trudnowski, "Use of ARMA block processing for estimating stationary low-frequency electromechanical modes of power systems," *IEEE Trans. Power Systems*, vol. 18, no. 1, pp. 167–173, 2003.

[7] H. Ghasemi, C. A. Cañizares, and A. Moshred, "Oscillatory stability limit prediction using stochastic subspace identification," *IEEE Trans. Power Systems*, vol. 21, no. 2, pp. 736–745, May 2006.

[8] M. G. Anderson, N. Zhou, J. W. Pierre, and R. W. Wies, "Bootstrap-based confidence interval estimates for electromechanical modes from multiple output analysis of measured ambient data," *IEEE Trans. Power Systems*, vol. 20, no. 2, pp. 943–950, May 2005.

[9] L. Ljung, *System Identification Theory for the User*. Englewood Cliffs, N.J.: Prentice Hall, 1999.

[10] P. Andersen, "Identification of civil engineering structures using vector ARMA models," Ph.D. dissertation, Aalborg University, Aalborg, Denmark, 1997.

[11] R. Söderström, *System Identification*. Cambridge, UK: Prentice Hall International (UK) Ltd., 1989.

[12] S. M. Kay, *Modern Spectral Estimation: Theory and Application*. Englewood Cliffs, N.J.: Prentice Hall, 1988.

[13] H. Ghasemi, "On-line monitoring and oscillatory stability margin prediction in power systems based on system identification," Ph.D. dissertation, University of Waterloo, Waterloo, ON, Canada, 2006.

[14] P. Kundur, *Power System Stability and Control*. New York: McGraw-Hill, 1994.

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